CHAPTER 3

Study Guide: Review

Objective: Help students organize and review key concepts and skills presented in Chapter 3.

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Answers

1. dependent
2. elimination
3. system of linear inequalities; feasible region
4. three-dimensional coordinate system; ordered triple
5. consistent
6. (5, 10)
7. (4, 2)
8. (−4, −1)
9. (0, −2)
10. independent; one solution
11. dependent; infinitely many solutions
12. inconsistent; no solution
13. independent; 1 solution
14. 3 locks

CHAPTER 3

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Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A consistent and ___?___ system has infinitely many solutions.
2. ___?___ involves adding or subtracting equations to get rid of one of the variables in a system.
3. In a linear programming problem, the solution to the ___?___ can be graphed as a(n) ___?___.
4. Each point in a(n) ___?___ can be represented by a(n) ___?___.
5. A(n) ___?___ system is a set of equations or inequalities that has at least one solution.

3-1 Using Graphs and Tables to Solve Linear Systems *(pp. 182–189)*

EXAMPLES

■ Solve $\begin{cases} x + y = 3 \\ 3x - 6y = -9 \end{cases}$ by using a graph and a table.

Solve each equation for *y*.

$$\begin{cases} y = -x + 3 \\ y = \frac{1}{2}x + \frac{3}{2} \end{cases}$$

Make a table of values.

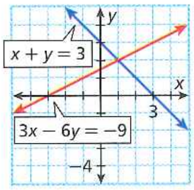
$y = -x + 3$

x	*y*
0	3
1	2
4	1

$y = \frac{1}{2}x + \frac{3}{2}$

x	*y*
0	1.5
1	2
4	2.5

Graph the lines.



The solution is (1, 2).

EXERCISES

Solve each system by using a graph and a table.

6. $\begin{cases} y = 2x \\ 3x - y = 5 \end{cases}$
7. $\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$
8. $\begin{cases} x - 6y = 2 \\ 2x - 5y = -3 \end{cases}$
9. $\begin{cases} x - 3y = 6 \\ 3x - y = 2 \end{cases}$

Classify each system and determine the number of solutions.

10. $\begin{cases} y = x - 7 \\ x + 9y = 16 \end{cases}$
11. $\begin{cases} \frac{1}{2}x + 2y = 3 \\ x + 4y = 6 \end{cases}$
12. $\begin{cases} 5x - 10y = 8 \\ x - 2y = 4 \end{cases}$
13. $\begin{cases} 4x - 3y = 21 \\ 2x - 2y = 10 \end{cases}$
14. **Security** A locksmith charges \$25 to make a house call and \$15 for each lock that is re-keyed. Another locksmith charges \$10 to make a house call and \$20 for each lock that is re-keyed. For how many locks will the total costs be the same?

3-2 Using Algebraic Methods to Solve Linear Systems *(pp. 190–197)*

EXAMPLES

- **Use substitution to solve** $\begin{cases} y = x + 6 \\ 4x - 5y = -18 \end{cases}$.

$4x - 5(x + 6) = -18$ *Substitute for y.*
$4x - 5x - 30 = -18 \rightarrow x = -12$

Substitute the x-value into either equation.

$y = x + 6 \rightarrow y = (-12) + 6 \rightarrow y = -6$

The solution to the system is $(-12, -6)$.

- **Use elimination to solve** $\begin{cases} 7x - 2y = 2 \\ 3x + 4y = 30 \end{cases}$.

Multiply the first equation by 2 to eliminate y.

$\begin{cases} 7x - 2y = 2 \\ 3x + 4y = 30 \end{cases} \rightarrow \begin{matrix} 2(7x - 2y = 2) \\ 3x + 4y = 30 \end{matrix} \rightarrow \begin{matrix} 14x - 4y = 4 \\ 3x + 4y = 30 \\ \hline 17x \quad = 34 \\ x \quad = 2 \end{matrix}$

Add the equations.
First part of the solution

Substitute the x-value into either equation.

$3x + 4y = 30 \rightarrow 3(2) + 4y = 30$

$\rightarrow y = 6$ *Second part of the solution*

The solution to the system is $(2, 6)$.

EXERCISES

Use substitution to solve each system of equations.

15. $\begin{cases} y = 3x \\ 2x - 3y = -7 \end{cases}$

16. $\begin{cases} y = x - 1 \\ 4x - y = 19 \end{cases}$

17. $\begin{cases} 4x - y = 0 \\ 6x - 3y = 12 \end{cases}$

18. $\begin{cases} 5x = -10y \\ 8x - 4y = 40 \end{cases}$

Use elimination to solve each system of equations.

