CHAPTER 4

Review

CHAPTER 4 REVIEW

portant Concepts

Important Facts and Formulas

When f is a polynomial and r is a real number that satisfies any of the following statements, then r satisfies all the statements.

- r is a zero of the polynomial function $y = f(x)$
- r is an x-intercept of the graph of f
- r is a solution, or root, of the equation $f(x) = 0$
- $x - r$ is a factor of the polynomial expression $f(x)$
- There is a one-to-one correspondence between the linear factors of $f(x)$ that have real coefficients and the x-intercepts of the graph of f.

A polynomial of degree n has at most n distinct real zeros.

All rational zeros of a polynomial have the form $\frac{r}{s}$, where r is a factor of the constant term and s is a factor of the leading coefficient.

The end behavior of the graph of a polynomial function is similar to the end behavior of the graph of the highest degree term of the polynomial.

Zeros of even multiplicity touch but do not cross the x-axis. Zeros of odd multiplicity cross the x-axis.

The number of local extrema of the graph of a polynomial function is at most one less than the degree of a polynomial.

The number of points of inflection of the graph of a polynomial function is at most two less than the degree of the polynomial.

The graph of a rational function has a vertical asymptote at every number that is a zero of the denominator and not a zero of the numerator.

The x-intercepts of the graph of a rational function occur at the numbers that are zeros of the numerator but are not zeros of the denominator.

Every complex number can be written in the standard form $a + bi$.

$i^2 = -1$ and $i = \sqrt{-1}$

If $a + bi$ is a zero of a polynomial with real coefficients, then its conjugate $a - bi$ is also a zero.

A polynomial of degree n has exactly n complex zeros counting multiplicities.

Every polynomial expression with real coefficients can be factored into linear and irreducible quadratic factors with real coefficients.

Every polynomial expression can be factored into linear factors with complex coefficients.

Review Exercises

Section 4.1

1. Which of the following are polynomials? **a, c, e, f**
   - a. $2^3 + x^2$
   - b. $x + \frac{1}{x}$
   - c. $x^3 - \frac{1}{\sqrt{2}}$
   - d. $\sqrt[3]{x^4}$
   - e. $\pi^3 - x$
   - f. $\sqrt{2} + 2x^2$
   - g. $\sqrt{x} + 2x^2$
   - h. $|x|$
2. What is the remainder when $x^4 + 3x^3 + 1$ is divided by $x^2 + 1$? **$-3x + 2$**
3. What is the remainder when $x^{112} - 2x^8 + 9x^5 - 4x^4 + x - 5$ is divided by $x - 1$? **0**
4. Is $x - 1$ a factor of $f(x) = 14x^{87} - 65x^{56} + 51$? Justify your answer. **$f(1) = 14 - 65 + 51 = 0.$**
5. Use synthetic division to show that $x - 2$ is a factor of $x^6 - 5x^5 + 8x^4 + x^3 - 17x^2 + 16x - 4$, and find the other factor.
6. Find a polynomial f of degree 3 such that $f(-1) = 0$, $f(1) = 0$, and $f(0) = 5$. **$f(x) = 5(x - 1)^2(x + 1)$**

Section 4.2

7. Find the zero(s) of $2\left(\frac{x}{5} + 7\right) - 3x - \frac{x + 2}{5} + 4$. **$x = \frac{44}{7}$**
8. Find the zeros of $3x^2 - 2x - 5$. **$x = \frac{5}{3}, -1$**
9. Factor the polynomial $x^3 - 8x^2 + 9x + 6$. *Hint:* 2 is a zero. **$f(x) = (x - 2)(x - (3 + 2\sqrt{3}))(x - (3 - 2\sqrt{3}))$**
10. Find all real zeros of $x^6 - 4x^3 + 4$. **$x = \sqrt[3]{2}$**

