Organizer

Objective: Help students organize and review key concepts and skills in Chapter 6.

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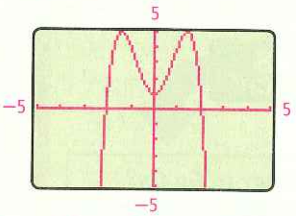
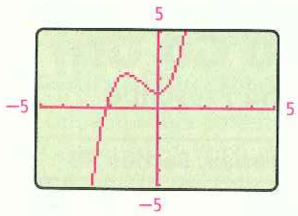
Answers

- monomial
- synthetic division
- multiplicity
- end behavior
- $-3x^3 + 4x^2 + 6x + 7$; -3; 3; 4; cubic polynomial with 4 terms
- $-x^5 + 2x^4 + 5x^3 + 8x$; -1; 5; 4; quintic polynomial with 4 terms
- $9x^2 - 11x + 1$; 9; 2; 3; quadratic trinomial
- $x^4 - 6x^2$; 1; 4; 2; quartic binomial
- $8x^3 + x^2 - 4x$
- $-5x^3 + 6x^2 + 10x - 1$
- $-6x^2 - x + 9$
- $-4x^4 - x^3 - 3$



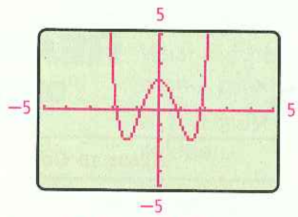
From left to right, it alternately increases and decreases, changing direction 3 times and crossing the x-axis 2 times. There appear to be 2 real zeros.

14.



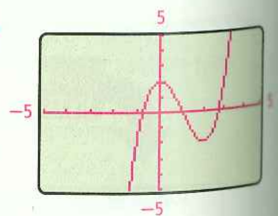
From left to right, it increases, decreases slightly, and then increases again. It crosses the x-axis 1 time. There appears to be 1 real zero.

15.



From left to right, it alternately decreases and increases, changing direction 3 times. It crosses the x-axis 4 times. There appear to be 4 real zeros.

16.



From left to right, it increases, decreases, and then increases again. It crosses the x-axis 3 times. There appear to be 3 real zeros.

CHAPTER 6 Study Guide: Review

Vocabulary

degree of a monomial 406	local maximum 455	polynomial 406
degree of a polynomial 406	local minimum 455	polynomial function 408
end behavior 453	monomial 406	synthetic division 423
leading coefficient 406	multiplicity 439	turning point 455

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a number or product of numbers and variables with whole number exponents.

2. A method of dividing a polynomial by a linear binomial of the form $x - a$ by using only the coefficients is ___?___.

3. The number of times $x - r$ is a factor of $P(x)$ is the ___?___ of r.

4. The ___?___ of a function is a description of the function values as x approaches positive infinity or negative infinity.

6-1 Polynomials *(pp. 406–412)*

EXAMPLES

■ **Subtract. Write your answer in standard form.**

$(6x - 2x^2 + 1) - (4x - 5x^2)$

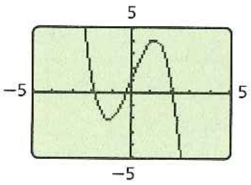
$(-2x^2 + 6x + 1) + (5x^2 - 4x)$ *Add the opposite.*

$(-2x^2 + 5x^2) + (6x - 4x) + 1$ *Combine like terms.*

$3x^2 + 2x + 1$

■ **Graph $f(x) = -x^3 + 4x + 1$ on a calculator. Describe the graph, and identify the number of real zeros.**

From left to right, the function decreases, increases, and then decreases again. It crosses the x-axis three times. There appear to be three real zeros.



EXERCISES

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

5. $4x^2 - 3x^3 + 6x + 7$ **6.** $5x^3 - x^5 + 8x + 2x^4$

7. $1 - 11x + 9x^2$ **8.** $-6x^2 + x^4$

Add or subtract. Write your answer in standard form.

9. $(8x^3 - 4x^2 - 3x + 1) - (1 - 5x^2 + x)$

10. $(6x^2 + 7x - 2) + (1 - 5x^3 + 3x)$

11. $(5x - 2x^2) - (4x^2 + 6x - 9)$

12. $(x^4 - x^2 + 4) + (x^2 - x^3 - 5x^4 - 7)$

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

13. $f(x) = -x^4 + 4x^2 + 1$

14. $f(x) = x^3 + 2x^2 + 1$

15. $f(x) = x^4 - 5x^2 + 2$

16. $f(x) = x^3 - 3x^2 + 2$

6-2 Multiplying Polynomials *(pp. 414–420)*

EXAMPLE

- Find the product.

