Organizer

Objective: Help students organize and review key concepts and skills in Chapter 6.

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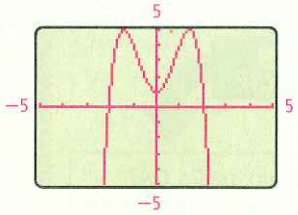
Answers

1. monomial
2. synthetic division
3. multiplicity
4. end behavior
5. $-3x^3 + 4x^2 + 6x + 7$; -3; 3; 4; cubic polynomial with 4 terms
6. $-x^5 + 2x^4 + 5x^3 + 8x$; -1; 5; 4; quintic polynomial with 4 terms
7. $9x^2 - 11x + 1$; 9; 2; 3; quadratic trinomial
8. $x^4 - 6x^2$; 1; 4; 2; quartic binomial
9. $8x^3 + x^2 - 4x$
10. $-5x^3 + 6x^2 + 10x - 1$
11. $-6x^2 - x + 9$
12. $-4x^4 - x^3 - 3$
13. 

From left to right, it alternately increases and decreases, changing direction 3 times and crossing the x-axis 2 times. There appear to be 2 real zeros.

Vocabulary

degree of a monomial 406
degree of a polynomial 406
end behavior 453
leading coefficient 406
local maximum 455
local minimum 455
monomial 406
multiplicity 439
polynomial 406
polynomial function 408
synthetic division 423
turning point 455

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a number or product of numbers and variables with whole number exponents.
2. A method of dividing a polynomial by a linear binomial of the form $x - a$ by using only the coefficients is ___?___.
3. The number of times $x - r$ is a factor of $P(x)$ is the ___?___ of r.
4. The ___?___ of a function is a description of the function values as x approaches positive infinity or negative infinity.

6-1 Polynomials *(pp. 406–412)*

EXAMPLES

■ **Subtract. Write your answer in standard form.**

$(6x - 2x^2 + 1) - (4x - 5x^2)$

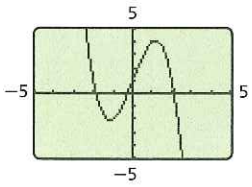
$(-2x^2 + 6x + 1) + (5x^2 - 4x)$ *Add the opposite.*

$(-2x^2 + 5x^2) + (6x - 4x) + 1$ *Combine like terms.*

$3x^2 + 2x + 1$

■ **Graph $f(x) = -x^3 + 4x + 1$ on a calculator. Describe the graph, and identify the number of real zeros.**

From left to right, the function decreases, increases, and then decreases again. It crosses the x-axis three times. There appear to be three real zeros.



EXERCISES

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

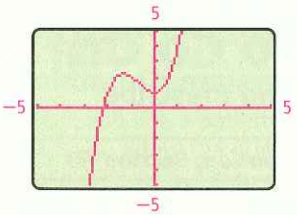
5. $4x^2 - 3x^3 + 6x + 7$
6. $5x^3 - x^5 + 8x + 2x^4$
7. $1 - 11x + 9x^2$
8. $-6x^2 + x^4$

Add or subtract. Write your answer in standard form.

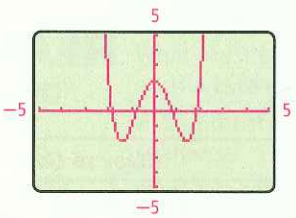
9. $(8x^3 - 4x^2 - 3x + 1) - (1 - 5x^2 + x)$
10. $(6x^2 + 7x - 2) + (1 - 5x^3 + 3x)$
11. $(5x - 2x^2) - (4x^2 + 6x - 9)$
12. $(x^4 - x^2 + 4) + (x^2 - x^3 - 5x^4 - 7)$

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

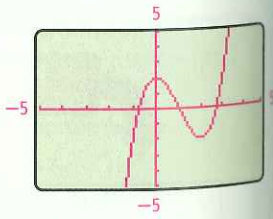
13. $f(x) = -x^4 + 4x^2 + 1$
14. $f(x) = x^3 + 2x^2 + 1$
15. $f(x) = x^4 - 5x^2 + 2$
16. $f(x) = x^3 - 3x^2 + 2$

14. 

From left to right, it increases, decreases slightly, and then increases again. It crosses the x-axis 1 time. There appears to be 1 real zero.

15. 

From left to right, it alternately decreases and increases, changing direction 3 times. It crosses the x-axis 4 times. There appear to be 4 real zeros.

16. 

From left to right, it increases, decreases, and then increases again. It crosses the x-axis 3 times. There appear to be 3 real zeros.

6-2 Multiplying Polynomials *(pp. 414–420)*

EXAMPLE

■ **Find the product.**
$(x-3)(5-x-2x^2)$

Multiply horizontally.

$(x-3)(-2x^2-x+5)$ *Write in standard form.*

$x(-2x^2)+x(-x)+x(5)-3(-2x^2)-3(-x)-3(5)$

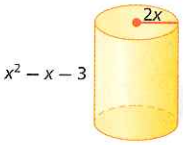
$-2x^3-x^2+5x+6x^2+3x-15$ *Multiply.*

$-2x^3+5x^2+8x-15$ *Combine like terms.*

EXERCISES

Find each product.

