

## 7-1

## Study Guide and Intervention

## Geometric Mean

**Geometric Mean** The **geometric mean** between two numbers is the square root of their product. For two positive numbers  $a$  and  $b$ , the geometric mean of  $a$  and  $b$  is the positive number  $x$  in the proportion  $\frac{a}{x} = \frac{x}{b}$ . Cross multiplying gives  $x^2 = ab$ , so  $x = \sqrt{ab}$ .

**Example**

Find the geometric mean between each pair of numbers.

**a. 12 and 3**

Let  $x$  represent the geometric mean.

$$\frac{12}{x} = \frac{x}{3}$$

Definition of geometric mean

$$x^2 = 36$$

Cross multiply.

$$x = \sqrt{36} \text{ or } 6$$

Take the square root of each side.

**b. 8 and 4**

Let  $x$  represent the geometric mean.

$$\frac{8}{x} = \frac{x}{4}$$

$$x^2 = 32$$

$$x = \sqrt{32}$$

$$\approx 5.7$$

**Exercises**

Find the geometric mean between each pair of numbers.

1. 4 and 4

2. 4 and 6

3. 6 and 9

4.  $\frac{1}{2}$  and 2

5.  $2\sqrt{3}$  and  $3\sqrt{3}$

6. 4 and 25

7.  $\sqrt{3}$  and  $\sqrt{6}$

8. 10 and 100

9.  $\frac{1}{2}$  and  $\frac{1}{4}$

10.  $\frac{2\sqrt{2}}{5}$  and  $\frac{3\sqrt{2}}{5}$

11. 4 and 16

12. 3 and 24

The geometric mean and one extreme are given. Find the other extreme.

13.  $\sqrt{24}$  is the geometric mean between  $a$  and  $b$ . Find  $b$  if  $a = 2$ .

14.  $\sqrt{12}$  is the geometric mean between  $a$  and  $b$ . Find  $b$  if  $a = 3$ .

Determine whether each statement is *always*, *sometimes*, or *never* true.

15. The geometric mean of two positive numbers is greater than the average of the two numbers.

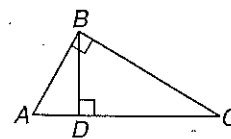
16. If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1.

## 7-1

## Study Guide and Intervention (continued)

## Geometric Mean

**Altitude of a Triangle** In the diagram,  $\triangle ABC \sim \triangle ADB \sim \triangle BDC$ . An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.



**Example 1** Use right  $\triangle ABC$  with  $\overline{BD} \perp \overline{AC}$ . Describe two geometric means.

a.  $\triangle ADB \sim \triangle BDC$  so  $\frac{AD}{BD} = \frac{BD}{CD}$ .

In  $\triangle ABC$ , the altitude is the geometric mean between the two segments of the hypotenuse.

b.  $\triangle ABC \sim \triangle ADB$  and  $\triangle ABC \sim \triangle BDC$ ,

so  $\frac{AC}{AB} = \frac{AB}{AD}$  and  $\frac{AC}{BC} = \frac{BC}{DC}$ .

In  $\triangle ABC$ , each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example 2**

Find  $x$ ,  $y$ , and  $z$ .

$$\frac{PR}{PQ} = \frac{PQ}{PS}$$

$$\frac{25}{15} = \frac{15}{x}$$

$$25x = 225$$

$$x = 9$$

$$PR = 25, PQ = 15, PS = x$$

Cross multiply.

Divide each side by 25.

Then

$$y = PR - SP$$

$$= 25 - 9$$

$$= 16$$

$$\frac{PR}{QR} = \frac{QR}{RS}$$

$$\frac{25}{z} = \frac{z}{y}$$

$$PR = 25, QR = z, RS = y$$

$$\frac{25}{z} = \frac{z}{16}$$

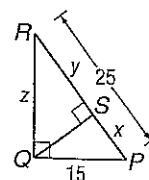
$$y = 16$$

$$z^2 = 400$$

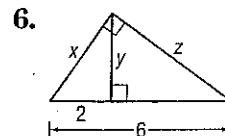
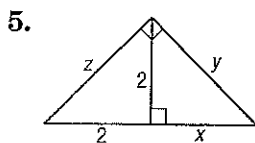
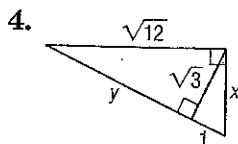
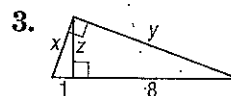
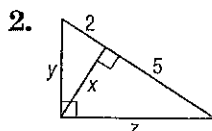
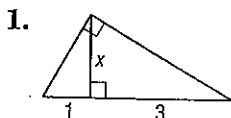
Cross multiply.

$$z = 20$$

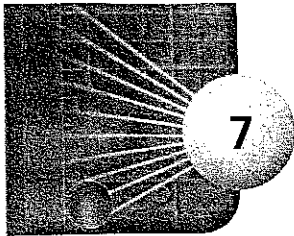
Take the square root of each side.

**Exercises**

Find  $x$ ,  $y$ , and  $z$  to the nearest tenth.



10. **CIVIL ENGINEERING** An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth.



## 7

**Reading to Learn Mathematics*****Proof Builder***

This is a list of key theorems and postulates you will learn in Chapter 7. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 7.1		
Theorem 7.2		
Theorem 7.3		
Theorem 7.4 <i>Pythagorean Theorem</i>		
Theorem 7.5 <i>Converse of the Pythagorean Theorem</i>		
Theorem 7.6		
Theorem 7.7		