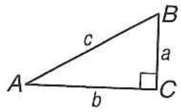
NAME ______________________________ DATE ____________ PERIOD ____

7-2 Study Guide and Intervention

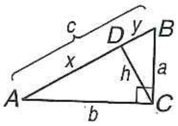
The Pythagorean Theorem and Its Converse

The Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.



$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

Example 1 **Prove the Pythagorean Theorem.**



With altitude $\overline{CD}$, each leg a and b is a geometric mean between hypotenuse c and the segment of the hypotenuse adjacent to that leg.

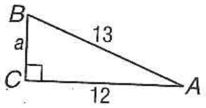
$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get

$a^2 + b^2 = cy + cx = c(y + x) = c^2$.

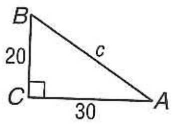
Example 2

a. Find *a*.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 12^2 = 13^2$	$b = 12$, $c = 13$
$a^2 + 144 = 169$	Simplify.
$a^2 = 25$	Subtract.
$a = 5$	Take the square root of each side.

b. Find *c*.

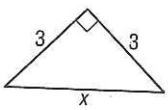


$a^2 + b^2 = c^2$	Pythagorean Theorem
$20^2 + 30^2 = c^2$	$a = 20$, $b = 30$
$400 + 900 = c^2$	Simplify.
$1300 = c^2$	Add.
$\sqrt{1300} = c$	Take the square root of each side.
$36.1 \approx c$	Use a calculator.

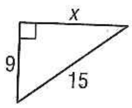
Exercises

Find *x*.

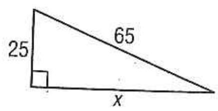
1.



2.



3.



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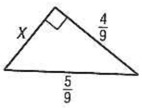

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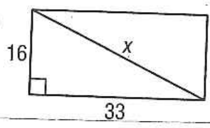

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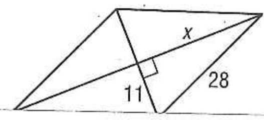
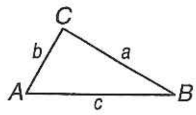


NAME ______________________ DATE __________ PERIOD _____

7-2 Study Guide and Intervention *(continued)*

The Pythagorean Theorem and Its Converse

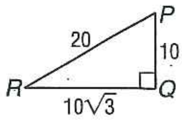
Converse of the Pythagorean Theorem If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.



If the three whole numbers a, b, and c satisfy the equation $a^2 + b^2 = c^2$, then the numbers a, b, and c form a **Pythagorean triple.**

If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.

Example **Determine whether $\triangle PQR$ is a right triangle.**



$a^2 + b^2 \stackrel{?}{=} c^2$	Pythagorean Theorem
$10^2 + (10\sqrt{3})^2 \stackrel{?}{=} 20^2$	$a = 10$, $b = 10\sqrt{3}$, $c = 20$
$100 + 300 \stackrel{?}{=} 400$	Simplify.
$400 = 400$ ✓	Add.

The sum of the squares of the two shorter sides equals the square of the longest side, so the triangle is a right triangle.

Exercises

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

1. 30, 40, 50
2. 20, 30, 40
3. 18, 24, 30
4. 6, 8, 9
5. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$
6. 10, 15, 20
7. $\sqrt{5}, \sqrt{12}, \sqrt{13}$
8. $2, \sqrt{8}, \sqrt{12}$
9. 9, 40, 41

A *family* of Pythagorean triples consists of multiples of known triples. For each Pythagorean triple, find two triples in the same family.

10. 3, 4, 5
11. 5, 12, 13
12. 7, 24, 25

Geometry Activity

A Preview of Lesson 7-2

The Pythagorean Theorem

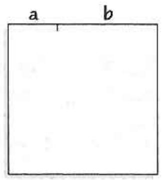
In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. Ancient cultures used the Pythagorean Theorem before it was officially named in 1909.

Use square pieces of patty paper and algebra. Then you too can discover this relationship among the measures of the sides of a right triangle.

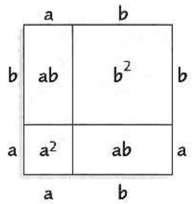
Activity

Use paper folding to develop the Pythagorean Theorem.

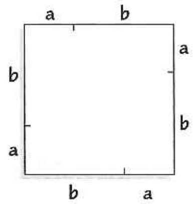
Step 1 On a piece of patty paper, make a mark along one side so that the two resulting segments are not congruent. Label one as a and the other as b.



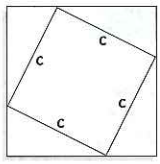
Step 2 Copy these measures on the other sides in the order shown at the right. Fold the paper to divide the square into four sections. Label the area of each section.



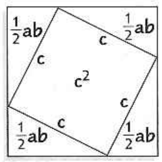
Step 3 On another sheet of patty paper, mark the same lengths a and b on the sides in the different pattern shown at the right.



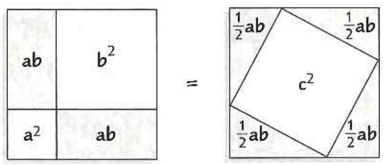
Step 4 Use your straightedge and pencil to connect the marks as shown at the right. Let c represent the length of each hypotenuse.



Step 5 Label the area of each section, which is $\frac{1}{2}ab$ for each triangle and c^2 for the square.



Step 6 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$.

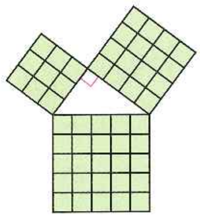


The parts that are not shaded tell us that $a^2 + b^2 = c^2$.

Model

1. Use a ruler to find actual measures for a, b, and c. Do these measures confirm that $a^2 + b^2 = c^2$? **yes**
2. Repeat the activity with different a and b values. What do you notice? **$a^2 + b^2 = c^2$**

Analyze the model



3. **Explain** why the drawing at the right is an illustration of the Pythagorean Theorem. **Sample answer: The sum of the areas of the two smaller squares is equal to the area of the largest square.**

Resource Manager

Teaching Geometry with Manipulatives
- p. 115 (student recording sheet)
- p. 17 (ruler)

Glencoe Mathematics Classroom Manipulative Kit
- rulers

Geometry Activity

A Preview of Lesson 7-2

Getting Started

Objective Physically explore side-length relationships in the Pythagorean Theorem.

Materials patty paper, ruler, pencil

Teach

- For Step 2, have students use a ruler and a pencil to draw the lines that the creases make.
- Advise students that the key to this activity is making sure measures a and b are exactly the same on both sheets of paper. Without using a ruler, students may use one marked edge of the first paper to mark accurate lengths on the second paper.
- After Step 6, some students may still be skeptical that the shaded areas of the two pieces of patty paper are equal. Point out that the two pieces of paper are the same size, so they have the same area. Then ask students to cut out all the shaded triangles from the second piece of paper and arrange them so that they fit over the shaded areas on the first piece of paper.

Assess

Exercises 1 and 2 provide students with actual measurements they can use to confirm the Pythagorean Theorem. In **Exercise 3**, students prove that three lengths meet the criteria for the Pythagorean Theorem.

Study Notebook

Ask students to summarize what they have learned about the Pythagorean Theorem.