17. $5x^2(3x-2)$
18. $-3t(2t^2-6t+1)$
19. $ab^2(a^2-a+ab)$
20. $(x-2)(x^2-2x-3)$
21. $(2x+5)(x^3-x^2+1)$
22. $(x-3)^3$
23. $(x+4)(x^4-3x^2+x)$
24. $(2x+1)^4$
25. A cylinder has a height of x^2-x-3 and a radius of $2x$ as shown. Express the volume of the cylinder as a sum of monomials.



6-3 Dividing Polynomials *(pp. 422–428)*

EXAMPLE

■ **Divide by using synthetic division.**
$(x^3-3x^2+8)\div(x+2)$

$a=-2$

x^3-3x^2+0x+8 *Write in standard form.*

−2	1	−3	0	8
		−2	10	−20
	1	−5	10	−12

Write the coefficients of the terms.

$\frac{x^3-3x^2+8}{x+2}=x^2-5x+10+\frac{-12}{x+2}$

EXERCISES

Divide by using long division.

26. $(x^3-5x^2+2x-7)\div(x+2)$
27. $(8x^4+6x^2-2x+4)\div(2x-1)$

Divide by using synthetic division.

28. $(x^3-4x^2+3x+2)\div(x-3)$
29. $(x^3+2x-1)\div(x-2)$
30. A spool of ribbon has a length of x^3+x^2 inches. Write an expression that represents the number of strips of ribbon with a length of $x-1$ inches that can be cut from one spool.

6-4 Factoring Polynomials *(pp. 430–435)*

EXAMPLES

Determine whether each binomial is a factor of the polynomial $P(x)=2x^2+x-10$.

■ $(x+5)$

−5	2	1	−10
		−10	45
	2	−9	35

$x+5$ is not a factor of $P(x)$.

■ $(x-2)$

2	2	1	−10
		4	10
	2	5	0

$x-2$ is a factor of $P(x)$.

EXERCISES

Determine whether the given binomial is a factor of the polynomial $P(x)$.

31. $(x+3)$; $P(x)=x^3+2x^2-5$
32. $(x-1)$; $P(x)=4x^4-5x^2+3x-2$
33. $(x-2)$; $P(x)=2x^3-3x^2+x-6$

Factor each expression.

34. $x^3-x^2-16x+16$
35. $4x^3-8x^2-x+2$
36. $3x^3+81$
37. $16x^3-2$

Answers

17. $15x^3-10x^2$
18. $-6t^3+18t^2-3t$
19. $a^3b^2-a^2b^2+a^2b^3$
20. x^3-4x^2+x+6
21. $2x^4+3x^3-5x^2+2x+5$
22. $x^3-9x^2+27x-27$
23. $x^5+4x^4-3x^3-11x^2+4x$
24. $16x^4+32x^3+24x^2+8x+1$
25. $4\pi x^4-4\pi x^3-12\pi x^2$
26. $x^2-7x+16-\frac{39}{x+2}$
27. $4x^3+2x^2+4x+1+\frac{5}{2x-1}$
28. $x^2-x+\frac{2}{x-3}$
29. $x^2+2x+6+\frac{11}{x-2}$
30. x^2+2x+2 in., remainder 2 in.
31. no
32. yes
33. yes
34. $(x-1)(x-4)(x+4)$
35. $(x-2)(2x-1)(2x+1)$
36. $3(x+3)(x^2-3x+9)$
37. $2(2x-1)(4x^2+2x+1)$

Answers

38. 1, 2
39. $-2, -2 \pm \sqrt{3}$
40. -1
41. $-3, 3, \pm\sqrt{3}$
42. $-1, \pm\sqrt{2}$
43. $1, 2 \pm 2\sqrt{2}$
44. $2m$
45. $P(x) = x^3 - 3x^2 - 10x + 24$
46. $P(x) = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$
47. $P(x) = x^3 + x^2 - 2x - 2$
48. $P(x) = x^3 + 3x^2 + x + 3$
49. $P(x) = x^4 - 5x^2 + 6$
50. $P(x) = x^4 - 2x^3 + 2x^2 - 8x - 8$
51. $1, -2i, 2i$
52. $-i, i, -\sqrt{2}, \sqrt{2}$
53. $\pm 4, \pm\frac{1}{2}i$
54. $\pm\sqrt{5}, -3$

6-5 Finding Real Roots of Polynomial Equations *(pp. 438–444)*

EXAMPLE

■ **Identify all of the real roots of $x^4 - 4x^3 + 4x^2 - 1 = 0$.**

By the Rational Root Theorem, possible roots are ± 1.

1	1	−4	4	0	−1
		1	−3	1	1
	1	−3	1	1	0

Try 1.

1	1	−3	1	1	
		1	−2	−1	
	1	−2	−1	0	

Try 1 again.

Factor $x^2 - 2x - 1$ by using the quadratic formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = 1 \pm \sqrt{2}$$

The roots are 1 with a multiplicity of 2, and $1 \pm \sqrt{2}$.

EXERCISES

Identify all of the real roots of each equation.