$(x-3)(5-x-2x^2)$

Multiply horizontally.

$(x-3)(-2x^2-x+5)$ *Write in standard form.*

$x(-2x^2)+x(-x)+x(5)-3(-2x^2)-3(-x)-3(5)$

$-2x^3-x^2+5x+6x^2+3x-15$ *Multiply.*

$-2x^3+5x^2+8x-15$ *Combine like terms.*

EXERCISES

Find each product.

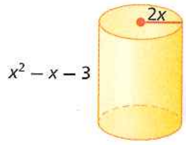
17. $5x^2(3x-2)$ **18.** $-3t(2t^2-6t+1)$

19. $ab^2(a^2-a+ab)$ **20.** $(x-2)(x^2-2x-3)$

21. $(2x+5)(x^3-x^2+1)$ **22.** $(x-3)^3$

23. $(x+4)(x^4-3x^2+x)$ **24.** $(2x+1)^4$

25. A cylinder has a height of x^2-x-3 and a radius of $2x$ as shown. Express the volume of the cylinder as a sum of monomials.



6-3 Dividing Polynomials *(pp. 422–428)*

EXAMPLE

- Divide by using synthetic division.

$(x^3-3x^2+8)\div(x+2)$

$a=-2$

x^3-3x^2+0x+8 *Write in standard form.*

Write the coefficients of the terms.

−2	1	−3	0	8
		−2	10	−20
	1	−5	10	−12

$\frac{x^3-3x^2+8}{x+2}=x^2-5x+10+\frac{-12}{x+2}$

EXERCISES

Divide by using long division.

26. $(x^3-5x^2+2x-7)\div(x+2)$

27. $(8x^4+6x^2-2x+4)\div(2x-1)$

Divide by using synthetic division.

28. $(x^3-4x^2+3x+2)\div(x-3)$

29. $(x^3+2x-1)\div(x-2)$

30. A spool of ribbon has a length of x^3+x^2 inches. Write an expression that represents the number of strips of ribbon with a length of $x-1$ inches that can be cut from one spool.

6-4 Factoring Polynomials *(pp. 430–435)*

EXAMPLES

Determine whether each binomial is a factor of the polynomial $P(x)=2x^2+x-10$.

- $(x+5)$

−5	2	1	−10
		−10	45
	2	−9	35

$x+5$ is not a factor of $P(x)$.

- $(x-2)$

2	2	1	−10
		4	10
	2	5	0

$x-2$ is a factor of $P(x)$.

EXERCISES

Determine whether the given binomial is a factor of the polynomial $P(x)$.

31. $(x+3)$; $P(x)=x^3+2x^2-5$

32. $(x-1)$; $P(x)=4x^4-5x^2+3x-2$

33. $(x-2)$; $P(x)=2x^3-3x^2+x-6$

Factor each expression.

34. $x^3-x^2-16x+16$ **35.** $4x^3-8x^2-x+2$

36. $3x^3+81$ **37.** $16x^3-2$

Answers

17. $15x^3-10x^2$
18. $-6t^3+18t^2-3t$
19. $a^3b^2-a^2b^2+a^2b^3$
20. x^3-4x^2+x+6
21. $2x^4+3x^3-5x^2+2x+5$
22. $x^3-9x^2+27x-27$
23. $x^5+4x^4-3x^3-11x^2+4x$
24. $16x^4+32x^3+24x^2+8x+1$
25. $4\pi x^4-4\pi x^3-12\pi x^2$
26. $x^2-7x+16-\frac{39}{x+2}$
27. $4x^3+2x^2+4x+1+\frac{5}{2x-1}$
28. $x^2-x+\frac{2}{x-3}$
29. $x^2+2x+6+\frac{11}{x-2}$
30. x^2+2x+2 in., remainder 2 in.
31. no
32. yes
33. yes
34. $(x-1)(x-4)(x+4)$
35. $(x-2)(2x-1)(2x+1)$
36. $3(x+3)(x^2-3x+9)$
37. $2(2x-1)(4x^2+2x+1)$

