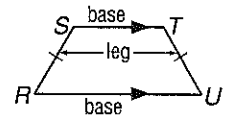


# 8-6 Study Guide and Intervention

## Trapezoids

**Properties of Trapezoids** A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases** and the nonparallel sides are called **legs**. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent.



$STUR$  is an isosceles trapezoid.  
 $\overline{SR} \cong \overline{TU}$ ;  $\angle R \cong \angle U$ ,  $\angle S \cong \angle T$

### Example

The vertices of  $ABCD$  are  $A(-3, -1)$ ,  $B(-1, 3)$ ,  $C(2, 3)$ , and  $D(4, -1)$ . Verify that  $ABCD$  is a trapezoid.

$$\text{slope of } \overline{AB} = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2$$

$$\text{slope of } \overline{AD} = \frac{-1 - (-1)}{4 - (-3)} = \frac{0}{7} = 0$$

$$\text{slope of } \overline{BC} = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$$

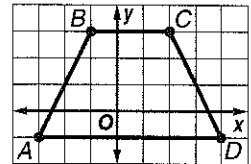
$$\text{slope of } \overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$$

$$AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

$$CD = \sqrt{(2 - 4)^2 + (3 - (-1))^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$



Exactly two sides are parallel,  $\overline{AD}$  and  $\overline{BC}$ , so  $ABCD$  is a trapezoid.  $AB = CD$ , so  $ABCD$  is an isosceles trapezoid.

### Exercises

In Exercises 1-3, determine whether  $ABCD$  is a trapezoid. If so, determine whether it is an isosceles trapezoid. Explain.

1.  $A(-1, 1)$ ,  $B(2, 1)$ ,  $C(3, -2)$ , and  $D(2, -2)$

2.  $A(3, -3)$ ,  $B(-3, -3)$ ,  $C(-2, 3)$ , and  $D(2, 3)$

3.  $A(1, -4)$ ,  $B(-3, -3)$ ,  $C(-2, 3)$ , and  $D(2, 2)$

4. The vertices of an isosceles trapezoid are  $R(-2, 2)$ ,  $S(2, 2)$ ,  $T(4, -1)$ , and  $U(-4, -1)$ . Verify that the diagonals are congruent.

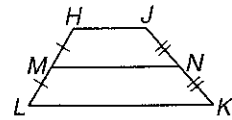
## 8-6

## Study Guide and Intervention (continued)

## Trapezoids

**Medians of Trapezoids** The **median** of a trapezoid is the segment that joins the midpoints of the legs. It is parallel to the bases, and its length is one-half the sum of the lengths of the bases.

In trapezoid  $HJKL$ ,  $MN = \frac{1}{2}(HJ + LK)$ .

**Example**

$\overline{MN}$  is the median of trapezoid  $RSTU$ . Find  $x$ .

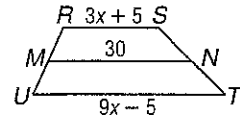
$$MN = \frac{1}{2}(RS + UT)$$

$$30 = \frac{1}{2}(3x + 5 + 9x - 5)$$

$$30 = \frac{1}{2}(12x)$$

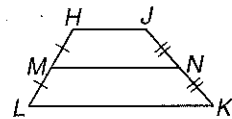
$$30 = 6x$$

$$5 = x$$

**Exercises**

$\overline{MN}$  is the median of trapezoid  $HJKL$ . Find each indicated value.

- Find  $MN$  if  $HJ = 32$  and  $LK = 60$ .
- Find  $LK$  if  $HJ = 18$  and  $MN = 28$ .
- Find  $MN$  if  $HJ + LK = 42$ .
- Find  $m\angle LMN$  if  $m\angle LHJ = 116$ .
- Find  $m\angle JKL$  if  $HJKL$  is isosceles and  $m\angle HLK = 62$ .
- Find  $HJ$  if  $MN = 5x + 6$ ,  $HJ = 3x + 6$ , and  $LK = 8x$ .
- Find the length of the median of a trapezoid with vertices  $A(-2, 2)$ ,  $B(3, 3)$ ,  $C(7, 0)$ , and  $D(-3, -2)$ .

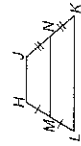


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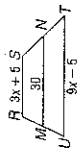
## 8-6 Study Guide and Intervention (continued)

### Trapezoids

**Medians of Trapezoids** The median of a trapezoid is the segment that joins the midpoints of the legs. It is parallel to the bases, and its length is one-half the sum of the lengths of the bases. In trapezoid  $HJKL$ ,  $MN = \frac{1}{2}(HJ + LK)$ .