38. $x^3 - 5x^2 + 8x - 4 = 0$
39. $x^3 + 6x^2 + 9x + 2 = 0$
40. $x^3 + 3x^2 + 3x + 1 = 0$
41. $x^4 - 12x^2 + 27 = 0$
42. $x^3 + x^2 - 2x - 2 = 0$
43. $x^3 - 5x^2 + 4 = 0$
44. A rectangular prism has length that is twice its width and height that is 4 meters longer than its width. The volume of the rectangular prism is 48 cubic meters. What is the width of the rectangular prism?

6-6 Fundamental Theorem of Algebra *(pp. 445–451)*

EXAMPLES

■ **Write the simplest polynomial function with roots −2, −1, and 4.**

$P(x) = 0$

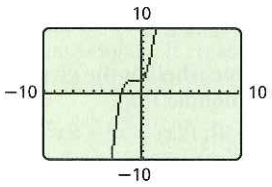
$a(x + 2)(x + 1)(x - 4) = 0$ *If r is a root of P(x), then x − r is a factor of P(x).*

$a(x^3 - x^2 - 10x - 8) = 0$ *Multiply. For the simplest equation, let a = 1.*

$x^3 - x^2 - 10x - 8 = 0$

■ **Solve $x^3 + 2x^2 + x + 2 = 0$ by finding all roots.**

The graphing calculator shows −2 as a root. Use synthetic division to write the equation as $(x + 2)(x^2 + 1) = 0$. Solve $x^2 + 1 = 0$ to find the remaining roots. The solutions are -2, i, and $-i$.



EXERCISES

Write the simplest polynomial function with the given roots.

45. $-3, 2, 4$
46. $-\frac{1}{2}, -2, 3$
47. $-\sqrt{2}, -1$
48. $-3, i$
49. $\sqrt{2}, \sqrt{3}$
50. $1 + \sqrt{3}, 2i$

Solve the equation by finding all roots.

51. $x^3 - x^2 + 4x - 4 = 0$
52. $x^4 - x^2 - 2 = 0$
53. $x^4 - \frac{63}{4}x^2 - 4 = 0$
54. $x^3 + 3x^2 - 5x - 15 = 0$

6-7 Investigating Graphs of Polynomial Functions *(pp. 453–459)*

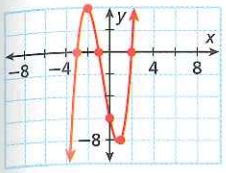
EXAMPLE

- **Graph the function $f(x) = x^3 + 2x^2 - 5x - 6$.**

Leading coefficient: 1; Degree: 3;
End behavior: $x \to -\infty, f(x) \to -\infty$
$x \to +\infty, f(x) \to +\infty$

The zeros are $-3, -1, 2$. *Factor to find the zeros.*

$f(0) = -6; f(-2) = 4; f(1) = -8$ *Evaluate f(x) at values between the roots. Plot these points.*



EXERCISES

Identify the leading coefficient, degree, and end behavior.

55. $-2x^3 + 5x^2 + 3$

56. $x^4 + 2x^3 - 3x + 1$

57. $-3x^6 + 9x^3 - 2x - 9$

58. $7x^5 + x^4 - 2x^2 + 5$

Graph each function.

59. $f(x) = x^3 - x^2 - 5x + 6$

60. $f(x) = x^4 - 10x^2 + 9$

61. $f(x) = -x^3 + 5x^2 + x - 5$

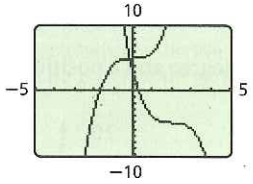
6-8 Transforming Polynomial Functions *(pp. 460–465)*

EXAMPLE

- **Write a function that transforms $f(x) = x^3 + 5$ by reflecting it across the x-axis and shifting it 2 units right. Support your solution by using a graphing calculator.**

$g(x) = -f(x - 2)$

$g(x) = -(x - 2)^3 - 5$



EXERCISES

Write a function that transforms $f(x) = x^4 - 6x^2 - 4$ in each of the following ways. Support your solution by using a graphing calculator.

62. Stretch vertically by a factor of 2, and move 9 units up.

63. Move 2 units down, and reflect across the x-axis.

64. Move 3 units right, and reflect across the y-axis.

6-9 Curve Fitting with Polynomial Models *(pp. 466–471)*

EXAMPLE

- **The table shows the profit for a company in thousands of dollars for the years shown. Write a polynomial function for the data.**

Year	1999	2000	2001	2002	2003
Profits	$286	$401	$507	$671	$960

First differences: 115 106 164 289
Second differences: −9 58 125
Third differences: 67 67 *Constant*
A cubic polynomial best describes the data. Use the cubic regression feature on your graphing calculator.