19. $\begin{cases} 4x + 5y = 41 \\ 7x + 5y = 53 \end{cases}$

20. $\begin{cases} -4x - y = -16 \\ -4x - 5y = -32 \end{cases}$

21. $\begin{cases} 2x - y = 8 \\ x + 2y = 9 \end{cases}$

22. $\begin{cases} 9x - 5y = 13 \\ 4x - 6y = 2 \end{cases}$

23. Mixtures A popular mixture of potpourri includes pine needles and lavender. If pine needles cost \$1.50 per ounce and lavender costs \$4.00 per ounce, how much of each ingredient should be mixed to make 80 oz of the potpourri that is worth \$200?

3-3 Solving Systems of Linear Inequalities *(pp. 199–204)*

EXAMPLE

- **The combined annual sales for a company's two divisions was almost \$12 million. One of the divisions accounted for at least 75% of the total sales. Write and graph a system of inequalities that can be used to determine the possible combinations of sales for both divisions of the company.**

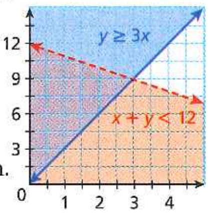
Let x be one division, and let y be the other division with 75% of the sales.

Write the system of inequalities.

$\begin{cases} x + y < 12 \\ y \geq 0.75(x + y) \end{cases} \rightarrow \begin{cases} x + y < 12 \\ y \geq 3x \end{cases}$ *dashed line* / *solid line*

Graph the boundary lines, and shade accordingly. Notice also that $x > 0$ and $y > 0$.

The overlapping region is the solution for the system.



EXERCISES

Graph each system of inequalities.

24. $\begin{cases} y + 1 > 4x \\ y \leq x + 1 \end{cases}$

25. $\begin{cases} y - 3x < 3 \\ 3y \geq x + 3 \end{cases}$

Graph the system of inequalities and classify the figure created by the solution region.

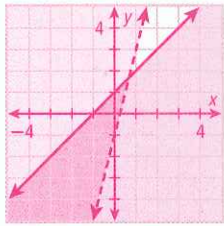
26. $\begin{cases} y \leq -x + 2 \\ x > -1 \\ y > -1 \end{cases}$

27. $\begin{cases} y \geq 2x \\ y < 4 \\ y > 2 \\ y \leq \frac{1}{2}x + 4 \end{cases}$

28. Business A coffee shop wants to make a maximum of 120 lb of a coffee mixture that costs less than \$10/lb. The shop will mix coffee that is sold at \$8/lb with coffee sold at \$11.50/lb. Write and graph a system of inequalities that shows the possible mixtures of the two coffee types.

Answers

15. (1, 3)
16. (6, 5)
17. (−2, −8)
18. (4, −2)
19. (4, 5)
20. (3, 4)
21. (5, 2)
22. (2, 1)
23. 48 oz pine; 32 oz lavender
24.



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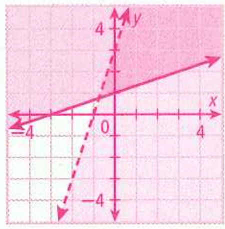
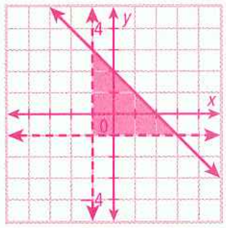
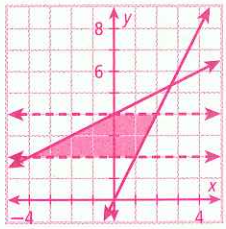


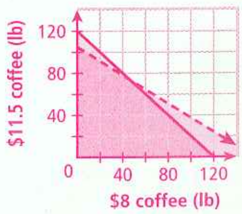
26. right triangle



27 trapezoid



28. $\begin{cases} x + y \leq 120 \\ 8x + 11.5y < 1200 \end{cases}$



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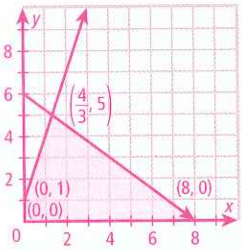
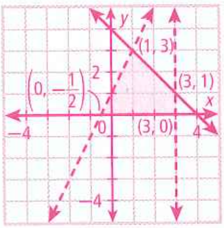
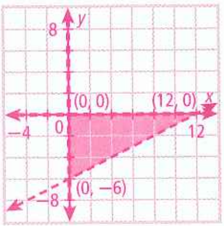
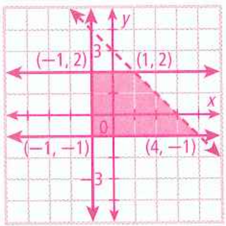
Answers

29. 

