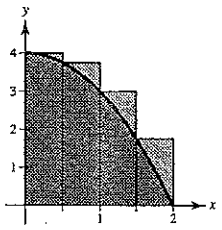
CHAPTER 1

Limits and Their Properties

1.1 A Preview of Calculus

1. Use the rectangles to approximate the area of the region in the first quadrant bounded by the graphs of $y = 4 - x^2$, $y = 0$, and $x = 0$.

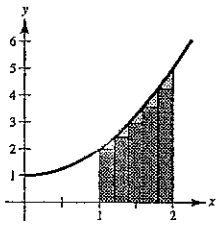


(a) $\frac{16}{3}$ (b) $\frac{25}{4}$ (c) $\frac{17}{4}$

(d) $\frac{7}{4}$ (e) None of these

2—M—Answer: b

2. Use the rectangles to approximate the area of the region in the first quadrant bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 1$, and $x = 2$.



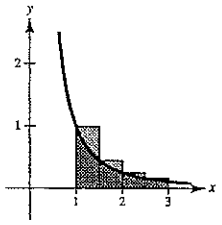
(a) 1.24 (b) 3.64 (c) $\frac{10}{3}$

(d) 3.04 (e) None of these

2—M—Answer: d

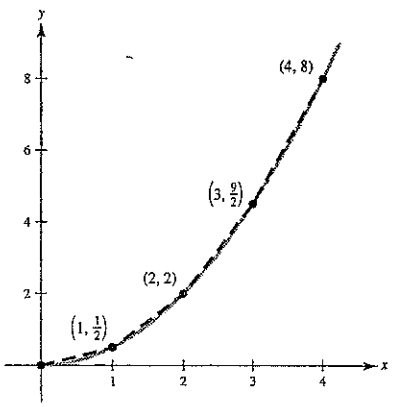
3. Use the rectangles to approximate the area of the region in the first quadrant bounded by the graphs of

$$y = \frac{1}{x^2},\ y = 0,\ x = 1,\ \text{and}\ x = 3.$$



2—O—Answer: $\frac{1669}{1800} = 0.927\overline{2}$

4. Approximate the length of the curve from the point $(0, 0)$ to the point $(4, 8)$ by finding the sum of the lengths of the four line segments as shown in the figure. Round your answer to three decimal places.

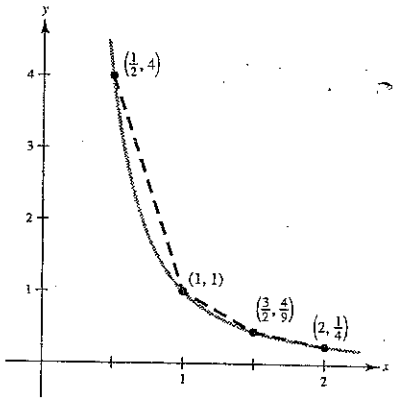


(a) 19.380 (b) 7.629 (c) 9.683

(d) 9.294 (e) None of these

2—M—Answer: c

5. Approximate the length of the curve from the point $\left(\frac{1}{2}, 4\right)$ to the point $\left(2, \frac{1}{4}\right)$ by finding the sum of the lengths of the three line segments as shown in the figure. Round your answer to three decimal places.

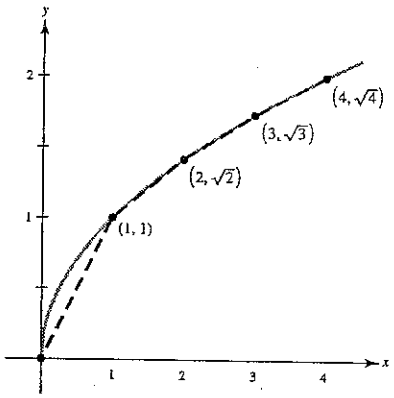


(a) 4.354 (b) 1.5 (c) 4.325

(d) 4.523 (e) None of these

2—M—Answer: c

6. Approximate the length of the curve from the point (0, 0) to the point $(4, \sqrt{4})$ by finding the sum of the lengths of the four line segments as shown in the figure. Round your answer to three decimal places.



2—O—Answer: 4.581

1.2 Finding Limits Graphically and Numerically

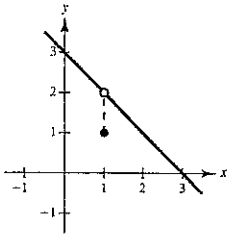
1. Use the graph to find $\lim_{x \to 1} f(x)$ for

$$f(x) = \begin{cases} 3 - x, & x \neq 1 \\ 1, & x = 1 \end{cases}$$

(a) 2 (b) 1 (c) $\frac{3}{2}$

(d) Does not exist (e) None of these

1—M—Answer: a

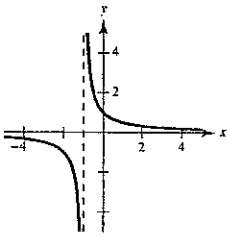


2. Use the graph to find $\lim_{x \to -1} f(x)$ for $f(x) = \dfrac{1}{x + 1}$.

(a) 0 (b) 1 (c) ∞

(d) Does not exist (e) None of these

1—M—Answer: d

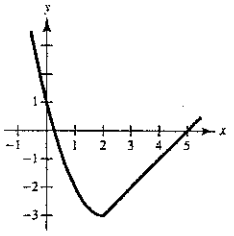


3. Use the graph to estimate $\lim_{x \to 2} f(x)$.

(a) Limit does not exist (b) 0 (c) -3

(d) 2 (e) None of these

1—M—Answer: c

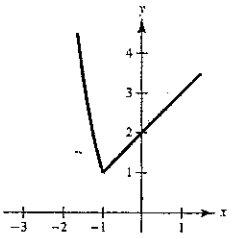


4. Use the graph to estimate $\lim_{x \to -1} f(x)$.

(a) 1 (b) -1 (c) 0

(d) Limit does not exist (e) None of these

1—M—Answer: a

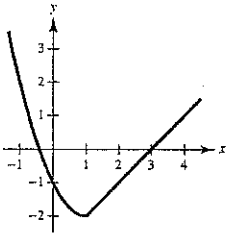


5. Use the graph to estimate $\lim_{x \to 1} f(x)$.

(a) 1 (b) Limit does not exist

(c) -2 (d) 0 (e) None of these

1—M—Answer: c



13. Let $f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ \frac{3}{2}x^2, & x > 1 \end{cases}$. Find the limit: $\lim_{x \to 1} f(x)$.

(a) 2 (b) $\frac{3}{2}$ (c) 2 and $\frac{3}{2}$

(d) The limit does not exist. (e) None of these

1—M—Answer: d

14. Let $f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ 2x^2, & x > 1 \end{cases}$. Find the limit: $\lim_{x \to 1} f(x)$.

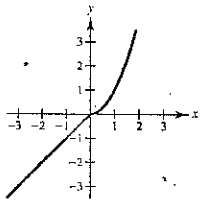
(a) 2 (b) 1 (c) 0

(d) The limit does not exist. (e) None of these

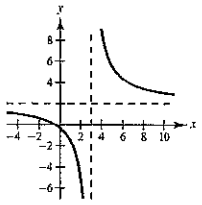
1—M—Answer: a

15. Let $f(x) = \begin{cases} x, & x \leq 0 \\ x^2, & x > 0 \end{cases}$. Sketch a graph of f and find the limit: $\lim_{x \to 0} f(x)$.

1—O—Answer: 0



16. Use the graph to find $\lim_{x \to 3} f(x)$ (if it exists) for $f(x) = \dfrac{2x + 1}{x - 3}$.



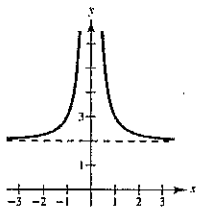
(a) 2 (b) 3

(c) $-\frac{1}{2}$ (d) The limit does not exist.

(e) None of these

1—M—Answer: d

17. Use the graph to find $\lim_{x \to 0} f(x)$ (if it exists) for $f(x) = \dfrac{1}{x^2} + 2$.



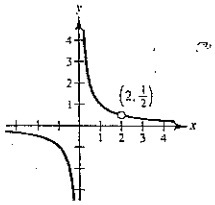
(a) The limit does not exist. (b) 2

(c) 0 (d) 3

(e) None of these

1—M—Answer: a

18. Use the graph to find $\lim_{x \to 2} f(x)$ (if it exists) for $f(x) = \dfrac{x-2}{x^2 - 2x}$.



(a) The limit does not exist. (b) 2

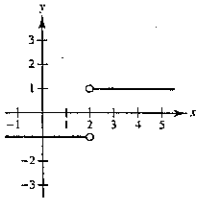
(c) $\frac{1}{2}$ (d) 0

(e) None of these.

1—M—Answer: c

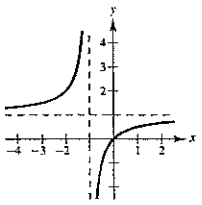
19. Sketch the graph of a function $y = f(x)$ such that $\lim_{x \to 2} f(x)$ does not exist.

1—O—Answer: Answers will vary.
One such answer is shown at the right.



20. Sketch the graph of a function $y = f(x)$ such that $\lim_{x \to -1} f(x)$ does not exist.

1—O—Answer: Answers will vary.
One such answer is shown at the right.



1.3 Evaluating Limits Analytically

1. Find the limit: $\lim_{x \to 2} (3x^2 + 5)$.

(a) 41 (b) 17 (c) 11

(d) 0 (e) None of these

1—M—Answer: b

2. Find the limit: $\lim_{x \to -3} (-2x^2 + 1)$.

(a) 37 (b) 19 (c) -17

(d) $\pm\sqrt{2}$ (e) None of these

1—M—Answer: c

3. Find the limit: $\lim_{x \to 2} (2x^2 - 6x + 1)$.

1—O—Answer: -3

4. Find the limit: $\lim_{x \to -2} (x^2 + 4x - 3)$.

1—O—Answer: -7

5. Find the limit: $\lim_{x \to -1} \dfrac{x^2 + 3x + 2}{x^2 + 1}$.

(a) 0 (b) ∞ (c) -1

(d) Does not exist (e) None of these

1—M—Answer: a

6. Find the limit: $\lim_{x \to -1} \dfrac{x^2 + 2x + 3}{x^2 + 1}$.

(a) 0 (b) 1 (c) ∞

(d) Does not exist (e) None of these

1—M—Answer: b

7. Find the limit: $\lim_{x \to 2} \dfrac{x^2 - 4x + 1}{x^2 + 4}$.

1—O—Answer: $-\frac{3}{8}$

8. Find the limit: $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x - 3}$.

1—O—Answer: 0

9. Find the limit: $\lim_{x \to 1} \dfrac{3x^3 - 4x^2 - 5x + 2}{x^2 - x - 2}$.

1—O—Answer: 2

10. Find the limit: $\lim_{x \to 1} \dfrac{x^2 - x - 2}{x - 3}$.

1—O—Answer: 1

11. Find the limit: $\lim_{x \to 3} \sqrt{x^2 - 4}$.

(a) 1 (b) 5 (c) -1

(d) $\sqrt{5}$ (e) None of these

1—M—Answer: d

12. Find the limit: $\lim_{x \to 3} (9 - x^2)^2$.

(a) 0 (b) $\sqrt{6}$ (c) $3\sqrt{2}$

(d) Does not exist (e) None of these

2—M—Answer: a

13. Find the limit: $\lim_{x \to 2} \sqrt{4x^2 + 9}$.

1—O—Answer: 5

14. Find the limit: $\lim_{x \to 1} f(x)$ for $f(x) = \begin{cases} x^2 + 4, & x \neq 1 \\ 2, & x = 1 \end{cases}$.

1—O—Answer: 5

15. Find the limit: $\lim_{x \to \pi} \dfrac{1 - \cos x}{x}$.

1—O—Answer: $\dfrac{2}{\pi} \approx 0.6366$

16. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c} \frac{f(x)}{g(x)}$.

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $-\frac{3}{4}$

(d) -3 (e) None of these

1—M—Answer: c

17. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c} [f(x)g(x)]$.

(a) $\frac{1}{6}$ (b) $-\frac{1}{3}$ (c) 1

(d) Does not exist (e) None of these

1—M—Answer: b

18. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c} [f(x) - g(x)]$.

1—O—Answer: $-\frac{7}{6}$

19. If $\lim_{x\to c} f(x) = -6$ and $\lim_{x\to c} g(x) = 3$, then find $\lim_{x\to c} ([f(x)]^2 - 2f(x)g(x) + [g(x)]^2)$.

(a) 63 (b) 81 (c) 45

(d) -9 (e) None of these

2—M—Answer: b

20. If $\lim_{x\to c} f(x) = 2$ and $\lim_{x\to c} g(x) = 6$, then find $\lim_{x\to c} ([f(x)]^2 - 2f(x)g(x) + [g(x)]^2)$.

(a) 40 (b) -4 (c) 16

(d) 28 (e) None of these

2—M—Answer: c

21. If $\lim_{x\to c} f(x) = 3$ and $\lim_{x\to c} g(x) = -5$, then find $\lim_{x\to c} ([f(x)]^2 - 2f(x)g(x) + [g(x)]^2)$.