$f(x) = 11.17x^3 - 38x^2 + 141.3x + 286$

EXERCISES

65. The chart shows the attendance for a new movie theater over five days. Write a polynomial function for the data.

Day	1	2	3	4	5
Attendance	248	298	318	388	428

66. The chart shows the population of a city for five years. Write a polynomial function for the data.

Year	1	2	3	4	5
Population (thousands)	1891	2674	3376	4480	6469

Answers

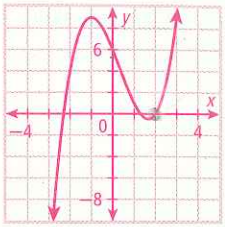
55. -2; 3; as $x \to -\infty$, $f(x) \to +\infty$; as $x \to +\infty$, $f(x) \to -\infty$

56. 1; 4; as $x \to \pm\infty$, $f(x) \to +\infty$

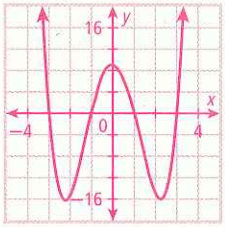
57. -3; 6; as $x \to \pm\infty$, $f(x) \to -\infty$

58. 7; 5; as $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$

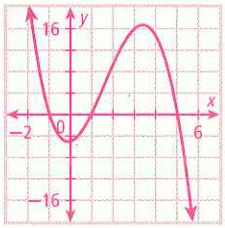
59.



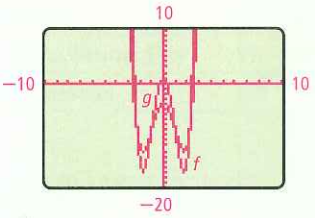
60.



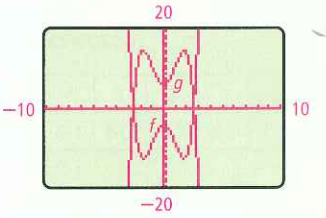
61.



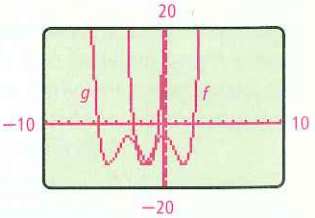
62. $g(x) = 2x^4 - 12x^2 + 1$



63. $g(x) = -x^4 + 6x^2 + 6$



64. $g(x) = (-x - 3)^4 - 6(-x - 3)^2 - 4$



65. $f(x) \approx -6\frac{2}{3}x^4 + 80x^3 - 328\frac{1}{3}x^2 + 575x - 72$

66. $f(x) \approx 80.5x^3 - 523.5x^2 + 1790x + 544$

Organizer

Objective: Help students organize and review key concepts and skills presented in Chapter 7.

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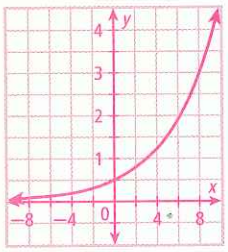
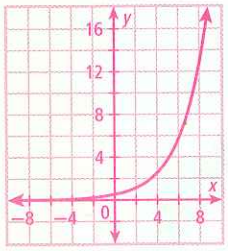
Answers

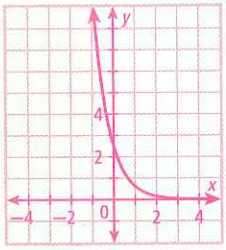
1. natural logarithmic function
2. asymptote
3. inverse relation
4. growth



5. growth



6. decay



Vocabulary

asymptote 490
base 490
common logarithm 506
exponential decay 490
exponential equation 522
exponential function 490
exponential growth 490
exponential regression 546
inverse function 499
inverse relation 498
logarithm 505
logarithmic equation 523
logarithmic function 507
logarithmic regression 546
natural logarithm 531
natural logarithmic function 532

Complete the sentences below with vocabulary words from the list above.

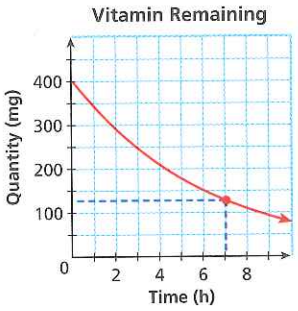
1. A(n) ___?___ has a base of e.
2. A(n) ___?___ is a line that a graphed function approaches but does not touch.
3. To graph a(n) ___?___, reflect each point in the relation across the line $y = x$.

7-1 Exponential Functions, Growth, and Decay *(pp. 490–496)*

EXAMPLE

A quantity of a certain vitamin is eliminated from the bloodstream at about 15% per hour.

- Will the function that represents this situation show growth or decay?
  It will show decay because the quantity decreases.
- Write a function to show the amount of the vitamin that remains t hours after the peak level of 400 mg.
  $f(x) = 400(0.85)^t$
- Graph the function. Use the graph to predict the amount remaining after 7 hours.
  After 7 hours, about 130 mg are left.



EXERCISES

Tell whether the function shows growth or decay. Then graph.

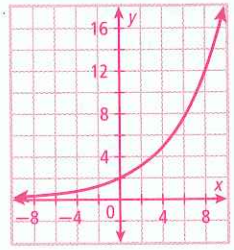
4. $f(x) = 0.5(1.25)^x$
5. $f(x) = 0.5\left(\frac{3}{2}\right)^x$
6. $f(x) = 2.5(0.25)^x$
7. $f(x) = 2(1 + 0.25)^x$

Use the following data to answer the questions.

The student population in a small resort town has increased by 2% per year for the last 5 years. This year's population is 765 students.