Review Exercises

ANSWERS

5.
2	1	−5	8	1	−17	16	−4
		2	−6	4	10	−14	4
	1	−3	2	5	−7	2	0

other factor: $x^5 - 3x^4 + 2x^3 + 5x^2 - 7x + 2$

11. Find all real zeros of $9x^3 - 6x^2 - 35x + 26$. *Hint:* Try $x = -2$. $-2, \frac{4 \pm \sqrt{3}}{3}$

12. Find all real zeros of $3y^3(y^4 - y^2 - 5)$. $0, \pm\sqrt{\frac{1 + \sqrt{21}}{2}}$

13. Find the rational zeros of $x^4 - 2x^3 - 4x^2 + 1$.
-1

14. Consider the polynomial $2x^3 - 8x^2 + 5x + 3$.
   a. List the only possible rational zeros. $\pm 3, \pm 1, \pm\frac{3}{2}, \pm\frac{1}{2}$
   b. Find one rational zero. 3
   c. Find all the zeros of the polynomial. $\frac{1 \pm \sqrt{3}}{2}$

15. a. Find all rational zeros of $x^3 + 2x^2 - 2x - 2$. There are no rational zeros
   b. Find two consecutive integers such that an irrational zero of $x^3 + 2x^2 - 2x - 2$ lies between them.
   -3 and -2, -1 and 0, or 1 and 2

16. How many distinct real zeros does $x^3 + 4x$ have?
1

17. How many distinct real zeros does $x^3 - 6x^2 + 11x - 6$ have?
3

18. Find the zeros of $x^4 - 11x^2 + 18$.
$\pm 3, \pm\sqrt{2}$

19. The polynomial $x^3 - 2x + 1$ has d
   a. no real zeros.
   b. only one real zero.
   c. three rational zeros.
   d. only one rational zero.
   e. none of the above.

20. Show that 5 is an upper bound for the real zeros of $x^4 - 4x^3 + 16x - 16$.

21. Show that -1 is a lower bound for the real zeros of $x^4 - 4x^3 + 15$.

In Exercises 22 and 23, find the real zeros of the polynomial.

22. $x^6 - 2x^5 - x^4 + 3x^3 - x^2 - x + 1$

23. $x^5 - 3x^4 - 2x^3 - x^2 - 23x - 20$

Section 4.3

24. List the zeros of the polynomial and the multiplicity of each zero.
$f(x) = 5(x - 4)^3(x - 2)(x + 17)^3(x^2 - 4)$

25. List the zeros of the polynomial and the multiplicity of each zero.
$f(x) = -2(x + 3)^2(x - 4)(x^2 - 9)$

26. Draw the graph of a function that could not possibly be the graph of a polynomial function, and explain why.

27. Draw a graph that could be the graph of a polynomial function of degree 5. You need not list a specific polynomial nor do any computation.

28. Which of the statements is not true about the polynomial function f whose graph is shown in the figure on the next page?

20. When $x^4 - 4x^3 + 16x - 16$ is divided by $x - 5$ synthetically, the last row, 1 1 5 41 189, is all positive. Therefore, 5 is an upper bound for the real zeros.

21. When $x^4 - 4x^3 + 15$ is divided by $x + 1$ synthetically, the last row, 1 −5 5 −5 20, has alternating signs. Therefore, −1 is a lower bound for the real zeros.

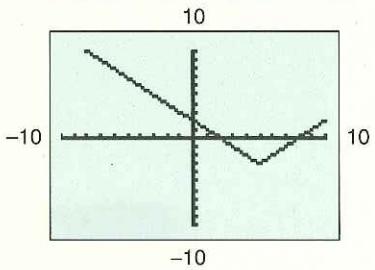
22. rational zeros: 1 and −1; irrational zeros: ≈ -0.86676 and 1.86676

23. rational zeros: −1 and 4; irrational zero: ≈ -1.328

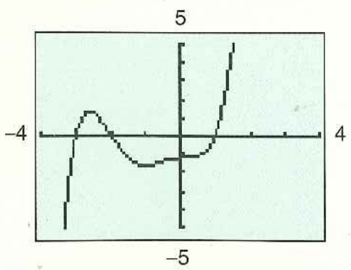
24. 4: multiplicity 3;
2: multiplicity 2;
−17: multiplicity 3;
−2: multiplicity 1