(a) 64 (b) 49 (c) 34

(d) 8 (e) None of these

2—M—Answer: a

22. If $\lim_{x\to c} f(x) = 9$ and $\lim_{x\to c} g(x) = 6$, then find $\lim_{x\to c} ([f(x)]^2 - 2f(x)g(x) + [g(x)]^2)$.

(a) 3 (b) 9 (c) 63

(d) 117 (e) None of these

2—M—Answer: b

23. Find the limit: $\lim_{x\to 2} \sec \frac{\pi x}{3}$.

(a) -2 (b) $\frac{2}{\sqrt{3}}$ (c) $-\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$ (e) None of these

1—M—Answer: a

24. Find the limit: $\lim_{x\to 5} \csc \frac{\pi x}{4}$.

(a) 1 (b) -1 (c) $-\sqrt{2}$

(d) $-\frac{1}{\sqrt{2}}$ (e) None of these

1—M—Answer: c

25. Find the limit: $\lim_{x\to 5} \cot \frac{\pi x}{6}$.

1—O—Answer: $-\sqrt{3}$

26. Find the limit: $\lim_{x\to \pi/2} \frac{\sin x}{x}$.

(a) 0 (b) $\frac{2}{\pi}$ (c) $-\frac{\pi}{2}$

(d) $\frac{2\sqrt{2}}{\pi}$ (e) None of these

1—M—Answer: b

27. Find the limit: $\lim_{x\to \pi} \tan 5x$.

(a) 5 (b) -5 (c) $-\frac{1}{\sqrt{3}}$

(d) 0 (e) None of these

1—M—Answer: d

28. Find the limit: $\lim_{x\to \pi} \frac{x}{\cos x}$.

1—O—Answer: $-\pi$

37. Find the limit: $\lim_{x \to -4} \dfrac{x^2 + 11x + 28}{x + 4}$.

(a) 7 (b) 3 (c) 0

(d) Limit does not exist (e) None of these

1—M—Answer: b

38. Find the limit: $\lim_{x \to 5} \dfrac{x^2 - 3x - 10}{x - 5}$.

(a) 2 (b) Limit does not exist (c) 0

(d) 7 (e) None of these

1—M—Answer: d

39. Find the limit: $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x}$.

(a) 1 (b) 0 (c) ∞

(d) Does not exist (e) None of these

1—M—Answer: b

40. Find the limit: $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x - 1}$.

1—O—Answer: 3

41. Find the limit: $\lim_{x \to 2} \dfrac{3x^3 - 4x^2 - 5x + 2}{x^2 - x - 2}$.

1—O—Answer: 5

42. Find the limit: $\lim_{x \to 3} \dfrac{x - 3}{|x - 3|}$.

(a) 0 (b) 1 (c) 3

(d) Does not exist (e) None of these

2—M—Answer: d

43. Find the limit: $\lim_{x \to 2} \dfrac{x - 2}{|x - 2|}$.

(a) 0 (b) 1 (c) 2

(d) Does not exist (e) None of these

2—M—Answer: d

44. Find the limit: $\lim_{x \to 1} \dfrac{x - 1}{|x - 1|}$.

2—O—Answer: Does not exist

45. Find the limit: $\lim_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$.

(a) 0 (b) $\frac{1}{4}$ (c) ∞

(d) 1 (e) None of these

2—M—Answer: b

46. Find the limit: $\lim_{x \to 0} \dfrac{\sqrt{x + 9} - 3}{x}$.

(a) 0 (b) 1 (c) ∞

(d) $\frac{1}{3}$ (e) None of these

1—M—Answer: e

47. Find the limit: $\lim_{x \to 1} \dfrac{1 - \sqrt{2x^2 - 1}}{x - 1}$.

1—O—Answer: -2

48. Find the limit: $\lim_{\Delta x \to 0} \dfrac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$.

1—O—Answer: $\dfrac{1}{2\sqrt{x}}$

49. Find the limit: $\lim_{\Delta x \to 0} \dfrac{\sqrt{(x + \Delta x) - 8} - \sqrt{x - 8}}{\Delta x}$.

(a) $\sqrt{x - 8}$ (b) 0 (c) None of these

(d) $\dfrac{1}{2\sqrt{x - 8}}$ (e) 1

1—M—Answer: d

50. Find the limit: $\lim_{\Delta x \to 0} \dfrac{\sqrt{(x + \Delta x) + 2} - \sqrt{x + 2}}{\Delta x}$.

(a) $\sqrt{x + 2}$ (b) None of these (c) 0

(d) $\dfrac{1}{2\sqrt{x + 2}}$ (e) 1

1—M—Answer: d

51. Find the limit: $\lim_{\Delta x \to 0} \dfrac{\sqrt{(x + \Delta x) - 9} - \sqrt{x - 9}}{\Delta x}$.

(a) None of these (b) $\sqrt{x - 9}$ (c) $\dfrac{1}{2\sqrt{x - 9}}$

(d) 0 (e) 1

1—M—Answer: c

52. Find the limit: $\lim_{x \to 0} \dfrac{x}{\tan x}$.

(a) 0 (b) $\dfrac{\pi}{4}$ (c) 1

(d) Does not exist (e) None of these

1—M—Answer: c

53. Find the limit: $\lim_{x \to 0} \dfrac{x}{\sin 3x}$.

1—O—Answer: $\frac{1}{3}$

54. Find the limit: $\lim_{x \to 0} \dfrac{\sin 5x}{x}$.

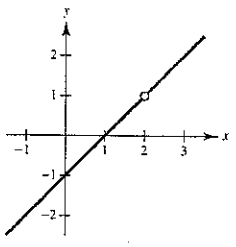
1—O—Answer: 5

55. Find the limit: $\lim_{x \to 0} \dfrac{1 - \cos x}{x}$.

1—O—Answer: 0

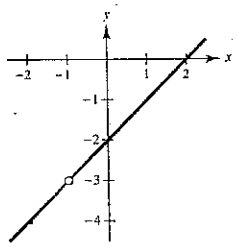
56. Sketch the graph of $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$.

2—O—Answer:

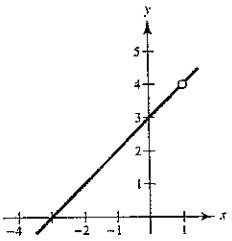


57. Sketch the graph of $f(x) = \dfrac{x^2 - x - 2}{x + 1}$.

2—O—Answer:



58. Match the graph with the correct function.



(a) $f(x) = \dfrac{x + 3}{x - 1}$ (b) $f(x) = x + 3$

(c) $f(x) = \dfrac{x - 1}{x^2 + 2x - 3}$ (d) $f(x) = \dfrac{x^2 + 2x - 3}{x - 1}$

(e) None of these

1—M—Answer: d .

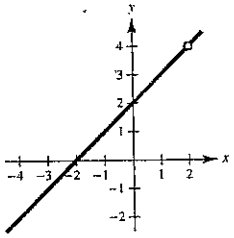
59. Let $f(x) = \dfrac{x^2 - 4}{x - 2}$.

a. Use a graphing utility to graph the function.

b. Use the graph to estimate $\lim_{x \to 2} f(x)$.

c. Find the limit by analytical methods.

2—O—T—Answer: a.



b. 4 **c.** 4

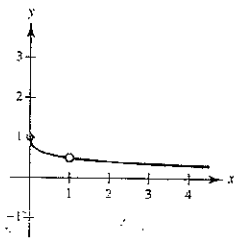
60. Let $f(x) = \dfrac{\sqrt{x} - 1}{x - 1}$.

a. Use a graphing utility to graph $f(x)$.

b. Use the graph to estimate $\lim_{x \to 1} f(x)$.

c. Find the limit by analytical methods.

2—O—T—Answer: a.



b. $\frac{1}{2}$ **c.** $\frac{1}{2}$

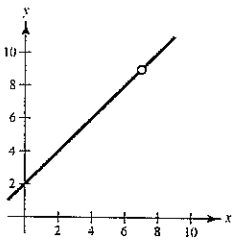
61. Let $f(x) = \dfrac{x^2 - 5x - 14}{x - 7}$.

a. Find $\lim_{x \to 7} f(x)$ (if it exists).

b. Identify another function that agrees with $f(x)$ at all but one point.

c. Sketch the graph of $f(x)$.

2—O—Answer: a. 9 **b.** $g(x) = x + 2$ **c.**



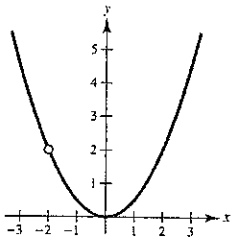
62. Let $f(x) = \dfrac{x^3 + 2x^2}{x + 2}$.

a. Find $\lim_{x \to -2} f(x)$ (if it exists).

b. Identify another function that agrees with $f(x)$ at all but one point.

c. Sketch the graph of $f(x)$.

2—O—Answer: a. 4 **b.** $g(x) = x^2$ **c.**



63. Find the limit: $\lim_{x\to 0} \dfrac{\dfrac{1}{x+3} - \dfrac{1}{3}}{x}$.

(a) $-\frac{1}{9}$ (b) 0 (c) $\frac{1}{9}$

(d) Limit does not exist (e) None of these

2—M—Answer: a

64. Find the limit: $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2 - 2x)}{\Delta x}$.

(a) $-4x$ (b) -2 (c) $2x - 2$

(d) Limit does not exist (e) None of these

2—M—Answer: c

65. Use a graphing utility to find the limit: $\lim_{x\to 0} \dfrac{\sin 4x}{x}$. Then verify your answer analytically.

2—O—T—Answer: 4; $\lim_{x\to 0} \dfrac{\sin 4x}{x} = \lim_{x\to 0} \dfrac{4 \sin 4x}{4x} = 4 \lim_{x\to 0} \dfrac{\sin 4x}{4x} = 4(1) = 4$

1.4 Continuity and One-Sided Limits

1. Find the limit: $\lim_{x \to 2^-} \sqrt{2x - 3}$.

(a) $1, -1$ (b) 1 (c) -1

(d) $\frac{1}{2}$ (e) None of these

1—M—Answer: b

2. Find the limit: $\lim_{x \to 3^+} \sqrt{2x - 5}$.

(a) 1 (b) 0 (c) $2i$

(d) Does not exist (e) None of these

1—M—Answer: a

3. Find the limit: $\lim_{x \to 2^+} \sqrt{2x - 1}$.

1—O—Answer: $\sqrt{3}$

4. Find the limit: $\lim_{x \to 1^+} \sqrt{x - 1}$.

1—O—Answer: 0

5. Find the limit: $\lim_{x \to 1^-} \sqrt{x - 1}$.

1—O—Answer: Does not exist

6. Find the limit: $\lim_{x \to 7^-} \frac{|8x - 56|}{7 - x}$.

(a) 8 (b) -1 (c) 1

(d) -8 (e) None of these

2—M—Answer: a

7. Find the limit: $\lim_{x \to 6^-} \frac{|3x - 18|}{6 - x}$.

(a) -1 (b) 1 (c) 3

(d) -3 (e) None of these

2—M—Answer: c

8. Find the limit: $\lim_{x \to 5^-} \dfrac{|3x - 15|}{5 - x}$.

(a) -3 (b) 3 (c) -1 (d) 1 (e) None of these

2—M—Answer: b

9. Find the limit: $\lim_{x \to 8^-} \dfrac{|7x - 56|}{8 - x}$.

(a) -1 (b) 7 (c) 1 (d) -7 (e) None of these

2—M—Answer: b

10. At which value(s) of x is $f(x) = \dfrac{x^2 - 2x - 3}{x - 2}$ discontinuous?

(a) 2 (b) $-1, 2, 3$ (c) 1 (d) $-1, \frac{3}{2}, 2, 3$ (e) None of these

1—M—Answer: a

11. At which value(s) of x is $f(x) = \dfrac{x - 4}{x^2 - x - 2}$ discontinuous?

(a) 4 (b) $-1, 2, 4$ (c) $-1, 2$

(d) $-1, 2, 4, -2$ (e) None of these

1—M—Answer: c

12. Find the x-values (if any) at which $f(x) = \dfrac{|x|}{x}$ is discontinuous. Are they removable or nonremovable?

2—O—Answer: $x = 0$, nonremovable

13. Find the x-values (if any) at which $f(x) = \dfrac{x^2 - 6x + 5}{x - 6}$ is discontinuous. Are they removable or nonremovable?

1—O—Answer: $x = 6$, nonremovable

14. Find the value(s) of x for which $f(x) = \dfrac{x - 2}{x^2 - 4}$ is discontinuous and label these discontinuities as removable or nonremovable?

1—O—Answer: $x = 2$, removable; $x = -2$, nonremovable

15. Let $f(x) = \dfrac{1}{x + 1}$ and $g(x) = x^2 - 5$. Find all values of x for which $f(g(x))$ is discontinuous.

(a) -1 (b) $-1, +\sqrt{5}$ (c) $+\sqrt{5}$

(d) $-2, 2$ (e) None of these

1—M—Answer: d

16. Let $f(x) = \dfrac{1}{|x|}$ and $g(x) = x - 1$. Find all values of x for which $f(g(x))$ is discontinuous.

(a) 0 (b) 1 (c) 0, 1

(d) $-1, 1$ (e) None of these

1—M—Answer: e

17. Let $f(x) = \dfrac{5}{x - 1}$ and $g(x) = x^4$.

a. Find $f(g(x))$.

b. Find all values of x for which $f(g(x))$ is discontinuous.