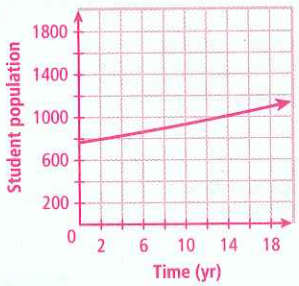
8. Will the function that represents this situation show growth or decay?
9. Suppose that the student population continues to follow the same trend. Write a function to show the number of students as a function of the year, starting with the current year.
10. Graph the function.
11. Use the graph to predict the number of students in 5 years.
12. When will the population exceed 1000 students?

7. growth



8. growth
9. $P(t) = 765(1.02)^t$

10.



11. ≈ 845
12. ≈ 13.5 yr

7-2 Inverses of Relations and Functions *(pp. 498–504)*

EXAMPLE

■ **Graph the function $f(x) = \frac{4}{5} - 3x$. Then write its inverse and graph.**

$y = -3x + \frac{4}{5}$ *Set $y = f(x)$ and graph*

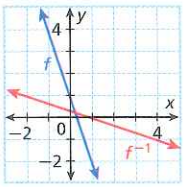
$x = -3y + \frac{4}{5}$ *Interchange x and y.*

$3y = -x + \frac{4}{5}$ *Solve for y.*

$y = -\frac{1}{3}x + \frac{4}{15}$

Write the inverse and graph.

$f^{-1}(x) = -\frac{1}{3}x + \frac{4}{15}$



EXERCISES

13. Graph the relation and connect the points. Then graph and write the inverse.

x	−1	0	1	2	3
y	1	0.2	0.04	0.008	0.001

This year the population of a species decreased by 3% from last year.

14. Write an expression for the size of the population this year P_T as a function of last year's population P_L.

15. Write an expression for the year as a function of the size of the population.

16. The formula $M = \frac{5}{8}K$ gives the approximate distance in miles as a function of kilometers. Write and use the inverse of this function to express 25 miles in kilometers.

7-3 Logarithmic Functions *(pp. 505–511)*

EXAMPLES

■ **Write the exponential equation $9^{1.5} = 27$ in logarithmic form.**

$9^{1.5} = 27$

$\log_9 27 = 1.5$ *A logarithm is an exponent.*

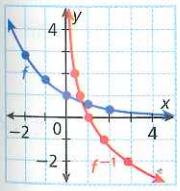
■ **Evaluate $\log_4 64$.**

Because $4^3 = 64$, $\log_4 64 = 3$.

■ **Graph $f(x) = 0.6^x$. Then graph its inverse. Describe the domain and range of the inverse function.**

x	−2	−1	0	1	2
f(x)	2.8	1.7	1	0.6	0.4

To graph the inverse, reverse each ordered pair.



For the inverse function, the domain is $\{x \mid x > 0\}$, and the range is $\mathbb{R}$.

EXERCISES

Write each exponential equation in logarithmic form.

17. $3^5 = 243$ 18. $1 = 9^0$ 19. $\left(\frac{1}{3}\right)^{-3} = 27$

Write each logarithmic equation in exponential form.

20. $\log_2 16 = 4$ 21. $\log 10 = 1$ 22. $2 = \log_{0.6} 0.36$

Evaluate by using mental math.

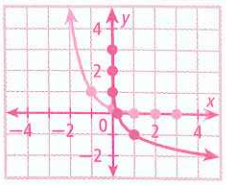
23. $\log_7 49$ 24. $\log_{0.5} 0.25$

25. $\log_{12}\left(\frac{1}{12}\right)$ 26. $\log 0.01$ 27. $\log_2 1$

28. Make a table of ordered pairs for $f(x) = \left(\frac{1}{2}\right)^x$. Graph the function and its inverse. Describe the domain and range of the inverse function.

Answers

13.



14. $P_T = P_L(1 - 0.03)$

15. $P_L = \frac{P_T}{0.97}$

16. $K = \frac{8}{5}M$; 40 km

17. $\log_3 243 = 5$

18. $\log_9 1 = 0$

19. $\log_{\frac{1}{3}} 27 = -3$

20. $2^4 = 16$

21. $10^1 = 10$

22. $0.6^2 = 0.36$

23. 2

24. 2

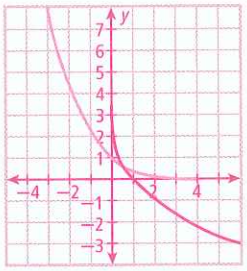
25. −1

26. −2

27. 0

28.

x	−2	−1	0	1	2
y	4	2	1	0.5	0.25



D: $\{x \mid x > 0\}$; R: $\mathbb{R}$

Answers

29. $\log_2 128 = 7$
30. $\log 1{,}000{,}000 = 6$
31. $\log_2 64 = 6$
32. $\log 100 = 2$
33. $\log_5 5^4 = 4$
34. $9 \log 10 = 9$
35. 10
36. -1
37. $x \geq -6$
38. $x > 10$
39. $17\frac{2}{3}$ yr
40a. $k = 0.0346$
b. ≈ 349

7-4 Properties of Logarithms *(pp. 512–519)*

EXAMPLES

Express as a single logarithm and simplify.