25. −3: multiplicity 2;
4: multiplicity 1;
−3: multiplicity 1;
3: multiplicity 1

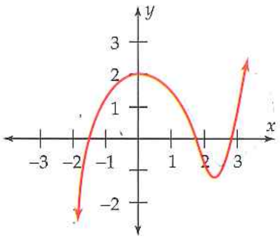
26. Answers may vary. Any graph that is not continuous is not the graph of a polynomial. Also, any graph that doesn't use all real numbers in its domain cannot be a polynomial. Another possibility is a graph with sharp corners like the one shown here. The graphs of polynomials are all smooth.



27. Answers may vary.

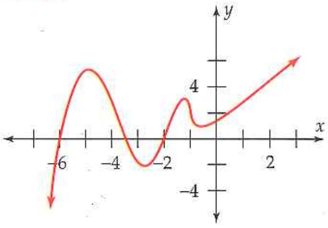


28. c



a. f has three zeros between -2 and 3
b. $f(x)$ could possibly be a fifth-degree polynomial
c. $(f \circ f)(0) > 0$
d. $f(2) - f(-1) < 3$
e. $f(x)$ is positive for all x in the interval $[-1, 0]$

29. Which of the statements **i**–**v** about the polynomial function f whose graph is shown in the figure below are false?
***i*, *iv*, and *v* are false.**



i f has 2 zeros in the interval $(-6, -3)$
ii $f(-3) - f(-6) < 0$
iii $f(0) < f(1)$
iv $f(2) - 2 = 0$
v f has degree ≤ 4

In Exercises 30–33, find a viewing window (or windows) that shows a complete graph of the function. Be alert for hidden behavior.

30. $f(x) = 0.5x^3 - 4x^2 + x + 1$

31. $g(x) = 0.3x^5 - 4x^4 + x^3 - 4x^2 + 5x + 1$

32. $h(x) = 4x^3 - 100x^2 + 600x$

33. $f(x) = 32x^3 - 99x^2 + 100x + 2$

In Exercises 34–37, sketch a complete graph of the function.

34. $f(x) = x^3 - 9x$

35. $g(x) = x^3 - 2x^2 + 3$

36. $h(x) = x^4 - x^3 - 4x^2 + 4x + 2$

37. $f(x) = x^4 - 3x - 2$

Section **4.3.A**

38. HomeArt makes plastic replicas of famous statues. Their total cost to produce copies of a particular statue is shown in the table on the next page.
a. Sketch a scatter plot of the data.
b. Use cubic regression to find a function $C(x)$ that models the data—that

30. Use $-3 \leq x \leq 9$ and $-35 \leq y \leq 15$

31. Use $-10 \leq x \leq 20$ and $-10{,}000 \leq y \leq 500$ for the overall graph and $-2 \leq x \leq 2$ and $-10 \leq y \leq 5$ for behavior around the origin.

32. Use $-2 \leq x \leq 18$ and $-500 \leq y \leq 1100$

33. Use $0.2 \leq x \leq 2$ and $20 \leq y \leq 40$; $-1 \leq x \leq 0.2$, $-20 \leq y \leq 20$

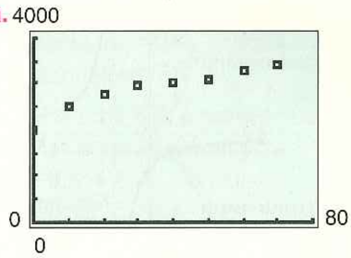
34. Use $-4 \leq x \leq 4$ and $-12 \leq y \leq 12$. There are x-intercepts at $(-3, 0)$, $(0, 0)$ and $(3, 0)$, a local maximum at $(-1.732, 10.392)$ and a local minimum at $(1.732, -10.392)$.