1—O—Answer: **a.** $\dfrac{5}{x^4 - 1}$ **b.** $-1, 1$

18. Determine the value of c so that $f(x)$ is continuous on the entire real line when $f(x) = \begin{cases} x - 2, & x \le 5 \\ cx - 3, & x > 5 \end{cases}$

(a) 0 (b) $\frac{6}{5}$ (c) 1

(d) $\frac{5}{6}$ (e) None of these

2—M—Answer: b

19. Determine the value of c so that $f(x)$ is continuous on the entire real line when $f(x) = \begin{cases} x + 3, & x \le -1 \\ 2x - c, & x > -1 \end{cases}$

(a) -4 (b) 4 (c) 0

(d) -1 (e) None of these

2—M—Answer: a

20. Determine the value of c so that $f(x)$ is continuous on the entire real line if $f(x) = \begin{cases} x^2, & x \le 3 \\ c/x, & x > 3 \end{cases}$

2—O—Answer: 27

21. Which of the following statements is *not* true of $f(x) = \sqrt{x^2 - 36}$?

(a) f is continuous on the interval $[6, \infty)$.

(b) f is continuous on the interval $[-6, 6]$.

(c) f is continuous on the interval $(-\infty, -6]$.

(d) f is continuous at $x = 12$.

(e) None of these

1—M—Answer: b

22. Which of the following statements is *not* true of $f(x) = \sqrt{x^2 - 49}$?

(a) f is continuous on the interval $(-\infty, -7]$.

(b) f is continuous at $x = 14$.

(c) f is continuous on the interval $[-7, 7]$.

(d) f is continuous on the interval $[7, \infty)$.

(e) None of these

1—M—Answer: c

23. Which of the following statements is *not* true of $f(x) = \sqrt{x^2 - 81}$?

(a) f is continuous on the interval $[-9, 9]$.

(b) f is continuous on the interval $(-\infty, -9]$

(c) f is continuous on the interval $[9, \infty)$.

(d) f is continuous at $x = 18$.

(e) None of these

1—M—Answer: a

24. Which of the following statements is *not* true of $f(x) = \sqrt{x^2 - 25}$?

(a) f is continuous at $x = 10$.

(b) f is continuous on the interval $(-\infty, -5]$.

(c) f is continuous on the interval $[5, \infty)$.

(d) f is continuous on the interval $[-5, 5]$.

(e) None of these

1—M—Answer: d

25. Find the x-values (if any) for which f is not continuous.

$$f(x) = \begin{cases} 3x + 2, & x < -1 \\ 2x^2 - 3x + 6, & x \geq -1 \end{cases}$$

1—O—Answer: -1

26. Find the x-values (if any) for which f is not continuous.

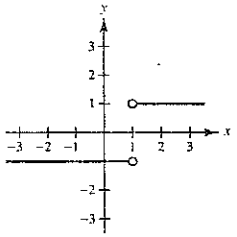
$$f(x) = \begin{cases} \dfrac{1}{(x-3)}, & x \leq 5 \\ \dfrac{1}{2}, & x > 5 \end{cases}$$

(a) 5 (b) $\frac{1}{2}$ (c) 3

(d) 3, 5 (e) None of these

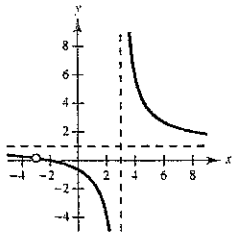
1—M—Answer: c

27. Use the graph of f at the right to find the x-values (if any) at which f is not continuous.



1—O—**Answer:** 1

28. Use the graph of f at the right to find the x-values (if any) at which f is not continuous.



(a) 3 (b) $-3, 3$

(c) $-2, -3$ (d) 2

(e) None of these

1—M—**Answer:** b

29. Use the Intermediate Value Theorem to show that the function $f(x) = x^4 - 2x^2 + 3x$ has a zero in the interval $[-2, -1]$.

2—O—**Answer:** $f(x)$ is continuous on the interval $(-\infty, \infty)$. $f(-2) = 2 > 0$ and $f(-1) = -4 < 0$. Therefore, there exists a real number c, $-2 < c < -1$, such that $f(c) = 0$.

30. Use the Intermediate Value Theorem to show that the function $f(x) = x^2 - 7x + 3$ has a zero in the interval $[0, 1]$.

2—O—**Answer:** $f(x)$ is continuous on the interval $(-\infty, \infty)$. $f(0) = 3 > 0$ and $f(1) = -3 < 0$. Therefore, there exists a real number c, $0 < c < 1$, such that $f(c) = 0$.

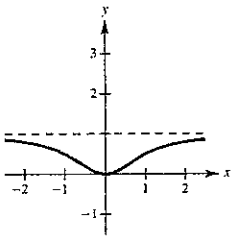
31. Use a graphing utility to graph $f(x) = x^3 - 2x - 5$. Then use the graph to find the interval for which the Intermediate Value Theorem guarantees the existence of at least one number c in that interval for which $f(c) = 0$.

(a) $[-1, 1]$ (b) $[1, 2]$ (c) $[2, 3]$

(d) $[3, 4]$ (e) None of these

1—M—T—**Answer:** c

32. Use the graph to find the interval(s) for which the function f is continuous.



(a) $[0, 1)$ (b) $(-\infty, \infty)$

(c) $(-\infty, 0)$ and $(0, \infty)$ (d) $[0, 1]$

(e) None of these

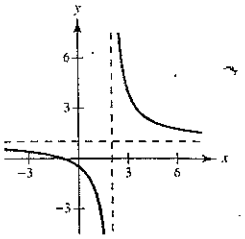
1—M—**Answer:** b

33. Use the graph to find the interval(s) for which the function f is continuous.

(a) $(-\infty, 2)$ and $(2, \infty)$ (b) $(-\infty, \infty)$

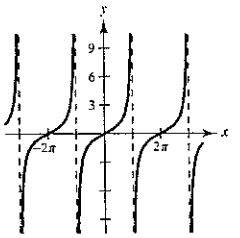
(c) $(-\infty, 1)$ and $(1, \infty)$ (d) $(1, 2)$

(e) None of these



1—M—Answer: a

34. Use the graph to find the interval(s) for which the function f is continuous.



1—O—Answer: $\big((2n-1)\pi,\ (2n+1)\pi\big)$, where n is an integer.

35. Use a graphing utility to graph $f(x) = \dfrac{x+2}{x^2-4}$. Then use the graph to determine x-values at which the function is not continuous.

1—O—T—Answer: -2 and 2

36. Use a graphing utility to graph $f(x) = \dfrac{\sqrt{x+3}-3}{x}$. Then use the graph to determine x-values greater than -3 at which the function is not continuous.

1—O—T—Answer: 0

37. Let $f(x) = \begin{cases} x^2+1, & x \le 0 \\ 2x-3, & x > 0 \end{cases}$. Find each limit (if it exists).

a. $\lim_{x\to 0^-} f(x)$ **b.** $\lim_{x\to 0^+} f(x)$ **c.** $\lim_{x\to 0} f(x)$

1—O—Answer: a. 1 **b.** -3 **c.** Limit does not exist

38. Let $f(x) = \dfrac{|x+3|}{x+3}$. Find each limit (if it exists).

a. $\lim_{x\to -3^-} f(x)$ **b.** $\lim_{x\to -3^+} f(x)$ **c.** $\lim_{x\to -3} f(x)$

1—O—Answer: a. -1 **b.** 1 **c.** Limit does not exist

1.5 Infinite Limits

1. Find the limit: $\lim_{x\to 2} \frac{1}{(x-2)^2}$.

(a) ∞ (b) $-\infty$ (c) 0

(d) $\frac{1}{4}$ (e) None of these

1—M—Answer: a

2. Find the limit: $\lim_{x\to 0} \left(2 + \frac{5}{x^2}\right)$.

(a) 7 (b) 2 (c) ∞

(d) $-\infty$ (e) None of these

1—M—Answer: c

3. Find the limit: $\lim_{x\to 2^-} \frac{1}{x-2}$.

(a) ∞ (b) $-\infty$ (c) 0

(d) $-\frac{1}{4}$ (e) None of these

1—M—Answer: b

4. Find the limit: $\lim_{x\to 0^+} \frac{1}{x}$.

(a) $+\infty$ (b) 0 (c) $-\infty$

(d) Does not exist (e) None of these

1—M—Answer: a

5. Find the limit: $\lim_{x\to 1} \frac{5}{(x-1)^2}$.

(a) 0 (b) $-\infty$ (c) $\frac{5}{4}$

(d) $+\infty$ (e) None of these

1—M—Answer: d

6. Find the limit: $\lim_{x\to 1} \left(2 - \frac{5}{(x-1)^2}\right)$.

(a) $-\infty$ (b) $+\infty$ (c) -3

(d) 2 (e) None of these

1—M—Answer: a

7. Find the limit: $\lim_{x \to -1^-} \frac{1}{x+1}$.

1—O—Answer: $-\infty$

8. Find the limit: $\lim_{x \to 3} \frac{3}{x^2 - 6x + 9}$.

1—O—Answer: ∞

9. Find the limit: $\lim_{x \to 0} \left(3x + 2 + \frac{1}{x^2}\right)$.

1—O—Answer: ∞

10. Find the limit: $\lim_{x \to 3^-} \frac{1}{x-3}$.

1—O—Answer: $-\infty$

11. Find the limit: $\lim_{x \to \pi/2^+} \tan x$.

1—O—Answer: $-\infty$

12. Find the limit: $\lim_{x \to 1^+} \frac{x^2 - x - 2}{x - 1}$.

1—O—Answer: $-\infty$

13. Find all vertical asymptotes of the graph of $f(x) = \frac{x-3}{x+2}$.

(a) $x = -2, x = 3$ (b) $x = -2$ (c) $x = 3$

(d) $x = 1$ (e) None of these

1—M—Answer: b

14. Find all vertical asymptotes of the graph of $g(x) = \frac{x+1}{x^2 - 1}$.

(a) $x = -1, x = 1$ (b) $x = -1$ (c) $x = 1$

(d) $y = 1$ (e) None of these

1—M—Answer: c

15. Find all vertical asymptotes of the graph of $f(x) = \frac{2x-1}{x+3}$.

(a) $x = 2$ (b) $x = \frac{1}{2}, x = -3$ (c) $x = -3$

(d) $x = \frac{1}{2}$ (e) None of these

1—M—Answer: c

16. Find all vertical asymptotes of the graph of $f(x) = \dfrac{x-2}{x^2-4}$.

(a) $x = -2,\ x = 2$ (b) $x = -2$ (c) $x = 0$

(d) $x = 2$ (e) None of these

1—M—Answer: b

17. Find the vertical asymptote: $f(x) = \dfrac{7}{x+2}$.

(a) $x = -2$ (b) $x = 2$ (c) $(0, -2)$

(d) $y = 0$ (e) None of these

1—M—Answer: a

18. Find the vertical asymptote(s): $f(x) = \dfrac{x-2}{x^2-3x-10}$

(a) $x = -2,\ x = 5$ (b) $y = 1$ (c) $y = 0$

(d) $y = 1,\ y = 0$ (e) None of these

1—M—Answer: a

19. Find all vertical asymptotes of the graph of $f(x) = \dfrac{x^2+3x-1}{x+7}$.

1—O—Answer: $x = -7$

20. Find all vertical asymptotes of the graph of $f(x) = \dfrac{2x-2}{(x-1)(x^2+x-1)}$.

1—O—Answer: $x = \dfrac{-1-\sqrt{5}}{2},\ x = \dfrac{-1+\sqrt{5}}{2}$

21. $f(x)$ decreases without bound as x approaches what value from the right?

$$f(x) = \frac{7}{(x-1)(7-x)}$$

(a) 1 (b) -1 (c) 7

(d) -7 (e) None of these

2—M—Answer: c

22. $f(x)$ decreases without bound as x approaches what value from the right?

$$f(x) = \frac{4}{(x-3)(5,-x)}$$

(a) 5 (b) -3 (c) -5

(d) 3 (e) None of these

2—M—Answer: a

23. $f(x)$ decreases without bound as x approaches what value from the right?

$$f(x) = \frac{9}{(x-4)(9-x)}$$

(a) -9 (b) 4 (c) 9

(d) -4 (e) None of these

2—M—Answer: c

24. $f(x)$ decreases without bound as x approaches what value from the right?

$$f(x) = \frac{6}{(x-2)(7-x)}$$

(a) -2 (b) -7 (c) 2

(d) 7 (e) None of these

2—M—Answer: d

25. Find the limit: $\lim_{x \to 3^-} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$.

(a) $\frac{1}{3}$ (b) $+\infty$ (c) $-\infty$

(d) 1 (e) None of these

1—M—Answer: c

26. Find the limit: $\lim_{x \to 4^+} \frac{x^2 - x}{(x-4)^2}$.

(a) 0 (b) $+\infty$ (c) $-\infty$

(d) 4 (e) None of these

1—M—Answer: b

27. Use a graphing utility to find the vertical asymptote(s) (if any) for $f(x) = \frac{\cot x}{x}$.

1—O—T—Answer: $x = n\pi$, where n is an integer.