Study Guide: Review 475

nswers

. 1, 2

. $-2, -2 \pm \sqrt{3}$

. -1

. $-3, 3, \pm\sqrt{3}$

. $-1, \pm\sqrt{2}$

. $1, 2 \pm 2\sqrt{2}$

. $2m$

. $P(x) = x^3 - 3x^2 - 10x + 24$

. $P(x) = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$

. $P(x) = x^3 + x^2 - 2x - 2$

. $P(x) = x^3 + 3x^2 + x + 3$

. $P(x) = x^4 - 5x^2 + 6$

. $P(x) = x^4 - 2x^3 + 2x^2 - 8x - 8$

. $1, -2i, 2i$

. $-i, i, -\sqrt{2}, \sqrt{2}$

. $\pm 4, \pm\frac{1}{2}i$

. $\pm\sqrt{5}, -3$

6-5 Finding Real Roots of Polynomial Equations *(pp. 438–444)*

EXAMPLE

- **Identify all of the real roots of $x^4 - 4x^3 + 4x^2 - 1 = 0$.**

By the Rational Root Theorem, possible roots are ± 1.

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 4 & 0 & -1 \\ & & 1 & -3 & 1 & 1 \\ \hline & 1 & -3 & 1 & 1 & \boxed{0} \end{array}$$

Try 1.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 1 & 1 \\ & & 1 & -2 & -1 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array}$$

Try 1 again.

Factor $x^2 - 2x - 1$ by using the quadratic formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = 1 \pm \sqrt{2}$$

The roots are 1 with a multiplicity of 2, and $1 \pm \sqrt{2}$.

EXERCISES

Identify all of the real roots of each equation.

38. $x^3 - 5x^2 + 8x - 4 = 0$

39. $x^3 + 6x^2 + 9x + 2 = 0$

40. $x^3 + 3x^2 + 3x + 1 = 0$

41. $x^4 - 12x^2 + 27 = 0$

42. $x^3 + x^2 - 2x - 2 = 0$

43. $x^3 - 5x^2 + 4 = 0$

44. A rectangular prism has length that is twice its width and height that is 4 meters longer than its width. The volume of the rectangular prism is 48 cubic meters. What is the width of the rectangular prism?

6-6 Fundamental Theorem of Algebra *(pp. 445–451)*

EXAMPLES

- **Write the simplest polynomial function with roots -2, -1, and 4.**

$P(x) = 0$

$a(x + 2)(x + 1)(x - 4) = 0$

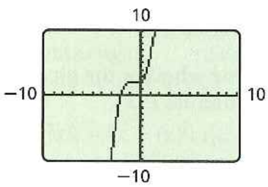
$a(x^3 - x^2 - 10x - 8) = 0$

$x^3 - x^2 - 10x - 8 = 0$

If r is a root of P(x), then x − r is a factor of P(x). Multiply. For the simplest equation, let a = 1.

- **Solve $x^3 + 2x^2 + x + 2 = 0$ by finding all roots.**

The graphing calculator shows -2 as a root. Use synthetic division to write the equation as $(x + 2)(x^2 + 1) = 0$. Solve $x^2 + 1 = 0$ to find the remaining roots. The solutions are -2, i, and $-i$.



EXERCISES

Write the simplest polynomial function with the given roots.

45. $-3, 2, 4$

46. $-\frac{1}{2}, -2, 3$

47. $-\sqrt{2}, -1$

48. $-3, i$

49. $\sqrt{2}, \sqrt{3}$

50. $1 + \sqrt{3}, 2i$

Solve the equation by finding all roots.

51. $x^3 - x^2 + 4x - 4 = 0$

52. $x^4 - x^2 - 2 = 0$

53. $x^4 - \frac{63}{4}x^2 - 4 = 0$

54. $x^3 + 3x^2 - 5x - 15 = 0$

6-7 Investigating Graphs of Polynomial Functions *(pp. 453–459)*

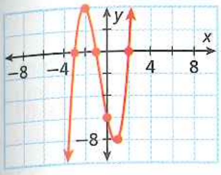
EXAMPLE

- Graph the function $f(x) = x^3 + 2x^2 - 5x - 6$.

Leading coefficient: 1; Degree: 3;
End behavior: $x \to -\infty, f(x) \to -\infty$
$x \to +\infty, f(x) \to +\infty$

The zeros are −3, −1, 2. *Factor to find the zeros.*

$f(0) = -6; f(-2) = 4; f(1) = -8$ *Evaluate f(x) at values between the roots. Plot these points.*



EXERCISES

Identify the leading coefficient, degree, and end behavior.

