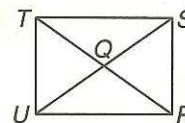


8-4

Study Guide and Intervention

Rectangles

Properties of Rectangles A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS, \angle TSR, \angle SRU,$ and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1

In rectangle $RSTU$ above, $US = 6x + 3$ and $RT = 7x - 2$. Find x .

The diagonals of a rectangle bisect each other, so $US = RT$.

$$\begin{aligned} 6x + 3 &= 7x - 2 \\ 3 &= x - 2 \\ 5 &= x \end{aligned}$$

Example 2

In rectangle $RSTU$ above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

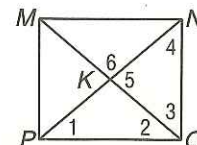
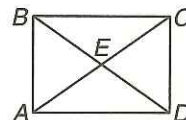
$$\begin{aligned} 8x + 3 + 16x - 9 &= 90 \\ 24x - 6 &= 90 \\ 24x &= 96 \\ x &= 4 \end{aligned}$$

$$m\angle STR = 8x + 3 = 8(4) + 3 \text{ or } 35$$

Exercises

$ABCD$ is a rectangle.

1. If $AE = 36$ and $CE = 2x - 4$, find x .
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y .
3. If $BC = 24$ and $AD = 5y - 1$, find y .
4. If $m\angle BEA = 62$, find $m\angle BAC$.
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$.
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD .
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$.
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y .
9. In rectangle $MNOP$, $m\angle 1 = 40$. Find the measure of each numbered angle.



8-4 Study Guide and Intervention *(continued)*

Rectangles

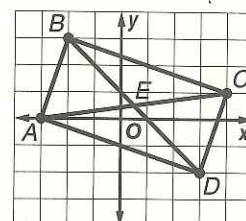
Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example

Determine whether $A(-3, 0)$, $B(-2, 3)$, $C(4, 1)$, and $D(3, -2)$ are the vertices of a rectangle.



Method 1: Use the Slope Formula.

$$\text{slope of } \overline{AB} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} \text{ or } 3 \quad \text{slope of } \overline{AD} = \frac{-2 - 0}{3 - (-3)} = \frac{-2}{6} \text{ or } -\frac{1}{3}$$

$$\text{slope of } \overline{CD} = \frac{-2 - 1}{3 - 4} = \frac{-3}{-1} \text{ or } 3 \quad \text{slope of } \overline{BC} = \frac{1 - 3}{4 - (-2)} = \frac{-2}{6} \text{ or } -\frac{1}{3}$$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so $ABCD$ is a rectangle.

Method 2: Use the Midpoint and Distance Formulas.

The midpoint of \overline{AC} is $\left(\frac{-3 + 4}{2}, \frac{0 + 1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and the midpoint of \overline{BD} is $\left(\frac{-2 + 3}{2}, \frac{3 - 2}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$. The diagonals have the same midpoint so they bisect each other.

Thus, $ABCD$ is a parallelogram.

$$AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} = \sqrt{49 + 1} \text{ or } \sqrt{50}$$

$$BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} = \sqrt{25 + 25} \text{ or } \sqrt{50}$$

The diagonals are congruent. $ABCD$ is a parallelogram with diagonals that bisect each other, so it is a rectangle.

Exercises

Determine whether $ABCD$ is a rectangle given each set of vertices. Justify your answer.

1. $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$ 2. $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$

3. $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$ 4. $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$

5. $A(-1, -5)$, $B(-3, 0)$, $C(2, 2)$, $D(4, -3)$ 6. $A(-1, -1)$, $B(0, 2)$, $C(4, 3)$, $D(3, 0)$

7. A parallelogram has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(5, -2)$. Find the coordinates of U so that $RSTU$ is a rectangle.

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8-4 Study Guide and Intervention Rectangles

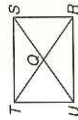
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Example 1

In rectangle $RSTU$ above, $US = 6x + 3$ and $RT = 7x - 2$. Find x .

The diagonals of a rectangle bisect each other, so $US = RT$.

$$6x + 3 = 7x - 2$$

$$3 = x - 2$$

$$5 = x$$

Example 2

In rectangle $RSTU$ above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.

$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$$8x + 3 + 16x - 9 = 90$$

$$24x - 6 = 90$$

$$24x = 96$$

$$x = 4$$

$$m\angle STR = 8x + 3 = 8(4) + 3 \text{ or } 35$$

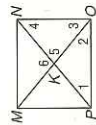
Example 3

$ABCD$ is a rectangle.



1. If $AE = 36$ and $CE = 2x - 4$, find x . **20**
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y . **2**
3. If $BC = 24$ and $AD = 5y - 1$, find y . **5**
4. If $m\angle BEA = 62$, find $m\angle BAC$. **59**
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$. **120**
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD . **52**
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$. **30**
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y . **10y**

9. In rectangle $MNOP$, $m\angle 1 = 40$. Find the measure of each numbered angle.



$m\angle 2 = 40$; $m\angle 3 = 50$; $m\angle 4 = 50$; $m\angle 5 = 80$; $m\angle 6 = 100$

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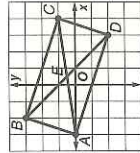
8-4 Study Guide and Intervention Rectangles

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slope of $\overline{AB} = \frac{3-0}{-2-(-3)} = \frac{3}{1}$ or 3 slope of $\overline{AD} = \frac{-2-0}{3-(-3)} = \frac{-2}{6}$ or $-\frac{1}{3}$
 slope of $\overline{CD} = \frac{-2-1}{3-4} = \frac{-3}{-1}$ or 3 slope of $\overline{BC} = \frac{1-3}{4-(-2)} = \frac{-2}{6}$ or $-\frac{1}{3}$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so $ABCD$ is a rectangle.

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$$AC = \sqrt{(-3-4)^2 + (0-1)^2} = \sqrt{49+1} \text{ or } \sqrt{50}$$

$$BD = \sqrt{(-2-3)^2 + (3-(-2))^2} = \sqrt{25+25} \text{ or } \sqrt{50}$$

The diagonals are congruent. $ABCD$ is a parallelogram with diagonals that bisect each other, so it is a rectangle.

Example 3

Determine whether $ABCD$ is a rectangle given each set of vertices. Justify your answer. **Sample justifications are given.**

1. $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$ **2. $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$**
Yes; consecutive sides are \perp . **No; consecutive sides are not \perp .**
3. $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$ **4. $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$**
No; consecutive sides are not \perp . **Yes; consecutive sides are \perp .**
5. $A(-1, -5)$, $B(-3, 0)$, $C(2, 2)$, $D(4, -3)$ **6. $A(-1, -1)$, $B(0, 2)$, $C(4, 3)$, $D(3, 0)$**
Yes; the diagonals are congruent and bisect each other. **No; the diagonals are not \cong .**
7. A parallelogram has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(5, -2)$. Find the coordinates of U so that $RSTU$ is a rectangle. **(3, -5)**