28. Use a graphing utility to find the vertical asymptote(s) (if any) for $f(x) = \frac{x^2 + 4}{x^2 - 4}$.

1—O—T—Answer: $x = 2$, $x = -2$

29. Find the limit: $\lim_{x \to 1^-} \frac{-2}{x - 1}$.

(a) ∞ (b) $-\infty$ (c) 0

(d) Limit does not exist (e) None of these

1—M—Answer: a

30. Find the vertical asymptote(s) of $f(x) = \dfrac{x^2 - x - 2}{x^2 + x - 6}$.

1—O—Answer: $x = -3$

31. Find the limit: $\lim\limits_{x \to 1} \dfrac{-2}{(1 - x)^2}$.

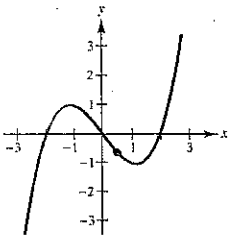
1—O—Answer: $-\infty$

CHAPTER TWO

Differentiation

2.1 The Derivative and the Tangent Line Problem

1. Determine whether the slope at the indicated point is positive, negative, or zero.



(a) Zero

(b) No slope

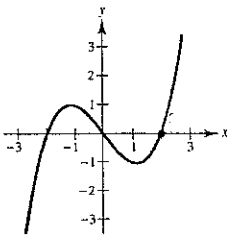
(c) Positive

(d) Negative

(e) None of these

1—M—Answer: d

2. Determine whether the slope at the indicated point is positive, negative, or zero.



(a) No slope

(b) Positive

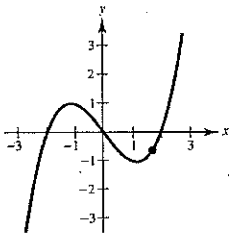
(c) Negative

(d) Zero

(e) None of these

1—M—Answer: b

3. Determine whether the slope at the indicated point is positive, negative, or zero.



(a) Positive

(b) Zero

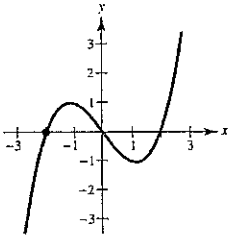
(c) Negative

(d) No slope

(e) None of these

1—M—Answer: a

4. Determine whether the slope at the indicated point is positive, negative, or zero.



(a) Negative

(b) Positive

(c) No slope

(d) Zero

(e) None of these

1—M—Answer: b

5. If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of $f(x)$?

(a) $\dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(b) $\lim\limits_{\Delta x \to 0} \dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(c) $\lim\limits_{\Delta x \to 0} \dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(d) $\dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(e) None of these

1—M—Answer: c

6. If $f(x) = -x^2 + x$, which of the following will calculate the derivative of $f(x)$?

(a) $\lim\limits_{\Delta x \to 0} \dfrac{(-x^2 + x + \Delta x) - (-x^2 + x)}{\Delta x}$

(b) $\lim\limits_{\Delta x \to 0} \left[\dfrac{-(x + \Delta x)^2 + (x + \Delta x)] - (-x^2 + x)}{\Delta x}\right]$

(c) $\dfrac{[-(x + \Delta x)^2 + (x + \Delta x)] - (-x^2 + x)}{\Delta x}$

(d) $\dfrac{(-x^2 + x + \Delta x) - (-x^2 + x)}{\Delta x}$

(e) None of these

1—M—Answer: b

7. Use the definition of a derivative to calculate the derivative of $f(x) = \dfrac{1}{x}$.

2—O—Answer: $-\dfrac{1}{x^2}$

8. Use the definition of a derivative to calculate the derivative of $f(x) = x^2 + x$.

2—O—Answer: $2x + 1$

9. Use the definition of a derivative to calculate the derivative of $f(x) = x^2 + 2$.

2—O—Answer: $-2x$

10. Use the definition of a derivative to calculate the derivative of $2\sqrt{x}$.

2—O—Answer: $\dfrac{1}{\sqrt{x}}$

11. If $f(2) = 3$ and $f'(2) = -1$, find an equation of the tangent line when $x = 2$.

(a) $y - 3 = 2(x + 1)$ (b) $y - 2 = 3(x + 1)$ (c) $y - 3 = -1(x - 2)$

(d) $y + 1 = 2(x - 2)$ (e) None of these

1—M—Answer: c

12. If $f(1) = 4$ and $f'(1) = 2$, find an equation of the tangent line when $x = 1$.

(a) $y - 4 = 2(x - 1)$ (b) $y + 4 = 2(x + 1)$ (c) $y - 1 = 4(x - 2)$

(d) $y - 2 = 4(x - 1)$ (e) None of these

1—M—Answer: a

13. Find the derivative of $y = 2$.

(a) 0 (b) 1 (c) 2 (d) $2 + \Delta x$ (e) None of these

1—M—Answer: a

14. Find the derivative of $y = \pi$.

(a) 1 (b) -1 (c) $\pi + \Delta x$ (d) 0 (e) None of these

1—M—Answer: d

15. Find the derivative of $y = -4$.

(a) -4 (b) 0 (c) 1 (d) $-4 + \Delta x$ (e) None of these

1—M—Answer: b

16. Find the slope of the tangent line to the graph of $f(x) = -2x + 4$ when $x = 2$.

1—O—Answer: -2

17. Find the slope of the tangent line to the graph of $f(x) = 3x - 1$ when $x = -1$.

1—O—Answer: 3

18. Find the slope of the tangent line to the graph of $f(x) = -x + 3$ when $x = 2$.

1—O—Answer: -1

19. Find the slope of the graph of $f(x) = x^2 - 2x$ at the point $(a, f(a))$.

(a) 0 (b) $2a - 2$ (c) $f(a)$

(d) $a^2 - 2a$ (e) None of these

1—M—Answer: b

20. Find an equation of the tangent line to the graph of $f(x) = x^2 - 2x - 3$ at the point $(-2, 5)$.

2—O—Answer: $6x + y + 7 = 0$

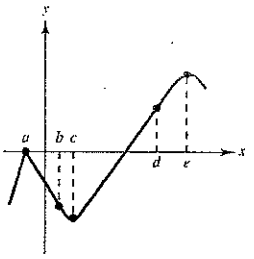
21. Find an equation of the tangent line to the graph of $f(x) = \dfrac{1}{x-1}$ at the point $(2, 1)$.

(a) $x + y + 3 = 0$ (b) $x - y = 1$ (c) $y - 1 = -\dfrac{(x-2)}{(x-1)^2}$

(d) $x + y = 6$ (e) None of these

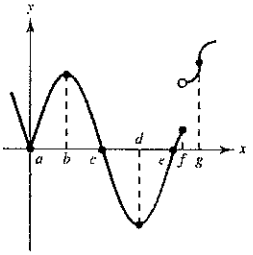
2—M—Answer: e

22. At each point indicated on the graph, determine whether the value of the derivative is positive, negative, zero, or the function has no derivative.



1—O—Answer: **a.** No derivative
b. Negative
c. Zero
d. Positive
e. Zero

23. Use the graph to determine all x-values at which the function is not differentiable.



(a) a, b, c, d, e, f, and g

(b) b and d

(c) a and f

(d) a, f, and g

(e) None of these

1—M—Answer: d

24. Consider $f(x) = \sqrt{x}$.

a. Use the definition of the derivative to calculate the derivative of f.

b. Find the slope of the tangent line to the graph of f at the point $(4, 2)$.

c. Write an equation of the tangent line in part **b**.

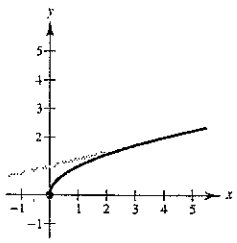
d. Use a graphing utility to graph f and the tangent line on the same axes. Then sketch the graphs.

2—O—T—Answer: **a.** $\dfrac{1}{2\sqrt{x}}$

b. $\dfrac{1}{4}$

c. $y = \dfrac{1}{4}x + 1$

d.



25. Let $f(x) = \dfrac{4}{x}$.

a. Use the definition of the derivative to calculate the derivative of f.

b. Find the slope of the tangent line to the graph of f at the point $(-2, -2)$.

c. Write an equation of the tangent line in part **b**.

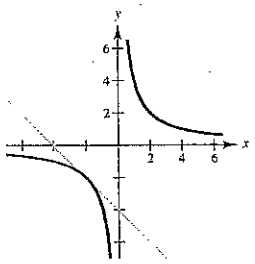
d. Use a graphing utility to graph f and the tangent line on the same axes. Then sketch the graphs.

2—O—T—Answer:. a. $-\dfrac{4}{x^2}$

b. -1

c. $y = -x - 4$

d.



2.2 Basic Differentiation Rules and Rates of Change

1. Find $f'(x)$: $f(x) = 4x^4 - 5x^3 + 2x - 3$.

(a) $4x^3 - 5x^2 + 2$ (b) $16x^3 - 15x^2 + 2$ (c) $16x^3 - 15x^2 + 2x - 3$

(d) $4x^4 - 5x^3 + 2x$ (e) None of these

1—M—Answer: b

2. Find $f'(x)$: $f(x) = 3x^4 - 6x^3 + 3x - 2$.

(a) $3x^3 - 6x^2 + 3$ (b) $12x^3 - 18x^2 + 3x - 2$ (c) $12x^3 - 18x^2 + 3$

(d) $3x^4 - 6x^3 + 3x$ (e) None of these

1—M—Answer: c

3. Find $f'(x)$: $f(x) = \frac{1}{x^2}$.

(a) $-\frac{1}{x^3}$ (b) $\frac{1}{x}$ (c) $-\frac{2}{x^3}$

(d) $\frac{2}{x}$ (e) None of these

1—M—Answer: c

4. Find $f'(x)$: $f(x) = \frac{1}{x}$.

(a) $-\frac{1}{x^2}$ (b) $\frac{1}{x}$ (c) $\frac{1}{x^2}$

(d) $-\frac{1}{x}$ (e) None of these

1—M—Answer: a

5. Find $f'(x)$: $f(x) = \frac{x^2 - 4x}{\sqrt{x}}$.

(a) $\frac{3x^{3/2} - 4}{2x^{1/2}}$ (b) $\frac{2x - 4}{\sqrt{x}}$ (c) $\frac{2x - 4}{1/(2\sqrt{x})}$

(d) $x^{3/2} - 4x^{1/2}$ (e) None of these

2—M—Answer: a

13. Let $g(x) = -7f(x)$ and let $f'(-7) = -9$. Find $g'(-7)$.

(a) -7 (b) -9 (c) 0

(d) 63 (e) None of these

1—M—Answer: d

14. Let $g(x) = -5f(x)$ and let $f'(-7) = 6$. Find $g'(-7)$.

(a) 6 (b) 0 (c) -30

(d) -5 (e) None of these

1—M—Answer: c

15. Find the instantaneous rate of change of w with respect to z for $w = \frac{1}{z} + \frac{z}{2}$.

(a) $\frac{3}{2}$ (b) -2 (c) $\frac{z^2 - 2}{2z^2}$

(d) $\frac{-1}{z^2}$ (e) None of these

1—M—Answer: c

16. Find the instantaneous rate of change of w with respect to z if $w = \frac{7}{3z^2}$

(a) $\frac{7}{6z}$ (b) $\frac{14}{3}z$ (c) $-\frac{14}{3z}$

(d) $-\frac{14}{3z^3}$ (e) None of these

1—M—Answer: d

17. Find the instantaneous rate of change of R with respect to x if $R = 2x^2 + \frac{1}{x}$.

1—O—Answer: $4x - \frac{1}{x^2}$

18. Find an equation for the tangent line to the graph of $f(x) = 2x^2 - 2x + 3$ at the point where $x = 1$.

(a) $y = 2x - 2$ (b) $y = 4x^2 - 6x + 5$ (c) $y = 2x + 1$

(d) $y = 4x^2 - 6x + 2$ (e) None of these

2—M—Answer: c

19. Find an equation for the tangent line to the graph of $f(x) = -2x^2 + 2x + 3$ at the point where $x = 1$.

(a) $y = -4x + 2$ (b) $2x + y - 1 = 0$ (c) $y = -4x^2 + 2x + 1$

(d) $2x + y = 5$ (e) None of these

2—M—Answer: d

20. Find an equation of the tangent line to the graph of $f(x) = 3x^3 + 2x$ when $x = 1$.

2—O—Answer: $y = 11x - 6$

21. Find the point(s) on the graph of the function $f(x) = x^3 - 2$ where the slope is 3.

(a) $(1, 3), (-1, 3)$ (b) $(1, -1), (-1, -3)$ (c) $(\sqrt[3]{2}, 0)$

(d) $(1, 3)$ (e) None of these

2—M—Answer: b

22. Find the values of x for all points on the graph of $f(x) = x^3 - 2x^2 + 5x - 16$ at which the slope of the tangent line is 4.

2—O—Answer: $\frac{1}{3}$, 1

23. Find all points on the graph of $f(x) = -x^3 + 3x^2 - 2$ at which there is a horizontal tangent line.

(a) $(0, -2), (2, 3)$ (b) $(0, -2)$ (c) $(1, 0), (0, -2)$

(d) $(2, 2)$ (e) None of these

2—M—Answer: a.