- $\log 25 + \log 40$
  $= \log(25 \cdot 40) = \log 1000 = 3$
- $\log_5 125 - \log_5 25$
  $= \log_5\left(\frac{125}{25}\right) = \log_5 5 = 1$
- $\log_3 8^2$
  $= 2\log_3 8 = 2 \cdot 2 = 4$
- **Evaluate $\log_5 16$.**
  $= \frac{\log 16}{\log 5}$ *Use the change of base formula.*
  $\approx \frac{1.2}{0.7} \approx 1.72$ *Use a calculator to evaluate.*

EXERCISES

Express as a single logarithm and simplify.

29. $\log_2 8 + \log_2 16$
30. $\log 100 + \log 10{,}000$
31. $\log_2 128 - \log_2 2$
32. $\log 10 - \log 0.1$
33. $\log_5 25^2$
34. $\log 10^5 + \log 10^4$
35. The apparent loudness of the music today at Sam's Café was 10 decibels greater than the loudness yesterday. Apparent loudness L is given by $L = 10 \log \frac{I}{I_0}$, where I is the intensity of sound, in W/m^2 and I_0 is the lowest intensity that the ear can detect. How many times more intense was the sound today than yesterday?

7-5 Exponential and Logarithmic Equations and Inequalities *(pp. 522–528)*

EXAMPLES

Solve.

- $5^x = 50$
  $\log 5^x = \log 50$
  $x \log 5 = \log 50$
  $x = \frac{\log 50}{\log 5} \approx 2.43$
- $\log_9 x^2 = 5$
  $2\log_9 x = 5$
  $\log_9 x = \frac{5}{2}$
  $x = 9^{\frac{5}{2}}$
  $x = (3^2)^{\frac{5}{2}} = 3^5 = 243$

EXERCISES

Solve and check.

36. $3^{x-1} = \frac{1}{9}$
37. $\left(\frac{1}{2}\right)^x \leq 64$
38. $\log x^{\frac{5}{2}} > 2.5$
39. $A = P(1 + r)^n$ gives amount A in an account after n years for an initial investment P that earns interest at an annual rate r. How long will it take for \$250 to increase to \$500 at 4% annual interest?

7-6 The Natural Base, e *(pp. 531–536)*

EXAMPLE

- **Simplify $e^{\ln(2s+1)}$.**
  $e^{\ln(2s+1)} = 2s + 1$ *e to the ln of a number is just the number.*
- **What is the total value of an investment of \$5000 that earned 6% interest compounded continuously for 5 years?**
  $A = 5000e^{0.06(5)}$ *Substitute in $A = Pe^{rt}$.*
  $A \approx 6749.29$ *Use a calculator.*
  The value is \$6749.29.

EXERCISES

40. The population of whooping cranes was about 22 in 1940 and grew at an exponential rate to about 194 in 2003.
    a. Use the exponential growth function $P(t) = P_0 e^{kt}$, where P_0 is the initial population and $P(t)$ is the population at time t, to determine the growth factor k.
    b. If the flock continues to grow at the same rate, how large will it be in 2020?

8-2 Multiplying and Dividing Rational Expressions *(pp. 577–582)*

EXAMPLES

- Simplify $\frac{4-x}{x^2-x-20}$. Identify any x-values for which the expression is undefined.

$$\frac{-1(x+4)}{(x-5)(x+4)} = \frac{-1}{x-5}$$

Factor. Then divide out common factors.

Undefined at $x = 5$ and $x = -4$

- Divide. Assume that all expressions are defined.

$$\frac{x^2-9}{x+2} \div \frac{x+3}{x^2+7x+10}$$

$$\frac{x^2-9}{x+2} \cdot \frac{x^2+7x+10}{x+3}$$

Rewrite as multiplication.

$$\frac{(x-3)(x+3)}{x+2} \cdot \frac{(x+2)(x+5)}{x+3} = (x-3)(x+5)$$

EXERCISES

Simplify. Identify any x-values for which the expression is undefined.

11. $\frac{24x^{14}}{9x^{16}}$
12. $\frac{6x^3}{3x+12}$
13. $\frac{x^2+x-12}{x^2+5x+4}$

Multiply. Assume that all expressions are defined.

14. $\frac{x+5}{3x+1} \cdot \frac{9x+3}{x^2-25}$
15. $\frac{x}{x-4} \cdot \frac{-x+2}{x^2+x-6}$
16. $\frac{x^2+2x-3}{x^2-x-2} \cdot \frac{x-2}{x+3}$
17. $\frac{9x^2-1}{x^2-9} \cdot \frac{x+3}{3x+1}$

Divide. Assume that all expressions are defined.

18. $\frac{x^3y}{4xy^4} \div \frac{x}{8y^2}$
19. $\frac{x^2+2x-15}{x-2} \div \frac{x^2-9}{2x-4}$
20. $\frac{3x-21}{3x} \div \frac{x^2-49}{x^2+7x}$
21. $\frac{x^2+4x+3}{x^2+2x-8} \div \frac{3x+3}{x-2}$

8-3 Adding and Subtracting Rational Expressions *(pp. 583–590)*

EXAMPLES

- Add. Identify any x-values for which the expression is undefined.

$$\frac{6x-3}{x^2-x-12} + \frac{x}{x+3}$$

$$\frac{6x-3}{(x-4)(x+3)} + \frac{x}{x+3}\left(\frac{x-4}{x-4}\right)$$

$$\frac{6x-3+x(x-4)}{(x-4)(x+3)}$$

Add the numerators.

$$\frac{x^2+2x-3}{(x-4)(x+3)}$$

Simplify the numerator.

$$\frac{(x+3)(x-1)}{(x-4)(x+3)} = \frac{x-1}{x-4}$$

Factor the numerator.