35. Use $-2 \leq x \leq 3$ and $-5 \leq y \leq 5$. There is an x-intercept at $(-1, 0)$, a local maximum at $(0, 3)$, and a local minimum at $\left(\frac{4}{3}, \frac{49}{27}\right)$.

36. Use $-3 \leq x \leq 3$ and $-5 \leq y \leq 5$. There are x-intercepts at approximately $(-1.908, 0)$ and $(-0.376, 0)$, local minima at approximately $(-1.326, -4.914)$ and $(1.607, 0.617)$, and a local maximum at approximately $(0.469, 2.941)$.

37. Use $-3 \leq x \leq 3$ and $-5 \leq y \leq 5$. There are x-intercepts at $x \approx -0.618, 1.618$ and a local minimum at $x \approx 0.909$.

38. a.

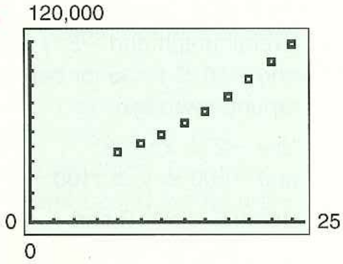


b. $c(x) = 0.008515x^3 - 1.10945x^2 + 56.25833x + 2017.25758$

38. c. $3466.47

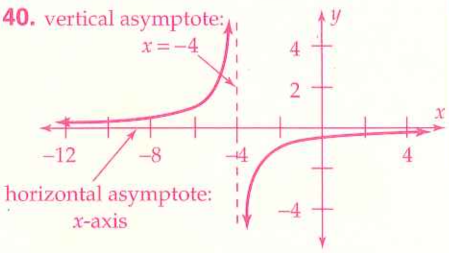
d. 35 statues: $85.49; 75 statues: $47.82

39. a.

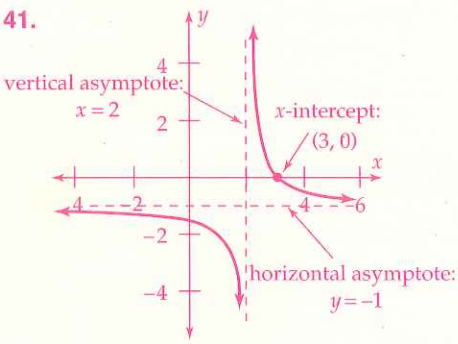


b. $y = 0.04x^4 + 0.19x^3 + 62.06x^2 + 1615.35x + 29{,}552.18$; $79,223 in 2007 and $127,317 in 2015

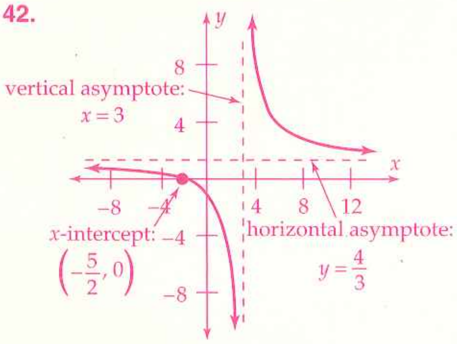
40.



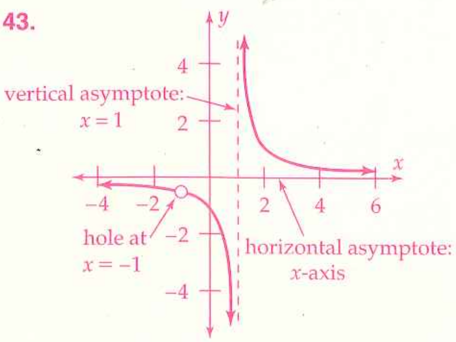
41.



42.



43.



is, the cost of making x statues. Assume C is reasonably accurate whe $x \le 100$.

c. Use C to estimate the cost of making the seventy-first statue.

d. Use C to approximate the average cost per statue when 35 are made, when 75 are made. Recall that the average cost of x statues is $\frac{C(x)}{x}$.