24. Find all points at which the graph of $f(x) = x^3 - 3x$ has horizontal tangent lines.

2—O—Answer: $(1, -2), (-1, 2)$

25. Suppose the position equation for a moving object is given by $s(t) = 3t^2 + 2t + 5$ where s is measured in meters and t is measured in seconds. Find the velocity of the object when $t = 2$.

(a) 13 m/sec (b) 14 m/sec (c) 10 m/sec

(d) 6 m/sec (e) None of these

1—M—Answer: b

26. Suppose the position equation for a moving object is given by $s(t) = 3t^2 - 2t + 5$ where s is measured in meters and t is measured in seconds. Find the velocity of the object when $t = 2$.

(a) 13 m/sec (b) 6 m/sec (c) 10 m/sec

(d) 14 m/sec (e) None of these

1—M—Answer: c

27. The position function for a particular object is $s = -12t^2 + 51t + 38$. Which statement is true?

(a) The velocity at time $t = 1$ is 27. (b) The velocity is a constant.

(c) The initial position is 51. (d) The initial velocity is -24.

(e) None of these

1—M—Answer: a

28. The position function for a particular object is $s = -\frac{35}{2}t^2 + 58t + 91$. Which statement is true?

(a) The initial velocity is -35.
(b) The velocity is a constant.
(c) The velocity at time $t = 1$ is 23.
(d) The initial position is $-\frac{35}{2}$.
(e) None of these

1—M—Answer: c

29. The position function for a particular object is $s = -\frac{29}{2}t^2 + 56t + 33$. Which statement is true?

(a) The velocity is a constant.
(b) The velocity at time $t = 1$ is 74.5.
(c) The initial position is 33.
(d) The initial velocity is -29.
(e) None of these

1—M—Answer: b

30. The position function for an object is given by $s(t) = 6t^2 + 240t$, where s is measured in feet and t is measured in seconds. Find the velocity of the object when $t = 2$ seconds.

1—O—Answer: 264 ft/sec

31. An object is dropped from a height of 128 feet.

a. When does it hit the ground?

b. What is the velocity of the object when it hits?

1—O—Answer: **a.** $2\sqrt{2}$ sec **b.** $-64\sqrt{2}$ ft/sec

32. Find the average rate of change of y with respect to x on the interval $[1, 4]$, where $y = x^2 + x + 1$.

1—O—Answer: 6

33. Find the average rate of change of y with respect to x on the interval $[2, 3]$, where $y = x^3 + 2$.

1—O—Answer: 19

34. Find the average rate of change of y with respect to x on the interval $[-1, 1]$, where $y = 2x^3 + 4x$.

1—O—Answer: 6

35. Find the average rate of change of y with respect to x on the interval $[0, 5]$, where $y = 2x^2 + x - 3$.

1—O—Answer: 11

36. The velocity function for an object is given by $s'(t) = -10t^2 + 4$, where s is measured in feet and t is measured in seconds. What is the instantaneous velocity when $t = 2$?

(a) -10 ft/sec
(b) -40 ft/sec
(c) -36 ft/sec
(d) 44 ft/sec
(e) None of these

1—M—Answer: c

37. The velocity function for an object is given by $s'(t) = -8t^2 + 6$, where s is measured in feet and t is measured in seconds. What is the instantaneous velocity when $t = 2$?

(a) -32 ft/sec (b) 262 ft/sec (c) -8 ft/sec

(d) -26 ft/sec (e) None of these

1—M—Answer: d

38. An object that is thrown (straight down) from the top of a 220-foot building with an initial velocity of 26 feet per second.

a. Write the position equation for the movement described.

b. What is the velocity at 1 second?

2—O—Answer: a. $s = -16t^2 - 26t + 220$ **b.** -58 ft/sec

39. An object that is thrown (straight down) from the top of a 300-foot building with an initial velocity of 30 feet per second.

a. Write the position equation for the movement described.

b. What is the velocity at 1 second?

2—O—Answer: a. $s = -16t^2 - 30t + 300$ **b.** -62 ft/sec

40. Find the value of the derivative $f(t) = \dfrac{t^3 + 2}{t}$ at the point $(-2, 3)$.

(a) $-\frac{9}{2}$ (b) $-\frac{7}{2}$ (c) 12

(d) $-\frac{11}{16}$ (e) None of these

2—M—Answer: a

41. Find the value of the derivative of $f(x) = -2(x - 3)^2$ at the point $(2, -2)$.

1—O—Answer: 4

42. Find $\dfrac{dy}{dx}$: $y = 4 \sin x - 5 \cos x + x$.

(a) $4 \cos x + 5 \sin x + 1$ (b) $-4 \cos x + 5 \sin x + 1$ (c) $4 \cos x - 5 \sin x + 1$

(d) $4 \cos x + 5 \sin x$ (e) None of these

1—M—Answer: a

43. Find an equation of the tangent line to the graph of the function $f(x) = 2 \sin x$ at the point where $x = \dfrac{\pi}{3}$.

(a) $y = x - \dfrac{\pi}{3}$ (b) $3x - 3y = \pi - 3\sqrt{3}$ (c) $3x - 6y = \pi - 6$

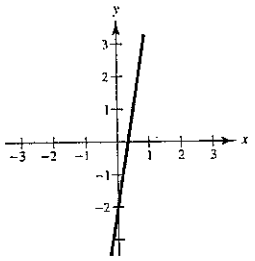
(d) $3\sqrt{3}x - 3y = \sqrt{3}\pi - 3$ (e) None of these

2—M—Answer: b

44. Find the point(s) on the graph of $y = \dfrac{1}{x}$ where the graph is parallel to the line $4x + 9y = 3$.

2—O—Answer: $\left(-\frac{3}{2}, -\frac{2}{3}\right)$ and $\left(\frac{3}{2}, \frac{2}{3}\right)$

45. The graph at the right represents the graph of the derivative of which of the following functions?



(a) $f(x) = 2x^2 + 1$

(b) $f(x) = 2x - 3$

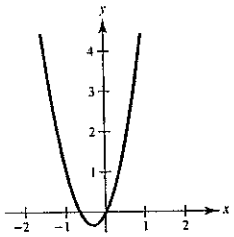
(c) $f(x) = 3x^2 - 2x - 1$

(d) $f(x) = x^3 + x^2$

(e) None of these

1—M—Answer: c

46. The graph at the right represents the graph of the derivative of which of the following functions?



(a) $f(x) = 2x^2 + 1$

(b) $f(x) = 2x - 3$

(c) $f(x) = 3x^2 - 2x - 1$

(d) $f(x) = x^3 + x^2$

(e) None of these

1—M—Answer: d

47. A coin is dropped from a height of 750 feet. The height, s (measured in feet), at time, t (measured in seconds), is given by

$$s = 16t^2 + 750.$$

a. Find the average velocity on the interval $[1, 3]$.

b. Find the instantaneous velocity when $t = 3$.

c. How long does it take for the coin to hit the ground?

d. Find the velocity of the coin when it hits the ground.

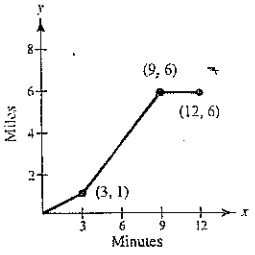
2—O—Answer: **a.** -64 ft/sec

b. -96 ft/sec

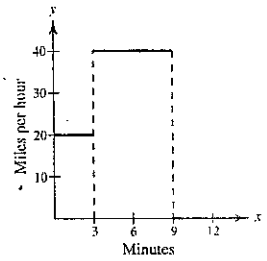
c. $\dfrac{5\sqrt{30}}{4}$ sec ≈ 6.85 sec

d. $-40\sqrt{30}$ ft/sec ≈ -219.09 ft/sec

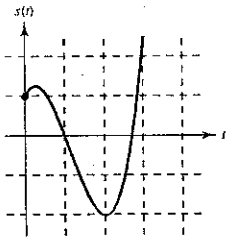
48. The given graph of a position function represents the distance in miles that a person drives during a 12-minute drive to school. Make a sketch of the corresponding velocity function.



2—O—Answer:



49. The position, $s(t)$ (measured in inches), at any time, t (measured in seconds), of an object is described in the graph. Use the graph to determine:



a. $s(0)$

b. $s(1)$

c. $v(2)$

d. Is $v(4) > 0$?

e. Is $v(1) > 0$?

1—O—Answer: a. 1 **b.** 0 **c.** 0 **d.** Yes **e.** No

50. The volume of a right circular cone of radius r and height r is given by $V = \frac{\pi}{3}r^3$.

How fast is the volume changing with respect to changes in r when the radius r is equal to 2 feet?

1—O—Answer: 4π ft^2.

51. Consider $f(x) = x^3 + 3x^2$. Use a graphing utility to graph f and f' on the same set of coordinate axes. Describe the behavior of f at those values of x that are zeros of f'.

1—O—T—Answer: At $x = 0$ and $x = -2$, f has horizontal tangent lines.

2.3 The Product and Quotient Rules and Higher-Order Derivatives

1. Differentiate: $y = \dfrac{3x}{x^2 + 1}$.

(a) $\dfrac{3}{1 + x^2}$ (b) $\dfrac{3}{2x}$ (c) $\dfrac{3x^2 - 3}{(1 + x^2)^3}$

(d) $\dfrac{3(1 - x^2)}{(1 + x^2)^2}$ (e) None of these

1—M—Answer: d

2. Differentiate: $y = \dfrac{1 + \cos x}{1 - \cos x}$.

(a) -1 (b) $-2 \csc x$ (c) $2 \csc x$

(d) $\dfrac{-2 \sin x}{(1 - \cos x)^2}$ (e) None of these

1—M—Answer: d

3. Differentiate: $y = \dfrac{2x}{1 - 3x^2}$.

1—O—Answer: $\dfrac{6x^2 + 2}{(1 - 3x^2)^2}$

4. Differentiate: $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

1—O—Answer: $\dfrac{4x}{(x^2 + 1)^2}$

5. Differentiate: $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$.

1—O—Answer: $\dfrac{8x}{(x^2 + 2)^2}$

6. Differentiate: $f(x) = x^2 + 2 \tan x$.

(a) $2x + 2 \tan x$ (b) $2x + \sec^2 x$ (c) $2 + \sec^2 x$

(d) $2x + 2 \sec^2 x$ (e) None of these

1—M—Answer: d

7. Differentiate: $f(x) = -x + \tan x$.

(a) $-1 + \tan^2 x$ (b) $\sec^2 x$ (c) $\tan^2 x$

(d) $-1 + \tan x$ (e) None of these

1—M—Answer: c

8. Find an equation of the tangent line to the graph of $f(x) = (x - 1)/(x + 1)$ when $x = 1$.

(a) $y = \dfrac{2}{(x+1)^2}(x - 1)$ (b) $x - 2y = 1$ (c) $y - 1 = \dfrac{1}{2}x$

(d) $y = 2(x - 1)$ (e) None of these

1—M—Answer: b

9. Find an equation of the tangent line to the graph of $f(x) = x \sin x$ when $x = 0$.

(a) $y = 0$ (b) $f'(x) = 0$ (c) $y = x \cos x + \sin x$

(d) $y = x$ (e) None of these

1—M—Answer: a

10. Find $\dfrac{d^2y}{dx^2}$ for $y = \dfrac{x + 3}{x - 1}$.

(a) 0 (b) $\dfrac{-8}{(x-1)^3}$ (c) $\dfrac{-4}{(x-1)^3}$

(d) $\dfrac{8}{(x-1)^3}$ (e) None of these

1—M—Answer: d

11. Find $\dfrac{d^2y}{dx^2}$ for $y = \dfrac{x + 2}{x - 3}$.

(a) $\dfrac{10}{(x-3)^3}$ (b) 0 (c) $\dfrac{-10}{(x-3)^3}$

(d) $\dfrac{2}{(x-3)^3}$ (e) None of these

1—M—Answer: a

12. Calculate $\dfrac{d^2y}{dx^2}$ if $y = \dfrac{1 - x}{2 - x}$.

1—O—Answer: $\dfrac{-2}{(2-x)^3}$

13. Find the derivative of $x^2 f(x)$.

(a) $x[xf'(x) + 2f(x)]$ (b) $2xf'(x)$ (c) $x[xf(x) + 2f'(x)]$

(d) $x^2 f'(x)$ (e) None of these

2—M—Answer: a

14. Find the derivative of $9x^2 f(x)$.

(a) $9x^2 f'(x)$ (b) $9x[xf(x) + 2f'(x)]$ (c) $18xf'(x)$

(d) $9x[xf'(x) + 2f(x)]$ (e) None of these

2—M—Answer: d

15. Let $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$ and $g'(7) = \frac{1}{7}$. Find $h'(7)$ if $h(x) = f(x)/g(x)$.