Undefined at $x = 4$ and $x = -3$

- Simplify. Assume that all expressions are defined.

$$\frac{\frac{x+2}{6x}}{\frac{x}{x-4}} = \frac{\frac{x+2}{6x}(6x)(x-4)}{\frac{x}{x-4}(6x)(x-4)}$$

The LCD is $(6x)(x-4)$.

$$\frac{(x+2)(x-4)}{x(6x)} = \frac{(x+2)(x-4)}{6x^2}$$

EXERCISES

Add. Identify any x-values for which the expression is undefined.

22. $\frac{4}{x^2+4} + \frac{x^2+8}{x^2+4}$
23. $\frac{1}{x+3} + \frac{1}{x-3}$
24. $\frac{x}{x^2-4} + \frac{1}{x-2}$
25. $\frac{2x-3}{3x+7} + \frac{6}{4x-1}$

Find the least common multiple for each pair.

26. $x^2 - 9$ and $x^2 - 6x + 9$
27. $x^2 + 2x - 35$ and $x^2 + 9x + 14$

Subtract. Identify any x-values for which the expression is undefined.

28. $\frac{2x}{x+4} - \frac{3}{x+4}$
29. $\frac{x}{x+5} - \frac{5}{x-5}$
30. $\frac{1}{x^2-x-6} - \frac{x}{x+2}$
31. $\frac{2x}{2x+1} - \frac{7}{3x-1}$

Simplify. Assume that all expressions are defined.

32. $\frac{\frac{x-6}{5}}{\frac{x+2}{8}}$
33. $\frac{\frac{x+3}{3x}}{\frac{x^2-9}{6x-9}}$
34. $\frac{\frac{x}{4} - \frac{1}{x}}{\frac{x+2}{x-2}}$

35. A jet's average speed is 520 mi/h when flying from Dallas to Chicago and 580 mi/h on the return trip. What is the jet's average speed for the entire trip?

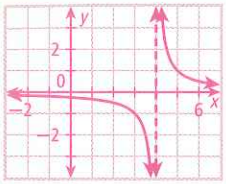
Answers

11. $\frac{8}{3x^2}$; $x \neq 0$
12. $\frac{2x^3}{x+4}$; $x \neq -4$
13. $\frac{x-3}{x+1}$; $x \neq -4$, $x \neq -1$
14. $\frac{3}{x-5}$
15. $\frac{-x}{(x-4)(x+3)}$
16. $\frac{x-1}{x+1}$
17. $\frac{3x-1}{x-3}$
18. $\frac{2x}{y}$
19. $\frac{2(x+5)}{x+3}$
20. 1
21. $\frac{x+3}{3(x+4)}$
22. $\frac{x^2+12}{x^2+4}$
23. $\frac{2x}{(x+3)(x-3)}$; $x \neq \pm 3$
24. $\frac{2(x+1)}{(x+2)(x-2)}$; $x \neq \pm 2$
25. $\frac{8x^2+4x+45}{(3x+7)(4x-1)}$; $x \neq -\frac{7}{3}$, $x \neq \frac{1}{4}$
26. $(x-3)^2(x+3)$
27. $(x-5)(x+2)(x+7)$
28. $\frac{2x-3}{x+4}$; $x \neq -4$
29. $\frac{x^2-10x-25}{(x+5)(x-5)}$; $x \neq \pm 5$
30. $\frac{-(x^2-3x-1)}{(x-3)(x+2)}$; $x \neq -2$, $x \neq 3$
31. $\frac{6x^2-16x-7}{(2x+1)(3x-1)}$; $x \neq -\frac{1}{2}$, $x \neq \frac{1}{3}$
32. $\frac{8(x-6)}{5(x+2)}$
33. $\frac{2x-3}{x(x-3)}$
34. $\frac{(x-2)^2}{4x}$
35. ≈ 548 mi/h

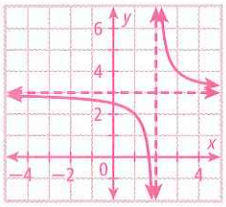
Study Guide: Review 639

Answers

36. g is f translated 4 units right.



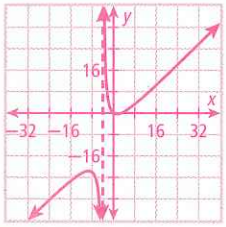
37. g is f translated 2 units right and 3 units up.