Number of statues	Total cost
0	$2,000
10	2,519
20	2,745
30	2,938
40	3,021
50	3,117
60	3,269
70	3,425

39. The following table gives the estimated cost of a college education at a public institution. Costs include tuition, fees, books, and room and boarc for four years.

a. Sketch a scatter plot of the data (with $x = 0$ corresponding to 1990).

b. Use quartic regression to find a function C that models the data. Estimate the cost of a college education in 2007 and in 2015.

Enrollment Year	Costs
1998	$46,691
2000	52,462
2002	58,946
2004	66,232
2006	74,418

Enrollment Year	Costs
2008	$ 83,61
2010	93,95
2012	105,56
2014	118,61

Source: Teachers Insurance and Annuity Association College Retirement Equities Fund

Section 4.4

In Exercises 40–43, sketch a complete graph of the function. Label the x-intercepts, all local extrema, holes, and asymptotes.

40. $g(x) = \frac{-2}{x + 4}$

41. $h(x) = \frac{3 - x}{x - 2}$

42. $k(x) = \frac{4x + 10}{3x - 9}$

43. $f(x) = \frac{x + 1}{x^2 - 1}$

In Exercises 44 and 45, list all asymptotes of the graph of the function.

44. $f(x) = \dfrac{x^2 - 1}{x^3 - 2x^2 - 5x + 6}$ **45.** $g(x) = \dfrac{x^4 - 6x^3 + 2x^2 - 6x + 2}{x^2 - 3}$

In Exercises 46–49, find a viewing window (or windows) that shows a complete graph of the function. Be alert for hidden behavior.

46. $f(x) = \dfrac{x - 3}{x^2 + x - 2}$ **47.** $g(x) = \dfrac{x^2 - x - 6}{x^3 - 3x^2 + 3x - 1}$

48. $h(x) = \dfrac{x^4 + 4}{x^4 - 99x^2 - 100}$ **49.** $k(x) = \dfrac{x^3 - 2x^2 - 4x + 8}{x - 10}$

50. Which of these statements is true about the graph of

$$f(x) = \frac{(x - 1)(x + 3)}{(x^2 + 1)(x^2 - 1)}?$$ d

a. The graph has two vertical asymptotes.
b. The graph touches the x-axis at $x = 3$.
c. The graph lies above the x-axis when $x < -1$.
d. The graph has a hole at $x = 1$.
e. The graph has no horizontal asymptotes.

Section 4.5

In Exercises 51–58, solve the equation in the complex number system.

51. $x^2 + 3x + 10 = 0$ $x = \dfrac{-3 \pm i\sqrt{31}}{2}$ **52.** $x^2 + 2x + 5 = 0$ $x = -1 \pm 2i$

53. $5x^2 + 2 = 3x$ $x = \dfrac{3 \pm i\sqrt{31}}{10}$ **54.** $-3x^2 + 4x - 5 = 0$ $x = \dfrac{2}{3} \pm \dfrac{\sqrt{11}}{3}i$

55. $3x^4 + x^2 - 2 = 0$ **56.** $8x^4 + 10x^2 + 3 = 0$

57. $x^3 + 8 = 0$ **58.** $x^3 - 27 = 0$

59. One zero of $x^4 - x^3 - x^2 - x - 2$ is i. Find all zeros. $\pm i, 2, -1$

60. One zero of $x^4 + x^3 - 5x^2 + x - 6$ is i. Find all zeros. $\pm i, -3, 2$

61. Give an example of a fourth-degree polynomial with real coefficients whose zeros include 0 and $1 + i$.
Possible answer: $f(x) = x^4 - 2x^3 + 2x^2$

62. Find a fourth-degree polynomial f whose only zeros are $2 + i$ and $2 - i$ such that $f(-1) = 50$.
$f(x) = \frac{1}{2}(x^4 - 8x^3 + 26x^2 - 40x + 25)$

Section 4.5.A

63. Find the orbit of 1 for $f(z) = \left(\dfrac{3}{5} + \dfrac{4}{5}i\right)z$.

64. Find the orbit of 0 for $f(z) = z^2 + c$ using the following values of c. State whether c is in the Mandelbrot set.

a. $c = -1$ **b.** $c = -0.5 + 0.6i$ **c.** $c = 0.3 + 0.5i$

Section 4.6

Factor each of the following over the set of real numbers and over the set of complex numbers.