(a) 98 (b) -14 (c) -2

(d) 14 (e) None of these

2—M—Answer: d

16. Let $f(-5) = 0$, $f'(-5) = -10$, $g(-5) = 1$ and $g'(-5) = -\frac{1}{5}$. Find $h'(-5)$ if $h(x) = f(x)/g(x)$.

(a) 10 (b) -2 (c) -10

(d) 50 (e) None of these

2—M—Answer: c

17. Let $f(3) = 0$, $f'(3) = 6$, $g(3) = 1$ and $g'(3) = \frac{1}{3}$. Find $h'(3)$ if $h(x) = f(x)/g(x)$.

(a) 18 (b) 6 (c) -6

(d) -2 (e) None of these

2—M—Answer: b

18. If $f''(x) = 3x^2 + 6x + 4$, find $f^{(4)}(x)$.

(a) 0 (b) 6 (c) $2x + 6$ (d) $6x + 6$ (e) None of these

1—M—Answer: b

19. If $f''(x) = 5x^2 + 9x + 8$, find $f^{(4)}(x)$.

(a) 10 (b) $10x + 9$ (c) $2x + 9$ (d) 0 (e) None of these

1—M—Answer: a

20. If $f''(x) = -2x^2 + 7x - 2$, find $f^{(4)}(x)$.

(a) 0 (b) -4 (c) $-2x + 7$ (d) $-4x + 7$ (e) None of these

1—M—Answer: b

21. Find $\frac{dy}{d\theta}$ for $y = \csc\theta - \cot\theta$.

(a) 0 (b) $-\cot^2\theta + \csc\theta\cot\theta$ (c) $\sec\theta\tan\theta - \sec^2\theta$

(d) $-\csc\theta\cot\theta + \csc^2\theta$ (e) None of these

1—M—Answer: d

22. Find the derivative: $f(x) = 5\sec x\tan x$.

2—O—Answer: $5\sec x(2\sec^2 x - 1)$

23. Find $f'(\theta)$: $f(\theta) = \theta\cot\theta$.

1—O—Answer: $\cot\theta - \theta\csc^2\theta$

24. Find y'' for $y = \dfrac{\csc x}{2}$.

2—O—Answer: $\dfrac{1}{2}(\csc x)(2\csc^2 x - 1)$

25. Find the derivative: $f(t) = \dfrac{t}{1 - \cos t}$.

1—O—Answer: $\dfrac{1 - \cos t - t\sin t}{(1 - \cos t)^2}$

26. Evaluate the derivative of $f(t) = \dfrac{t}{\cos t}$ at the point $\left(\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right)$.

2—O—Answer: $\dfrac{6 + 2\sqrt{3}\pi}{3}$

27. Find an equation of the tangent line to the graph of $f(\theta) = \tan\theta$ at the point $\left(\dfrac{\pi}{4}, 1\right)$.

(a) $4x - 4y = \pi - 4$ (b) $4\sqrt{2}x - 4y = \pi - 4$ (c) $4x - 2y = \pi - 2$

(d) $y = x$ (e) None of these

2—M—Answer: c

28. Find $f'(x)$ if $f(x) = \sin x(\sin x + \cos x)$.

2—O—Answer: $\sin 2x + \cos 2x$

29. Let $f(x) = \dfrac{x^2 + 4}{x}$.

a. Calculate $f'(x)$.

b. Use a graphing utility to graph f and f' on the same axes.

c. Use the graphs to determine those point(s) where f has a horizontal tangent line.

d. Give the value of f' at each of the points found in part **c**.

2—O—T—Answer: **a.** $1 - \dfrac{4}{x^2}$ **c.** $(-2, -4)$ and $(2, 4)$ **d.** 0

30. Let $f(x) = x^5 - 5x$.

a. Calculate $f'(x)$.

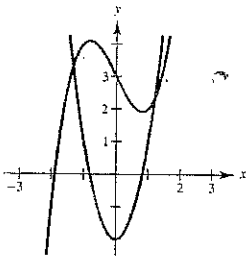
b. Use a graphing utility to graph f and f' on the same axes. Sketch the graphs.

c. Use the graphs to determine those point(s) where f has a horizontal tangent line.

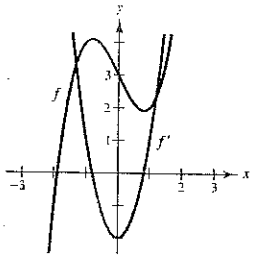
d. Give the value of f' at each of the points found in part **c**.

2—O—T—Answer: **a.** $5x^4 - 5$ **c.** $(-1, 4)$ and $(1, -4)$ **d.** 0

31. The graphs of a function f and its derivative f' are given on the same coordinate axes. Label the graphs as f or f' and state the reasons for your choice.

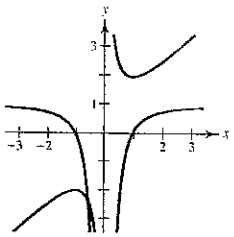


1—O—Answer:

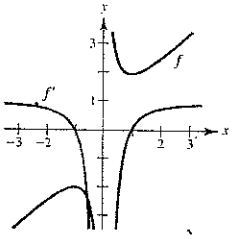


The derivative of f is zero if its tangent line is horizontal.

32. The graphs of a function f and its derivative f' are given on the same coordinate axes. Label the graphs as f or f' and state the reasons for your choice.



1—O—Answer:



The derivative of f is zero if its tangent line is horizontal.

2.4 The Chain Rule

1. Find $\frac{dy}{dx}$ for $y = x^3\sqrt{x+1}$.

(a) $\frac{3x^2}{2\sqrt{x+1}}$ (b) $\frac{x^2(7x+6)}{2\sqrt{x+1}}$ (c) $3x^2\sqrt{x+1}$

(d) $\frac{7x^3+x^2}{2\sqrt{x+1}}$ (e) None of these

2—M—Answer: b

2. Find $f'(x)$ for $f(x) = (2x^2+5)^7$.

(a) $7(4x)^6$ (b) $(4x)^7$ (c) $28x(2x^2+5)^6$

(d) $7(2x^2+5)^6$ (e) None of these

1—M—Answer: c

3. Find $\frac{dy}{dx}$ for $y = \sqrt{x}(3x-1)$.

(a) $\frac{9x-1}{2\sqrt{x}}$ (b) $\frac{9}{2}\sqrt{x}-1$ (c) $3\sqrt{x}$

(d) $\frac{3}{2\sqrt{x}}$ (e) None of these

2—M—Answer: a

4. Find $\frac{dy}{dx}$ for $y = \sqrt{2x+1}(x^3)$.

2—O—Answer: $\frac{x^2(7x+3)}{\sqrt{2x+1}}$

5. Find the derivative of $f(x) = -4(1-x)^2 - 7(1-x) + 3$.

(a) $8(1-x)+7$ (b) $-8(1-x)+7$ (c) $-8x+8$

(d) $15(1-x)$ (e) None of these

1—M—Answer: a

6. Find the derivative of $f(x) = 2(1-x)^2 + 9(1-x) + 6$.

(a) $4(1-x)-9$ (b) $-13(1-x)$ (c) $-4(1-x)-9$

(d) $4x-4$ (e) None of these

1—M—Answer: c

7. Find the derivative of $f(x) = -8(1 - x)^2 + 7(1 - x) + 2$.

(a) $-16x + 16$ (b) $16(1 - x) - 7$ (c) $9(1 - x)$

(d) $-16(1 - x) - 7$ (e) None of these

1—M—Answer: b

8. Find the derivative of $y = \sqrt[3]{x^2 + x}$.

1—O—Answer: $\dfrac{2x + 1}{3(x^2 + x)^{2/3}}$

9. Find the derivative of $y = (x^2 + 2x + 5)^6$.

1—O—Answer: $12(x + 1)(x^2 + 2x + 5)^5$

10. Differentiate: $y = \sec^2 x + \tan^2 x$.

(a) 0 (b) $\tan x + \sec^4 x$ (c) $\sec^2 x(\sec^2 x + \tan^2 x)$

(d) $4 \sec^2 x \tan x$ (e) None of these

1—M—Answer: d

11. Find the derivative: $s(t) = \csc \dfrac{t}{2}$.

(a) $-\csc \dfrac{t}{2} \cot \dfrac{t}{2}$ (b) $-\dfrac{1}{2} \cot^2 \dfrac{t}{2}$ (c) $\dfrac{1}{2} \csc \dfrac{t}{2} \cot \dfrac{t}{2}$

(d) $\dfrac{1}{2} \cot^2 \dfrac{t}{2}$ (e) None of these

1—M—Answer: e

12. Find $f'(x)$ if $f(x) = \sin^3 4x$.

(a) $4 \cos^3 4x$ (b) $3 \sin^2 4x \cos 4x$ (c) $\cos^3 4x$

(d) $12 \sin^2 4x \cos 4x$ (e) None of these

1—M—Answer: d

13. Differentiate: $y = \csc^2 \theta + \cot^2 \theta$.

(a) $\cot \theta + \csc^4 \theta$ (b) 0 (c) $-4 \csc^2 \theta \cot \theta$

(d) $-\csc^2 \theta(\csc^2 \theta + \cot^2 \theta)$ (e) None of these

1—M—Answer: c

14. Find the derivative: $s(t) = \sec \sqrt{t}$.

(a) $\tan^2 \sqrt{t}$ (b) $\dfrac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$ (c) $\sec \dfrac{1}{2\sqrt{t}} \tan \dfrac{1}{2\sqrt{t}}$

(d) $\sec \sqrt{t} \tan \sqrt{t}$ (e) None of these

1—M—Answer: b

15. Find the derivative: $f(\theta) = \sec \theta^2$.

1—O—Answer: $2\theta \sec \theta^2 \tan \theta^2$

16. Differentiate: $y = \sin^2 t - \cos^2 t$.

(a) 0 (b) 1 (c) $2 \sin 2t$

(d) $-4 \sin t \cos t$ (e) None of these

1—M—Answer: c,

17. Find the derivative of $y = \sin x^2$.

1—O—Answer: $2x \cos x^2$

18. Find the derivative of $y = \sin^2 x$.

1—O—Answer: $\sin 2x$

19. Find the derivative of $y = \tan\left(x^2 - \frac{\pi}{4}\right)$.

1—O—Answer: $2x \sec^2\left(x^2 - \frac{\pi}{4}\right)$

20. Find the derivative of $y = \sin^2 x + \cos^2 x$.

1—O—Answer: 0

21. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x+1}$ at the point where $x = 3$.

2—O—Answer: $x - 4y = -5$.

22. Find $f'(x)$ if $f(x) = \cot^3\sqrt{x}$.

1—O—Answer: $\dfrac{-3 \cot^2\sqrt{x}\left(\csc^2\sqrt{x}\right)}{2\sqrt{x}}$

23. The position equation for the movement of a particle is given by $s = (t^2 - 1)^3$ when s is measured in feet and t is measured in seconds. Find the acceleration at two seconds.

(a) 342 units/sec^2 (b) 18 units/sec^2 (c) 288 units/sec^2

(d) 90 units/sec^2 (e) None of these

2—M—Answer: a

24. A particle moves along the curve given by $y = \sqrt{t^3 + 1}$. Find the acceleration when $t = 2$ seconds.

(a) 3 units/sec^2 (b) $\frac{2}{3}$ units/sec^2 (c) $-\frac{1}{108}$ units/sec^2

(d) $-\frac{1}{9}$ units/sec^2 (e) None of these

2—M—Answer: b

25. The position equation for the movement of a particle is given by $s = (t^3 + 1)^2$ where s is measured in feet and t is measured in seconds. Find the acceleration of this particle at 1 second.

2—O—Answer: 42 ft/sec^2

26. Find the derivative: $f(x) = \dfrac{1}{\sqrt[3]{3 - x^3}}$.

(a) $\dfrac{-1}{3(3 - x^3)^{4/3}}$ (b) $\dfrac{x^2}{(3 - x^3)^{4/3}}$ (c) $\dfrac{-x^2}{(3 - x^3)^{2/3}}$

(d) $\dfrac{-x^2}{(3 - x^3)^{4/3}}$ (e) None of these

1—M—Answer: b

27. Find the derivative: $f(x) = \dfrac{1}{1 - x}$.

(a) -1 (b) $\dfrac{-1}{(1 - x)^2}$ (c) 1

(d) $\dfrac{1}{(1 - x)^2}$ (e) None of these

1—M—Answer: d

28. Find the derivative: $f(x) = \sqrt{\dfrac{x}{x - 1}}$.

1—O—Answer: $\dfrac{-1}{2|x - 1|\sqrt{x(x - 1)}}$

29. Find the derivative: $f(\theta) = \sqrt{\sin 2\theta}$.

(a) $\dfrac{\cos 2\theta}{\sqrt{\sin 2\theta}}$ (b) $\sqrt{\sec 2\theta}$ (c) $\dfrac{\cos 2\theta}{2\sqrt{\sin 2\theta}}$

(d) $\cos \theta$ (e) None of these

1—M—Answer: a

30. Find $f''(x)$ if $f(x) = \sin x^2$.

(a) $2(\cos x^2 - 2x^2 \sin x^2)$ (b) $-4x \sin x^2$ (c) $2x \cos x^2$

(d) $2(\cos x^2 - x \sin x^2)$ (e) None of these

2—M—Answer: a

31. Find $f''(x)$: $f(x) = \sqrt{2x^2 + 5}$.

2—O—Answer: $\dfrac{10}{(2x^2 + 5)^{3/2}}$

32. Find an equation for the tangent line to the graph of $f(x) = \tan 2x$ at the point $\left(\frac{\pi}{8}, 1\right)$.

(a) $8x + 2y = \pi + 2$ (b) $8x - 16y = \pi - 16$ (c) $8x - 2y = \pi - 2$

(d) $8x - 4y = \pi - 4$ (e) None of these

2—M—Answer: c

33. Let $f(x) = \sqrt{x^2 + 1}$.

a. Calculate $f'(x)$.

b. Use a graphing utility to graph f and f' on the same axes.

c. Use the graph to determine those point(s) where f has a horizontal tangent line.

d. Give the value of f' at each of the points found in part **c**.