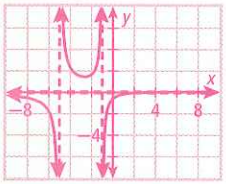
38. asymptotes: $x = 1$, $y = -3$; D: $\{x \mid x \neq 1\}$; R: $\{y \mid y \neq -3\}$

39. asymptotes: $x = -2$, $y = 1$; D: $\{x \mid x \neq -2\}$; R: $\{y \mid y \neq 1\}$

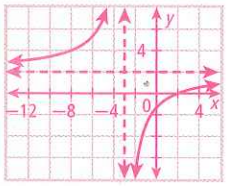
40. zeros: 0, 3; asymptote: $x = -4$



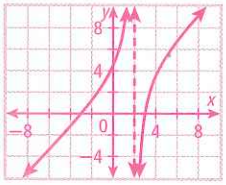
41. zero: 3; asymptotes: $x = -5$, $x = -1$, $y = 0$



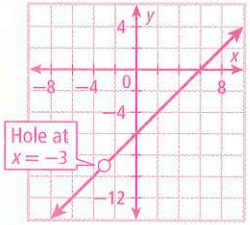
42. zero: 2; asymptotes: $x = -3$, $y = 2$



43. zeros: -3, 3; asymptote: $x = 2$



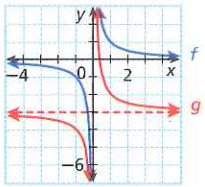
44. hole at $x = -3$



8-4 Rational Functions *(pp. 592–599)*

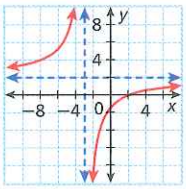
EXAMPLES

■ Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph $g(x) = \frac{1}{x} - 3$.



Because $k = -3$, translate f down 3 units.

■ Identify the zeros and asymptotes of $f(x) = \frac{2x-4}{x+3}$. Then graph.



Zero: 2
Vertical asymptote: $x = -3$
Horizontal asymptote: $y = 2$

EXERCISES

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

36. $g(x) = \frac{1}{x-4}$ 37. $g(x) = \frac{1}{x-2} + 3$

Identify the asymptotes, domain, and range of each function.

38. $f(x) = \frac{2}{x-1} - 3$ 39. $f(x) = \frac{3}{x+2} + 1$

Identify the zeros and asymptotes of each function. Then graph.

40. $f(x) = \frac{x^2 - 3x}{x+4}$ 41. $f(x) = \frac{x-3}{x^2+6x+5}$

42. $f(x) = \frac{2x-4}{x+3}$ 43. $f(x) = \frac{x^2-9}{x-2}$

44. Identify holes in the graph of $f(x) = \frac{x^2-3x-18}{x+3}$. Then graph.

8-5 Solving Rational Equations and Inequalities *(pp. 600–607)*

EXAMPLE

■ Solve the equation $\frac{30}{x+1} + x = 10$.

$\frac{30}{x+1}(x+1) + x(x+1) = 10(x+1)$

$30 + x^2 + x = 10x + 10$ *Simplify. $x \neq -1$*

$x^2 - 9x + 20 = 0$ *Write in standard form.*

$(x-4)(x-5) = 0$ *Factor.*

$x = 4$ or $x = 5$ *Solve for x.*

EXERCISES

Solve each equation.

45. $x - \frac{6}{x} = 1$ 46. $\frac{4x}{x-5} = \frac{3x+5}{x-5}$

47. $\frac{3x}{x+2} = \frac{2x+2}{x+2}$ 48. $\frac{x}{x+4} + \frac{x}{2} = \frac{2x}{2x+8}$

Solve each inequality.

49. $\frac{x+4}{x} > -2$ 50. $\frac{2}{x-3} < 4$

8-6 Radical Expressions and Rational Exponents *(pp. 610–617)*

EXAMPLES

Simplify each expression. Assume that all variables are positive.

■ $\sqrt[3]{-8x^9} = \sqrt[3]{(-2^3)} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} = -2x^3$

■ $\sqrt[4]{8x^6} \cdot \sqrt[4]{2x^2} = \sqrt[4]{16x^8} = \sqrt[4]{2^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^4} = 2x^2$

■ Write the expression $(\sqrt{16})^3$ by using rational exponents.

$16^{\frac{3}{2}}$ $(\sqrt[n]{a})^m = a^{\frac{m}{n}}$

EXERCISES

Simplify each expression. Assume that all variables are positive.

51. $\sqrt[3]{27x^6}$ 52. $\sqrt[4]{81x^{12}}$ 53. $\sqrt[3]{\frac{8x^3}{3}}$

Write each expression by using rational exponents.

54. $(\sqrt[3]{-27})^2$ 55. $\sqrt[4]{16^3}$ 56. $(\sqrt{9})^3$

Simplify each expression.

57. $17^{\frac{1}{3}} \cdot 17^{\frac{2}{3}}$ 58. $(9^4)^{\frac{1}{2}}$ 59. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

45. $x = -2$ or $x = 3$
46. no solution
47. $x = 2$
48. $x = 0$
49. $x < -\frac{4}{3}$ or $x > 0$
50. $x < 3$ or $x > \frac{7}{2}$
51. $3x^2$
52. $3x^3$
53. $2x\frac{\sqrt[3]{9}}{3}$
54. $(-27)^{\frac{2}{3}}$
55. $16^{\frac{3}{4}}$
56. $9^{\frac{3}{2}}$
57. 17
58. 81
59. $\frac{1}{2}$