**Example**  $\overline{MN}$  is the median of trapezoid  $RSTU$ . Find  $x$ .



$$MN = \frac{1}{2}(RS + UT)$$

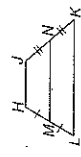
$$30 = \frac{1}{2}(3x + 5 + 9x - 5)$$

$$30 = \frac{1}{2}(12x)$$

$$30 = 6x$$

$$5 = x$$

**Example**  $\overline{MN}$  is the median of trapezoid  $HJKL$ . Find each indicated value.



- Find  $MN$  if  $HJ = 32$  and  $LK = 60$ .  
46
- Find  $LK$  if  $HJ = 18$  and  $MN = 28$ .  
38
- Find  $MN$  if  $HJ + LK = 42$ .  
21
- Find  $m\angle LMN$  if  $m\angle LHJ = 116$ .  
116
- Find  $m\angle JKL$  if  $HJKL$  is isosceles and  $m\angle HLK = 62$ .  
62
- Find  $HJ$  if  $MN = 5x + 6$ ,  $HJ = 3x + 6$ , and  $LK = 8x$ .  
24

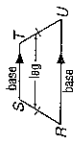
- Find the length of the median of a trapezoid with vertices  $A(-2, 2)$ ,  $B(3, 3)$ ,  $C(7, 0)$ , and  $D(-3, -2)$ .  
 $\frac{3\sqrt{26}}{2}$

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## 8-6 Study Guide and Intervention

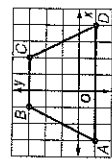
### Trapezoids

**Properties of Trapezoids** A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called bases and the nonparallel sides are called legs. If the legs are congruent, the trapezoid is an isosceles trapezoid. In an isosceles trapezoid both pairs of base angles are congruent.



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 $SR \parallel UT$ ;  $\angle R \cong \angle U$ ,  $\angle S \cong \angle T$

**Example** The vertices of  $ABCD$  are  $A(-3, -1)$ ,  $B(-1, 3)$ ,  $C(2, 3)$ , and  $D(4, -1)$ . Verify that  $ABCD$  is a trapezoid.



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$$\text{slope of } \overline{BC} = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$$

$$\text{slope of } \overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$$

Exactly two sides are parallel,  $\overline{AD}$  and  $\overline{BC}$ , so  $ABCD$  is a trapezoid.  $AB = CD$ , so  $ABCD$  is an isosceles trapezoid.

In Exercises 1-3, determine whether  $ABCD$  is a trapezoid. If so, determine whether it is an isosceles trapezoid. Explain.

- $A(-1, 1)$ ,  $B(2, 1)$ ,  $C(3, -2)$ , and  $D(2, -2)$   
Slope of  $\overline{AB} = 0$ , slope of  $\overline{DC} = 0$ , slope of  $\overline{AD} = -1$ , slope of  $\overline{BC} = -3$ . Exactly two sides are parallel, so  $ABCD$  is a trapezoid.  $AD = 3\sqrt{2}$  and  $BC = \sqrt{10}$ ;  $AD \neq BC$ , so  $ABCD$  is not isosceles.
- $A(3, -3)$ ,  $B(-3, -3)$ ,  $C(-2, 3)$ , and  $D(2, 3)$   
Slope of  $\overline{AB} = 0$ , slope of  $\overline{DC} = 0$ , slope of  $\overline{BC} = 6$ , slope of  $\overline{AD} = -6$ . Exactly 2 sides are  $\parallel$ , so  $ABCD$  is a trapezoid.  $BC = \sqrt{37}$  and  $AD = \sqrt{37}$ ;  $BC = AD$ , so  $ABCD$  is isosceles.
- $A(1, -4)$ ,  $B(-3, -3)$ ,  $C(-2, 3)$ , and  $D(2, 2)$   
Slope of  $\overline{AB} = -\frac{1}{4}$ , slope of  $\overline{DC} = -\frac{1}{4}$ , slope of  $\overline{BC} = 6$ , slope of  $\overline{AD} = 6$ . Both pairs of opposite sides are parallel, so  $ABCD$  is not a trapezoid.

- The vertices of an isosceles trapezoid are  $R(-2, 2)$ ,  $S(2, 2)$ ,  $T(4, -1)$ , and  $U(-4, -1)$ . Verify that the diagonals are congruent.  
 $RT = \sqrt{((4 - (-2))^2 + (-1 - 2)^2)} = \sqrt{45}$  or  $3\sqrt{5}$   
 $SU = \sqrt{((-4 - 2))^2 + (-1 - 2)^2} = \sqrt{45}$  or  $3\sqrt{5}$