30. 

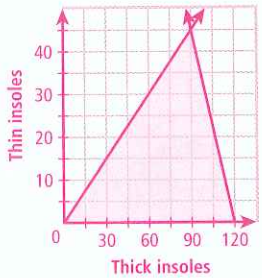
31. 

32. 

33. 58

34. −4.5

35. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 6x + 4y \leq 720 \\ x \geq 2y \end{cases}$



36. $P = 8x + 9y$

37. $1125

38. 25 phones with contracts and 5 without contracts

3-4 Linear Programming *(pp. 205–211)*

EXAMPLE

■ **A café sells cold sandwiches and hot entrées. The range of items sold is shown in the table. The café has never sold more than a total of 125 sandwiches and entrées in one day. If the café makes a profit of $0.75 on each sandwich and $1 on each hot entrée, how many of each item would maximize the café profit?**

Menu Item	Minimum Sold	Maximum Sold
Cold sandwiches	60	80
Hot entrées	40	60

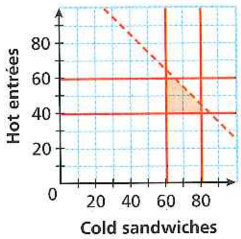
Let x be the number of cold sandwiches, and let y be the number of hot entrées.

Write the constraints.

$$\begin{cases} 60 \leq x \leq 80 & \textit{Number of sandwiches} \\ 40 \leq y \leq 60 & \textit{Number of hot entrées} \\ x + y < 125 & \textit{Number of items sold} \end{cases}$$

Graph the feasible region and identify vertices.

The feasible region has five vertices at (60, 40), (60, 60), (65, 60), (80, 45), and (80, 40).



Write the objective function.

The objective function is $P = 0.75x + y$.
$P(0, 0) = 18(0) + 25(0) = 0$

Evaluate the objective function at each vertex.

$P(60, 40) = 0.75(60) + 40 = 85$
$P(60, 60) = 0.75(60) + 60 = 105$
$P(65, 60) = 0.75(65) + 60 = 108.75$
$P(80, 45) = 0.75(80) + 45 = 105$
$P(80, 40) = 0.75(80) + 40 = 100$

The objective function is maximized at (65, 60). The maximum profit of $108.75 is obtained when 65 cold sandwiches and 60 hot entrées are sold.

EXERCISES

Graph each feasible region.

29. $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 3x + 1 \\ y \leq -\frac{3}{4}x + 6 \end{cases}$

30. $\begin{cases} x < 3 \\ y \geq 0 \\ y < 2x + 1 \\ y \leq -x + 4 \end{cases}$

31. $\begin{cases} x > 0 \\ y < 0 \\ y > \frac{1}{2}x - 6 \end{cases}$

32. $\begin{cases} x \leq 2 \\ y \geq -1 \\ x \geq -1 \\ y \leq -x + 3 \end{cases}$

Maximize or minimize each objective function.

33. Maximize $P = 6x + 10y$ for the constraints from Exercise 29.

34. Minimize $P = 14x + 9y$ for the constraints from Exercise 30.

Manufacturing A shoe insole company produces two models of insoles: an extra thick insole for sports shoes and a thinner insole for dress shoes. The thick insole requires 6 min of manufacturing time and generates a profit of $8. The thin insole requires 4 min of manufacturing time and generates a profit of $9. The manufacturing line runs at most 12 h a day, or 720 min. Because of demand, the company manufactures at least twice as many thick insoles as thin insoles.

35. Write the constraints, and graph the feasible region.

36. Write the objective function for the company's profit.

37. What is the maximum profit that can be generated in one day?

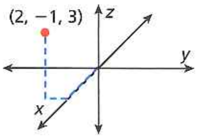
38. **Sales** Each day, a cell phone stand sells between 10 and 25 cell phones with new service contracts, and between 5 and 10 cell phones without contracts. The stand never sells more than 30 new cell phones per day. The cell phone stand makes a commission of $35 for each phone with a contract and $5 for each phone without a contract. How many of each option would maximize the stand's profit?

3-5 Linear Equations in Three Dimensions *(pp. 214–218)*

EXAMPLES

- **Graph (2, −1, 3) in three-dimensional space.**

From the origin, move 2 units forward along the x-axis, 1 unit left, and 3 units up.