2—O—T—Answer: **a.** $\frac{x}{\sqrt{x^2 + 1}}$ **c.** $(0, 1)$ **d.** 0

2.5 Implicit Differentiation

1. Find $\frac{dy}{dx}$ if $y^2 - 3xy + x^2 = 7$.

(a) $\frac{2x + y}{3x - 2y}$ (b) $\frac{3y - 2x}{2y - 3x}$ (c) $\frac{2x}{3 - 2y}$

(d) $\frac{2x}{y}$ (e) None of these

1—M—Answer: b

2. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 2xy$.

(a) $\frac{x}{1 - y}$ (b) $\frac{y + x}{y - x}$ (c) 1

(d) $-\frac{x}{y}$ (e) None of these

1—M—Answer: c

3. Find $\frac{dy}{dx}$ if $y = \frac{x}{x + y}$.

1—O—Answer: $\frac{y}{(x + y)^2 + x}$

4. Find $\frac{dy}{dx}$ for $2x^2 + xy + 3y^2 = 0$.

(a) $-\frac{4x + y}{x + 6y}$ (b) $-\frac{4x + y}{6y}$ (c) $4x + y + 6y$

(d) $\frac{4x + 6y}{-x}$ (e) None of these

1—M—Answer: a

5. Find $\frac{dy}{dx}$ for $5x^2 - 2xy + 7y^2 = 0$.

(a) $\frac{5x + 7y}{x}$ (b) $\frac{y - 5x}{7y}$ (c) $10x - 2y + 14y$

(d) $\frac{y - 5x}{7y - x}$ (e) None of these

1—M—Answer: d

6. Find $\frac{dy}{dx}$ for $7x^2 + 6xy + 9y^2 = 0$.

(a) $-\frac{7x + 3y}{9y}$ (b) $-\frac{7x + 3y}{3x + 9y}$ (c) $\frac{7x + 9y}{-3x}$

(d) $14x + 6y + 18y$ (e) None of these

1—M—Answer: b

7. Find $\frac{dy}{dx}$ for the equation $x^3 - 2x^2y + 3xy^2 = 38$.

1—O—Answer: $\frac{3x^2 - 4xy + 3y^2}{2x^2 - 6xy}$

8. Find $\frac{dy}{dx}$ for the equation $x^3 - 2x^2y + 4xy^2 = 20$.

1—O—Answer: $\frac{3x^2 - 4xy + 4y^2}{2x^2 - 8xy}$

9. Find $\frac{dy}{dx}$ if $y = \sin(x + y)$.

(a) 0 (b) $\frac{\cos(x + y)}{1 - \cos(x + y)}$ (c) $\cos(x + y)$

(d) 1 (e) None of these

1—M—Answer: b

10. Find $\frac{dy}{dx}$ if $x = \tan(x + y)$.

(a) $-\sin^2(x + y)$ (b) $\sec^2(x + y)$ (c) $-\tan^2(x + y)$

(d) $\frac{1 - \sec^2 x}{\sec^2 y}$ (e) None of these

1—M—Answer: a

11. Use implicit differentiation to find $\frac{dy}{dx}$ if $x = \cos y$.

1—O—Answer: $-\csc y$

12. Find an equation of the tangent line to the graph of $x^2 + 2y^2 = 3$ at the point $(1, 1)$.

(a) $y - 1 = -\frac{x}{zy}(x - 1)$ (b) $y + 1 = -\frac{1}{2}(x + 1)$ (c) $y - 1 = \frac{1}{2}(x - 1)$

(d) $x + 2y = 3$ (e) None of these

2—M—Answer: d

13. Find an equation of the tangent line to the graph of $x^2 + 3y^2 = 4$ at the point $(1, 1)$.

(a) $y + 1 = -\frac{1}{3}(x + 1)$ (b) $y - 1 = -\frac{x}{3y}(x - 1)$ (c) $x + 3y = 2$

(d) $y - 1 = -\frac{1}{3}(x - 1)$ (e) None of these

2—M—**Answer:** d

14. Determine the slope of the graph of $2x^2 - 3xy + y^3 = -1$ at the point $(2, -3)$.

(a) $-\frac{17}{21}$ (b) $\frac{5}{7}$ (c) $-\frac{1}{3}$

(d) $\frac{4}{3}$ (e) None of these

1—M—**Answer:** a

15. Find $\frac{d^2y}{dx^2}$ in terms of x and y: $y^3 - xy = 5$.

2—O—**Answer:** $\frac{-2xy}{(3y^2 - x)^3}$

16. Find $\frac{d^2y}{dx^2}$ in terms of x and y: $x \sin y = 3$.

2—O—**Answer:** $\frac{\tan y(\tan^2 y + 2)}{x^2}$

17. Let $4x^2 + 9y^2 = 144$.

a. Find two explicit functions by solving for y in terms of x.

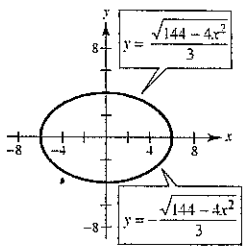
b. Sketch the graphs of the explicit functions on the same axes and label each.

c. Differentiate the explicit functions.

d. Find $\frac{dy}{dx}$ implicitly and show that the result is equivalent to that of part **c**.

2—O—**Answer:** **a.** $y = \pm\frac{\sqrt{144 - 4x^2}}{3}$

b.



c. $\mp\frac{4x}{3\sqrt{144 - 4x^2}}$

d. $-\frac{4x}{9y}$

18. Find the point(s) (if any) of horizontal tangent lines:

$x^2 + xy + y^2 = 6.$

2—O—Answer: $(\sqrt{2}, -2\sqrt{2})$ and $(-\sqrt{2}, 2\sqrt{2})$

19. Find $\frac{dy}{dx}$ if $xy^2 - y = x^2$.

(a) $\frac{2x - y^2}{2xy - 1}$ (b) $\frac{2x}{2y - 1}$ (c) $2x - y^2$

(d) y (e) None of these

1—M—Answer: a

20. Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 5$.

1—O—Answer: $\frac{-2x - y}{x + 2y}$

21. Find $\frac{dy}{dx}$ if $x\sqrt{y} + y^2 = x$.

1—O—Answer: $\frac{2\sqrt{y} - 2y}{2y\sqrt{y} + x}$

22. Find $\frac{dy}{dx}$ if $x^2y + y^2 = x$.

(a) $\frac{1}{2x + 2y}$ (b) $\frac{1 - 2xy - 2y}{x^2}$ (c) $\frac{1 - 2xy}{x^2 + 2y}$

(d) $\frac{1}{x}$ (e) None of these

1—M—Answer: c

23. Find y' for $xy^2 + y = x^2$.

(a) $\frac{2x - y^2}{2xy + 1}$ (b) $\frac{2x}{2y + 1}$ (c) $2x - y^2$

(d) y (e) None of these

1—M—Answer: a

24. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(0, 2)$: $x^2 - 2xy = y^2 - 4$.

(a) -1 (b) 3 (c) 1 (d) $\frac{1}{2}$ (e) None of these

1—M—Answer: a

25. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(0, 2)$: $x^2 - 2xy^2 = y^3 - 8$.

(a) 1 (b) $-\frac{5}{4}$ (c) $\frac{2}{3}$ (d) -1 (e) None of these

1—M—Answer: c

26. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(0, 2)$: $5x^2 - 3xy + y = 2$.

(a) -2 (b) $-\frac{5}{4}$ (c) 0 (d) 6 (e) None of these

1—M—Answer: d

27. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(0, 0)$: $2x - 5x^3y^2 + 4y = 0$.

(a) 3 (b) $-\frac{1}{2}$ (c) undefined (d) 6 (e) None of these

1—M—Answer: b

28. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(1, -1)$: $x^4 + 4x^2y^3 + y^2 = 2y$.

(a) 3 (b) 0 (c) undefined (d) $\frac{1}{2}$ (e) None of these

1—M—Answer: d.

29. Find $\frac{dy}{dx}$, then evaluate the derivative at the point $(0, -2)$: $x^2 - y^2 - 2x - 4y - 4 = 0$.

(a) 3 (b) 0 (c) undefined (d) $\frac{1}{2}$ (e) None of these

1—M—Answer: c

30. Find the slope of the line tangent to the graph of $4y^2 - xy = 3$ at the point $(-1, -1)$.

(a) $\frac{2}{3}$ (b) $\frac{1}{7}$ (c) 0

(d) undefined (e) None of these

1—M—Answer: b

31. Find the slope of the line tangent to the graph of $4y^2 - xy = 3$ at the point $\left(-1, \frac{3}{4}\right)$.

(a) $\frac{3}{28}$ (b) $\frac{1}{7}$ (c) $\frac{4}{71}$

(d) undefined (e) None of these

1—M—Answer: a

32. Find the slope of the line tangent to the graph of $y^3 - 2xy^2 = x^2$ at the point $(-1, -1)$.

(a) $\frac{3}{28}$ (b) $\frac{4}{71}$ (c) undefined

(d) 0 (e) None of these

1—M—Answer: d

33. Find the slope of the curve $y^4 - xy^2 = x$ at the point $\left(\frac{1}{2}, 1\right)$.

1—O—Answer: $\frac{2}{3}$

34. Find an equation of the line tangent to the curve $x^2 + y^2 = 9$ at the point $(3, 0)$.

(a) $x = -1$ (b) $x = 3$ (c) $y = 0$

(d) $y = x - 3$ (e) None of these

2—M—Answer: b

35. Find an equation of the line tangent to the curve $2x^2 - y^2 = 1$ at the point $(5, 7)$.

2—O—Answer: $y = \frac{10}{7}x - \frac{1}{7}$

36. Find an equation of the line tangent to the curve $2y^2 - x^2 = 1$ at the point $(7, -5)$.

2—O—Answer: $y = -\frac{7}{10}x - \frac{17}{2}$

37. The air pollution y (in parts per million) x miles away is given by $y + 2xy + x^2y = 600$. Find the rate of pollution 10 miles away:

(a) 3.0924 parts per million (b) -0.9016 parts per million (c) 1.0020 parts per million

(d) -1.997 parts per million (e) None of these

1—M—Answer: b

2.6 Related Rates

1. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 1.8 inches? [Hint: $V = \pi r^2 h$]

   (a) -2.195 in./sec (b) 39.51 in./sec (c) 2.195 in./sec

   (d) -43.90 in./sec (e) None of these

   1—M—Answer: c

2. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 2.5 inches? [Hint: $V = \pi r^2 h$]

   (a) 20.48 in./sec (b) -0.8192 in./sec (c) -16.38 in./sec

   (d) 0.8192 in./sec (e) None of these

   1—M—Answer: d

3. A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 1.5 inches? [Hint: $V = \pi r^2 h$]

   (a) -75.85 in./sec (b) 56.89 in./sec (c) 3.793 in./sec

   (d) -3.793 in./sec (e) None of these

   1—M—Answer: c

4. As a balloon in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?

   1—O—Answer: $\frac{1}{\pi}$ in./sec

5. The radius of a circle is increasing at the rate of 5 inches per minute. At what rate is the area increasing when the radius is 10 inches?

   1—O—Answer: 100π in.^{2}/min

6. Sand is falling off a conveyor onto a conical pile at the rate of 15 cubic feet per minute. The diameter of the base of the cone is approximately twice the altitude. At what rate is the height of the pile changing when it is 10 feet high?

   1—O—Answer: 0.048 ft/min

7. A point moves along the curve $y = 2x^2 + 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = \frac{3}{2}$?

(a) Increasing $\frac{1}{3}$ unit/sec (b) Decreasing $\frac{1}{3}$ unit/sec (c) Decreasing $\frac{7}{2}$ unit/sec

(d) Increasing $\frac{7}{2}$ unit/sec (e) None of these

1—M—Answer: b

8. A point moves along the curve $y = 2x^2 - 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = -\frac{3}{2}$?

(a) Decreasing $\frac{7}{2}$ unit/sec (b) Increasing $\frac{7}{2}$ unit/sec (c) Increasing $\frac{1}{3}$ unit/sec

(d) Decreasing $\frac{1}{3}$ unit/sec (e) None of these

1—M—Answer: c

9. A balloon rises at the rate of 8 feet per second from a point on the ground 60 feet from an observer. Find the rate of the angle of elevation when the balloon is 25 feet above the ground.

2—O—Answer: $\frac{96}{845}$ rad/sec

10. The volume of a cube is changing at the rate of 18 cubic centimeters per second. How fast is the edge of the cube expanding when each edge is 2 centimeters?

(a) $\sqrt[3]{18}$ cm/sec (b) $\frac{3}{2}$ cm/sec (c) 3 cm/sec

(d) $\sqrt[3]{9}$ cm/sec (e) None of these

1—M—Answer: b

11. The formula for the volume of a tank is $V = \pi r^3$ where r is the radius of the tank. If water is flowing in at the rate of 15 cubic feet per minute, find the rate at which the radius is changing when the radius is 3 feet.