65. $x^3 - 6x^2 + 11x - 6$ **66.** $x^3 + 3x^2 + 3x + 2$

67. $x^4 - x^3 - x^2 - x - 2$ **68.** $2x^3 + 3x^2 + 9x + 4$

69. $x^4 + 2x^2 + 1$ **70.** $9x^5 + 30x^4 + 43x^3 + 114x^2 + 28x - 24$

44. vertical asymptotes: $x = 3$ and $x = -2$; horizontal asymptote: $y = 0$; hole at $\left(1, -\frac{1}{3}\right)$

45. vertical asymptotes: $x = \pm\sqrt{3}$; parabolic asymptote: $y = x^2 - 6x + 5$

46. Use $-4.7 \le x \le 4.7$ and $-5 \le y \le 5$. For hidden behavior use $2 \le x \le 30$ and $-0.2 \le y \le 0.1$.

47. Use $-4.7 \le x \le 4.7$ and $-10 \le y \le 10$. vertical asymptote $x = 1$; horizontal asymptote $y = 0$; x-intercepts at $x = -2$, $x = 3$; for hidden behavior use $-15 \le x \le 10$, $-0.5 \le y \le 0.5$

48. Use $-18.8 \le x \le 18.8$ and $-8 \le y \le 8$.

49. Use $-30 \le x \le 30$ and $-1000 \le y \le 1000$. For hidden behavior use $-7 \le x \le 7$, $-5 \le y \le 5$.

55. $x = \pm\sqrt{\frac{2}{3}}$ or $\pm i$

56. $x = \pm\frac{1}{\sqrt{2}}i$ or $\pm\frac{\sqrt{3}}{2}i$

57. $x = -2$ or $1 \pm i\sqrt{3}$

58. $x = 3$ or $-\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$

63. a fixed orbit of one point: $\left(\frac{3}{5}, \frac{4}{5}\right)$

64. a. $f^1(0) = -1$, $f^2(0) = 0$, $f^3(0) = -1$, $f^4(0) = 0, \ldots$ The orbit has a period of 2; therefore, -1 is in the Mandelbrot set.

b. The orbit has a period of 6 that is approximately $-0.5 + 0.6i$, -0.61, $-0.13 + 0.6i$, $-0.84 + 0.45i$, $0.01 - 0.15i$, $-0.52 + 0.60i$; therefore, $-0.5 + 0.6i$ is in the Mandelbrot set.

c. The orbit has a period of 5 that is approximately $0.3 + 0.5i$, $0.14 + 0.8i$, $-0.32 + 0.72i$, $-0.12 + 0.36i$, $0.31 + 0.49i$; therefore, $0.3 + 0.5i$ is in the Mandelbrot set.

65. $(x - 1)(x - 2)(x - 3)$

6. $(x + 2)(x^2 + x + 1)$;
$(x + 2)\left(x - \left(\frac{-1 + i\sqrt{3}}{2}\right)\right) \cdot \left(x - \left(\frac{-1 - i\sqrt{3}}{2}\right)\right)$

7. $(x + 1)(x - 2)(x^2 + 1)$;
$(x + 1)(x - 2)(x + i)(x - i)$

68. $(2x + 1)(x^2 + x + 4)$;
$(2x + 1)\left(x - \left(\frac{-1 + i\sqrt{15}}{2}\right)\right) \cdot \left(x - \left(\frac{-1 - i\sqrt{15}}{2}\right)\right)$

69. $(x^2 + 1)(x^2 + 1)$;
$(x + i)(x - i)(x + i)(x - i)$

70. $(x + 3)(3x + 2)(3x - 1)(x^2 + 4)$;
$(x + 3)(3x + 2)(3x - 1)(x + 2i)(x - 2i)$