55. $-2x^3 + 5x^2 + 3$ **56.** $x^4 + 2x^3 - 3x + 1$

57. $-3x^6 + 9x^3 - 2x - 9$ **58.** $7x^5 + x^4 - 2x^2 + 5$

Graph each function.

59. $f(x) = x^3 - x^2 - 5x + 6$

60. $f(x) = x^4 - 10x^2 + 9$

61. $f(x) = -x^3 + 5x^2 + x - 5$

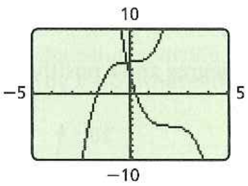
6-8 Transforming Polynomial Functions *(pp. 460–465)*

EXAMPLE

- Write a function that transforms $f(x) = x^3 + 5$ by reflecting it across the x-axis and shifting it 2 units right. Support your solution by using a graphing calculator.

$g(x) = -f(x - 2)$

$g(x) = -(x - 2)^3 - 5$



EXERCISES

Write a function that transforms $f(x) = x^4 - 6x^2 - 4$ in each of the following ways. Support your solution by using a graphing calculator.

62. Stretch vertically by a factor of 2, and move 9 units up.

63. Move 2 units down, and reflect across the x-axis.

64. Move 3 units right, and reflect across the y-axis.

6-9 Curve Fitting with Polynomial Models *(pp. 466–471)*

EXAMPLE

- The table shows the profit for a company in thousands of dollars for the years shown. Write a polynomial function for the data.

Year	1999	2000	2001	2002	2003
Profits	\$286	\$401	\$507	\$671	\$960

First differences: 115 106 164 289
Second differences: −9 58 125
Third differences: 67 67 *Constant*

A cubic polynomial best describes the data. Use the cubic regression feature on your graphing calculator.

$f(x) = 11.17x^3 - 38x^2 + 141.3x + 286$

EXERCISES

65. The chart shows the attendance for a new movie theater over five days. Write a polynomial function for the data.

Day	1	2	3	4	5
Attendance	248	298	318	388	428

66. The chart shows the population of a city for five years. Write a polynomial function for the data.

Year	1	2	3	4	5
Population (thousands)	1891	2674	3376	4480	6469

Answers

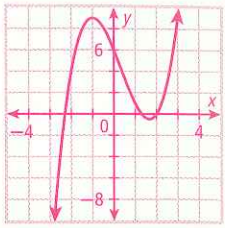
55. −2; 3; as $x \to -\infty, f(x) \to +\infty$; as $x \to +\infty, f(x) \to -\infty$

56. 1; 4; as $x \to \pm\infty, f(x) \to +\infty$

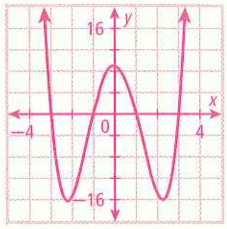
57. −3; 6; as $x \to \pm\infty, f(x) \to -\infty$

58. 7; 5; as $x \to -\infty, f(x) \to -\infty$; as $x \to +\infty, f(x) \to +\infty$

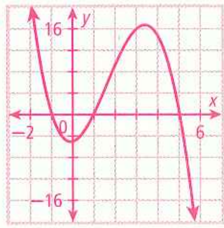
59.



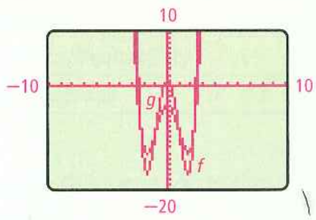
60.



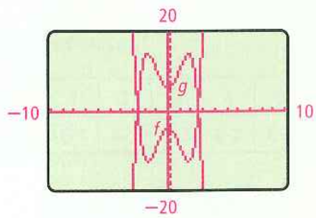
61.



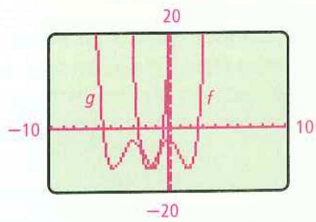
62. $g(x) = 2x^4 - 12x^2 + 1$



63. $g(x) = -x^4 + 6x^2 + 6$



64. $g(x) = (-x - 3)^4 - 6(-x - 3)^2 - 4$



65. $f(x) \approx -6\frac{2}{3}x^4 + 80x^3 - 328\frac{1}{3}x^2 + 575x - 72$

66. $f(x) \approx 80.5x^3 - 523.5x^2 + 1790x + 544$