- **Graph the linear equation $3x + 6y - z = -6$ in three-dimensional space.**

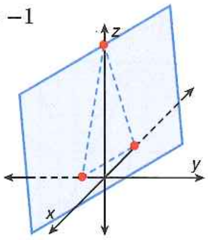
Find the intercepts.

x-intercept: $3x = -6 \rightarrow x = -2$

y-intercept: $6y = -6 \rightarrow y = -1$

z-intercept: $-z = -6 \rightarrow z = 6$

Plot the points (−2, 0, 0), (0, −1, 0), and (0, 0, 6). Sketch a plane through the three points.



EXERCISES

Graph each point in three-dimensional space.

39. (−1, 0, 3) **40.** (2, −2, 1)

41. (0, −1, 1) **42.** (3, 1, 0)

Graph each linear equation in three-dimensional space.

43. $x - 3y + 2z = 6$ **44.** $2x - 4y - 2z = 4$

45. $-x + y - 5z = 5$ **46.** $3x + 2y + z = -6$

47. Consumer Economics Lee has $35 to purchase a combination of drinks, pizza, and ice cream for a party. Each drink costs $2, each pizza costs $9, and each quart of ice cream costs $4. Write a linear equation in three variables to represent this situation.

3-6 Solving Linear Systems in Three Variables *(pp. 220–226)*

EXAMPLES

- **Use elimination to solve** $\begin{cases} 3x + 2y - z = -1 \\ x + 3y - z = -10 \\ 2x - y - 3z = -3 \end{cases}$.

First, eliminate z to obtain a 2-by-2 system.

$$\begin{array}{r} 3x + 2y - z = -1 \\ \underline{-(x + 3y - z = -10)} \\ 2x - y \quad = 9 \end{array} \qquad \begin{array}{r} 3(x + 3y - z = -10) \\ \underline{2x - y - 3z = -3} \\ x + 10y \quad = -27 \end{array}$$

The resulting 2-by-2 system is $\begin{cases} 2x - y = 9 \\ x + 10y = -27 \end{cases}$.

Eliminate x.

$$\begin{array}{r} 2x - \quad y = 9 \\ \underline{-2(x + 10y = -27)} \\ -21y = 63 \rightarrow y = -3 \end{array}$$

Substitute to solve for x and then z.

$2x - y = 9 \rightarrow 2x - (-3) = 9 \rightarrow x = 3$

$3x + 2y - z = -1 \rightarrow 3(3) + 2(-3) - z = -1 \rightarrow z = 4$

The solution to the system is (3, −3, 4).

EXERCISES

Use elimination to solve each system of equations.

48. $\begin{cases} x + 3y + 2z = 13 \\ 2x + 2y - z = 3 \\ x - 2y + 3z = 6 \end{cases}$

49. $\begin{cases} x + y + z = 2 \\ 3x + 2y - z = -1 \\ 3x - y = 4 \end{cases}$

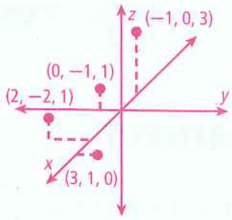
Classify each system as consistent or inconsistent, and determine the number of solutions.

50. $\begin{cases} x + y + z = -2 \\ -x + 2y - 5z = 4 \\ 3x + 3y + 3z = 5 \end{cases}$

51. $\begin{cases} -x - y + 2z = -3 \\ 4x + 4y - 8z = 12 \\ 2x + y - 3z = -2 \end{cases}$

Answers

39–42.



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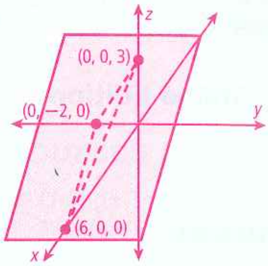

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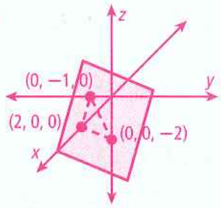

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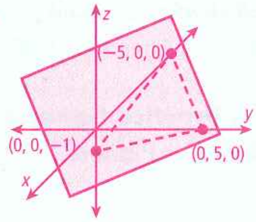

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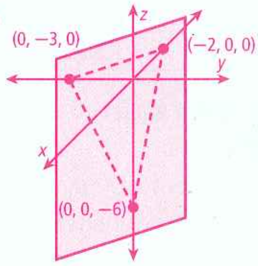

47. $2d + 9p + 4c = 35$, where d = drinks, p = pizza, c = ice cream

48. (1, 2, 3)

49. (1, −1, 2)

50. inconsistent, no solution

51. dependent, infinitely many solutions