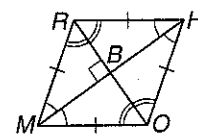


8-5 Study Guide and Intervention

Rhombi and Squares

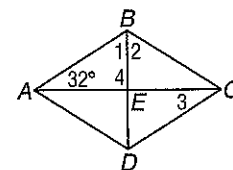
Properties of Rhombi A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	\overline{MH} bisects $\angle RMO$ and $\angle RHO$. \overline{RO} bisects $\angle MRH$ and $\angle MOH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If $RHOM$ is a parallelogram and $\overline{RO} \perp \overline{MH}$, then $RHOM$ is a rhombus.

Example In rhombus $ABCD$, $m\angle BAC = 32$. Find the measure of each numbered angle.

$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58 . The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

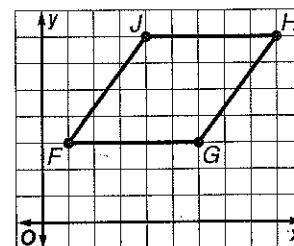
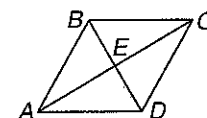


A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

$ABCD$ is a rhombus.

- If $m\angle ABD = 60$, find $m\angle BDC$.
- If $AE = 8$, find AC .
- If $AB = 26$ and $BD = 20$, find AE .
- Find $m\angle CEB$.
- If $m\angle CBD = 58$, find $m\angle ACB$.
- If $AE = 3x - 1$ and $AC = 16$, find x .
- If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y .
- If $AD = 2x + 4$ and $CD = 4x - 4$, find x .
- What is the midpoint of \overline{FH} ?
 - What is the midpoint of \overline{GJ} ?
 - What kind of figure is $FGHJ$? Explain.
 - What is the slope of \overline{FH} ?
 - What is the slope of \overline{GJ} ?
 - Based on parts **c**, **d**, and **e**, what kind of figure is $FGHJ$? Explain.



8-5

Study Guide and Intervention *(continued)***Rhombi and Squares**

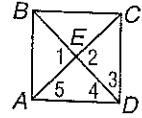
Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

Example

Find the measure of each numbered angle of square $ABCD$.

Using properties of rhombi and rectangles, the diagonals are perpendicular and congruent. $\triangle ABE$ is a right triangle, so $m\angle 1 = m\angle 2 = 90$.

Each vertex angle is a right angle and the diagonals bisect the vertex angles, so $m\angle 3 = m\angle 4 = m\angle 5 = 45$.

**Exercises**

Determine whether the given vertices represent a *parallelogram*, *rectangle*, *rhombus*, or *square*. Explain your reasoning.

✓ 1. $A(0, 2), B(2, 4), C(4, 2), D(2, 0)$

✓ 3. $A(-2, -1), B(0, 2), C(2, -1), D(0, -4)$

↓ ✓ 4. $A(-3, 0), B(-1, 3), C(5, -1), D(3, -4)$

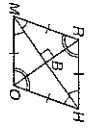
7. Square $RSTU$ has vertices $R(-3, -1), S(-1, 2)$, and $T(2, 0)$. Find the coordinates of vertex U .

8-5 Study Guide and Intervention

Rhombi and Squares

Properties of Rhombi A rhombus is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

The diagonals are perpendicular.	$MH \perp FO$
Each diagonal bisects a pair of opposite angles.	MH bisects $\angle FMO$ and $\angle FHO$. FO bisects $\angle MHR$ and $\angle MCH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If FHO is a parallelogram and $FO \perp MH$, then FHO is a rhombus.

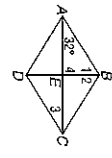


EXAMPLE 2

In rhombus $ABCD$, $m\angle BAC = 32^\circ$. Find the measure of each numbered angle.

$ABCD$ is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90^\circ$ and $m\angle 1 = 90^\circ - 32^\circ = 58^\circ$. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58^\circ$.

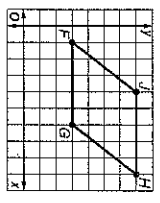
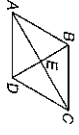
A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32^\circ$.



EXAMPLE 3

$ABCD$ is a rhombus.

- If $m\angle ABD = 60^\circ$, find $m\angle BDC$. 60
- If $AE = 8$, find AC . 16
- If $AB = 26$ and $BD = 20$, find AE . 24
- Find $m\angle CEB$. 90
- If $m\angle CBD = 58^\circ$, find $m\angle ACB$. 32
- If $AE = 3x - 1$ and $AC = 16$, find x . 3
- If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y . 10
- If $AD = 2x + 4$ and $CD = 4x - 4$, find x . 4
- What is the midpoint of \overline{FH} ? $(5, 5)$
 - What is the midpoint of \overline{GJ} ? $(5, 5)$
 - What kind of figure is $FGHJ$? Explain. $FGHJ$ is a parallelogram because the diagonals bisect each other.
 - What is the slope of \overline{FH} ? 1
 - What is the slope of \overline{GJ} ? -2
 - Based on parts c, d, and e, what kind of figure is $FGHJ$? Explain. $FGHJ$ is a parallelogram with perpendicular diagonals, so it is a rhombus.



8-5 Study Guide and Intervention (continued)

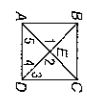
Rhombi and Squares

Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

EXAMPLE 4

Find the measure of each numbered angle of square $ABCD$.

Using properties of rhombi and rectangles, the diagonals are perpendicular and congruent. $\triangle ABE$ is a right triangle, so $m\angle 1 = m\angle 2 = 90^\circ$. Each vertex angle is a right angle and the diagonals bisect the vertex angles, so $m\angle 3 = m\angle 4 = m\angle 5 = m\angle 6 = 45^\circ$.



EXAMPLE 5

Determine whether the given vertices represent a parallelogram, rectangle, rhombus, or square. Explain your reasoning.

- $A(0, 2)$, $B(2, 4)$, $C(4, 2)$, $D(2, 0)$
Square; the four sides are \cong and consecutive sides are \perp .
- $D(-2, 1)$, $E(-1, 3)$, $F(3, 1)$, $G(2, -1)$
Rectangle; both pairs of opposite sides are \parallel and consecutive sides are \perp .
- $A(-2, -1)$, $B(0, 2)$, $C(2, -1)$, $D(0, -4)$
Rhombus; the four sides are \cong and consecutive sides are not \perp .
- $A(-3, 0)$, $B(-1, 3)$, $C(5, -1)$, $D(3, -4)$
Rectangle; both pairs of opposite sides are \parallel and consecutive sides are \perp .
- $S(-1, 4)$, $T(3, 2)$, $U(1, -2)$, $V(-3, 0)$
Square; the four sides are \cong and consecutive sides are \perp .
- $R(-1, 0)$, $G(1, 3)$, $H(4, 1)$, $K(2, -2)$
Square; the four sides are \cong and consecutive sides are \perp .
- Square $RSTU$ has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(2, 0)$. Find the coordinates of vertex U . $(0, -3)$