(a) $\frac{5}{9\pi}$ ft/min (b) $\frac{5}{3\pi}$ ft/min (c) $\frac{5}{9}$ ft/min

(d) $\frac{5}{3}\pi$ ft/min (e) None of these

1—M—Answer: a

12. The formula for the volume of a tank is $V = 2\pi r^3$ where r is the radius of the tank. If the radius is increasing at the rate of $\frac{3}{2}$ feet per minute, find the rate at which the volume is increasing when the radius is 3 feet.

1—O—Answer: 81π ft^3/min

13. As a balloon in the shape of a sphere is being blown up, the radius in increasing $\frac{1}{\pi}$ inches per second. At what rate is the volume increasing when the radius is 1 inch?

(a) 4π in.^{3}/sec (b) 3 in.^{3}/sec (c) 4 in.^{3}/sec

(d) 3π in.^{3}/sec (e) None of these

1—M—Answer: c

14. Air is being pumped into a spherical balloon at a rate of 28 cubic feet per minute. At what rate is the radius changing when the radius is 3 feet?

$\left(V = \frac{4}{3}\pi r^3\right)$

1—O—Answer: $\frac{7}{9\pi}$ ft/min

15. Assume x and y are both differentiable functions of t. Find $\frac{dx}{dt}$ given $y = -1$ and $\frac{dy}{dt} = 12 : 2x^3 + y^2 = 3$.

1—O—Answer: $\frac{dx}{dt} = 4$

16. Assume x and y are both differentiable functions of t. Find $\frac{dx}{dt}$ given $y = 1$ and $\frac{dy}{dt} = \frac{1}{9} : 3x^2 + 2y^3 = 57$.

(a) -1 (b) $\frac{1}{3}$ (c) 6 (d) -54 (e) None of these

1—M—Answer: a

17. A particle moves on the curve $y = \frac{4}{3 - x^2}$ such that $\frac{dy}{dt} = 6$. Find the instantaneous rate of change of x with respect to t when $x = 5$.

1—O—T—Answer: 72.6

18. A particle moves on the curve $y = \frac{3}{x^2 + 4}$ such that $\frac{dy}{dt} = 6$. Find the instantaneous rate of change of x with respect to t when $x = 2$.

(a) -128 (b) -32 (c) 32 (d) 128 (e) None of these

1—M—Answer: b

19. A point moves along the curve $y = \frac{-x^2 + 4x - 3}{10}$ so that the y value is decreasing at a rate of 3 units per second. Find the instantaneous rate of change of x with respect to time at the point on the curve where $x = 5$.

1—O—Answer: 5 units/sec

20. A side of a square is increasing at the rate of 2 feet per minute. Find the rate at which the area is increasing when the side is 7 feet.

(a) 28 ft^2/min (b) 49 ft^2/min (c) 14 ft^2/min

(d) 28π ft^2/min (e) None of these

2—M—Answer: a

21. A spherical balloon is inflated at the rate of 16 cubic feet per minute. How fast is the radius of the balloon changing at the instant the radius is 2 feet?

(a) 4π ft/min (b) $\frac{32}{3}\pi$ ft/min (c) $\frac{1}{\pi}$ ft/min

(d) 2π ft/min (e) None of these

2—M—Answer: c

22. A spherical balloon is inflated at the rate of 12 cubic feet per minute. How fast is the radius of the balloon changing at the instant the radius is 3 feet?

(a) $\frac{1}{3\pi}$ ft/min (b) $\frac{3}{\pi}$ ft/min (c) 27π ft/min

(d) 3π ft/min (e) None of these

2—M—Answer: a

23. A metal cube contracts when it is cooled. If the edge of the cube is decreasing at a rate of 0.2 cm/hr, how fast is the volume changing when the edge is 60 centimeters?

2—O—T—Answer: -36.0 cm^3/hr

24. A 5-meter-long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at a rate of 0.4 m/sec. Determine how fast the top of the ladder is descending when the foot of the ladder is 3 meters from the house.

(a) 1.2 m/sec (b) -1.2 m/sec (c) 0.3 m/sec

(d) -0.3 m/sec (e) None of these

1—M—Answer: d

25. Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the two boats changing after 2 hours?

(a) 19.2 knots/hr (b) 26.8 knots/hr (c) 17.7 knots/hr

(d) 38.4 knots/hr (e) None of these

2—M—Answer: a

26. Two boats leave the same port at the same time with one boat traveling north at 35 knots per hour and the other boat traveling east at 40 knots per hour. How fast is the distance between the two boats changing after 2 hours?

(a) 106.4 knots/hr (b) 429.4 knots/hr (c) 53.2 knots/hr

(d) 57.9 knots/hr (e) None of these

2—M—Answer: c

27. The area of a circle is decreasing at a rate of 2 square centimeters per minute. Find the rate of change of the radius with respect to time when the radius is 4 centimeters.

(a) $\frac{1}{4\pi}$ cm/min (b) $-\frac{1}{8\pi}$ cm/min (c) $-\frac{1}{2\pi}$ cm/min

(d) $\frac{1}{4\pi}$ cm/min (e) None of these

2—M—Answer: e

28. The height of a cylinder with a radius of 4 cm is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume of the cylinder with respect to time when the height is 10 centimeters.

(a) 16π cm^3/min (b) $\frac{1}{16\pi}$ cm^3/min (c) $\frac{5}{8\pi}$ cm^3/min

(d) 160π cm^3/sec (e) None of these

2—M—Answer: e

29. The radius of a circle is increasing at the rate of 2 feet per minute. Find the rate at which the area is increasing when the radius is 7 feet.

(a) 28 ft^2/min (b) 49π ft^2/min (c) 14π ft^2/min

(d) 28π ft^2/min (e) None of these

2—M—Answer: d

CHAPTER THREE

Applications of Differentiation

3.1 Extrema on an Interval

1. Find all critical numbers for the function: $f(x) = \dfrac{x-1}{x+3}$.

(a) 1 (b) $1, -3$ (c) -3

(d) $1, -1$ (e) None of these

1—M—Answer: e

2. Find all critical numbers for the function: $f(x) = (9 - x^2)^{3/5}$.

(a) 0 (b) 3 (c) $-3, 3$

(d) $-3, 0, 3$ (e) None of these

1—M—Answer: d

3. Find all critical numbers for the function: $f(x) = x\sqrt{2x+1}$.

1—O—Answer: $-\frac{1}{3}, -\frac{1}{2}$

4. Find all critical numbers for the function: $f(x) = 3x^4 - 4x^3$.

1—O—Answer: $x = 0,\ 1$

5. Find all critical numbers for the function: $f(x) = x^3 - 12x^2$.

1—O—Answer: $x = 0,\ x = 8$

6. Find all extrema in the interval $[0,\ 2\pi]$ for $y = x + \sin x$.

(a) $\left(-1,\ -1 + \dfrac{3\pi}{2}\right)$ (b) $(\pi,\ \pi)$ (c) $(-1, 0)$

(d) $\left(\dfrac{3\pi}{2},\ 0\right)$ (e) None of these

2—M—Answer: b

7. Find all extrema in the interval $[0, 2\pi]$ for $y = x - \cos x$.

(a) $\left(-1, -1 + \frac{3\pi}{2}\right)$ (b) $(\pi, \pi + 1)$ (c) $(-1, 0)$

(d) $\left(\frac{3\pi}{2}, \frac{2\pi}{2}\right)$ (e) None of these

2—M—Answer: e

8. Find all extrema in the interval $[0, 2\pi]$ for $y = \sin x + \cos x$.

2—O—Answer: $\left(\frac{\pi}{4}, \sqrt{2}\right), \left(\frac{5\pi}{4}, -\sqrt{2}\right)$

9. Find the absolute maximum and absolute minimum of f on the interval $(0, 3]$.

$$f(x) = \frac{x^3 - 4x^2 + 7x}{x}$$

(a) Maximum: None Minimum: (3, 4) (b) Maximum: (0, 7) Minimum: (3, 4)

(c) Maximum: None Minimum: (2, 3) (d) Maximum: (0, 7) Minimum: (2, 3)

(e) None of these

2—M—Answer: c

10. Find the absolute maximum and absolute minimum of f on the interval $(-1, 2]$.

$$f(x) = \frac{-x^3 + x^2 + 3x + 1}{x + 1}$$

(a) Maximum: $(1, -2)$ Minimum: $(-1, 2)$ (b) Maximum: $(1, -2)$ Minimum: None

(c) Maximum: None Minimum: None (d) Maximum: None Minimum: $(-1, 2)$

(e) None of these

2—M—Answer: b

11. Find the absolute maximum and absolute minimum of f on the interval $(1, 4]$.

$$f(x) = \frac{x^3 - 7x^2 + 12x - 6}{x - 1}$$

(a) Maximum: None Minimum: $(3, -3)$ (b) Maximum: (1, 1) Minimum: $(4, -2)$

(c) Maximum: (1, 1) Minimum: $(3, -3)$ (d) Maximum: None Minimum: $(4, -2)$

(e) None of these

2—M—Answer: a

12. Find the absolute maximum and absolute minimum of f on the interval $(-4, -1]$.

$$f(x) = \frac{x^3 + 8x^2 + 19x + 12}{x + 4}$$

(a) Maximum: None Minimum: $(-2, -1)$

(b) Maximum: $(-4, 3)$ Minimum: $(-1, 0)$

(c) Maximum: $(-4, 3)$ Minimum: $(-2, -1)$

(d) Maximum: None Minimum: $(-1, 0)$

(e) None of these

2—M—Answer: a

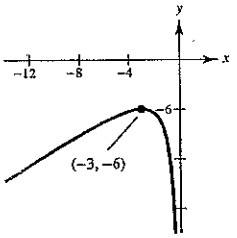
13. Find the absolute minimum and the absolute maximum for $f(x) = \dfrac{10}{(x^2 + 1)}$ on the interval $[-1, 2]$.

2—O—Answer: Maximum at $(0, 10)$; Minimum at $(2, 2)$

14. Find the minimum and maximum values of $f(x) = x^2 - 2x + 1$ on the interval $[0, 3]$.

2—O—Answer: Minimum at $(1, 0)$; Maximum at $(3, 4)$

15. Find the value of the derivative (if it exists) at the indicated extremum.



(a) 2

(b) -6

(c) 0

(d) The derivative does not exist.

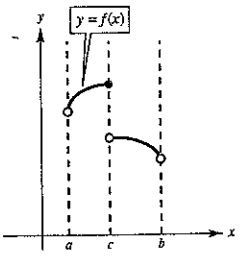
(e) None of these

1—M—Answer: c

16. Use a graphing utility to graph $f(x) = |x - 2|$. Use the graph to find the value of the derivative (if it exists) at the extremum.

1—O—T—Answer: The derivative does not exist at the point $(2, 0)$.

17. Determine from the graph whether f possesses extrema on the interval (a, b).



(a) Maximum at $x = c$, minimum at $x = b$

(b) Maximum at $x = c$, no minimum

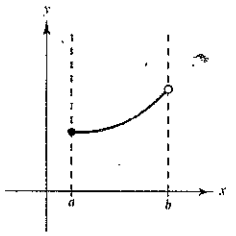
(c) No maximum, minimum at $x = b$

(d) No extrema

(e) None of these

1—M—Answer: b

18. Determine from the graph whether f possesses extrema on the interval (a, b).



(a) Maximum at $x = a$; minimum at $x = b$

(b) Maximum at $x = b$; minimum at $x = a$

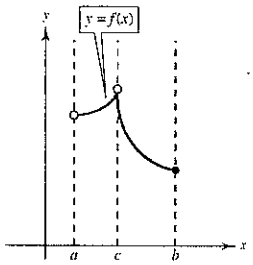
(c) No extrema

(d) No maximum; minimum at $x = a$

(e) None of these

1—M—Answer: d

19. Determine from the graph whether f possesses extrema on the interval (a, b).



1—O—Answer: No maximum, minimum at $x = b$

20. Explain why $f(x) = \dfrac{1}{x}$ has a minimum on the interval $[1, 2]$ but not on the interval $[-1, 1]$.

2—O—Answer: $f(x) = \dfrac{1}{x}$ is continuous on the closed interval $[1, 2]$ and, therefore, the Extreme Value Theorem guarantees the existence of extrema on that interval. However, $f(x)$ is not continuous on the interval $[-1, 1]$ and $\lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty$.

21. Find all critical numbers for the function $f(x) = (x + 2)^3(x - 1)^4$.

(a) -2 and 1 (b) $-2, -\frac{7}{5}, 1$ (c) -2

(d) $-2, 1, -\frac{5}{7}$ (e) None of these

2—M—Answer: d

22. Find all critical numbers for the function $f(x) = (x + 2)^4(x - 1)^3$.

(a) 2 and -1 (b) $-2, -\frac{2}{7}, 1$ (c) -2 and 1

(d) $-2, -\frac{7}{2}, 1$ (e) None of these

2—M—Answer: b

23. Find all critical numbers for the function $f(x) = x^3(x + 3)^2$.

2—O—Answer: $-3, -\frac{9}{5}$, and 0