8.2 Inverse Trigonometric Functions

Inverse Sine Function

For each v with $-1 \le v \le 1$,

$\sin^{-1} v$ is the unique number u in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is v; that is,

$$\sin^{-1} v = u \quad \text{exactly when} \quad \sin u = v.$$

Properties of Inverse Sine

$$\sin^{-1}(\sin u) = u \quad \text{if} \quad -\frac{\pi}{2} \le u \le \frac{\pi}{2}$$

$$\sin(\sin^{-1} v) = v \quad \text{if} \quad -1 \le v \le 1$$

Inverse Cosine Function

For each v with $-1 \le v \le 1$,

$\cos^{-1} v$ is the unique number u in the interval $[0, \pi]$ whose cosine is v; that is,

$$\cos^{-1} v = u \quad \text{exactly when} \quad \cos u = v.$$

Properties of Inverse Cosine

$$\cos^{-1}(\cos u) = u \quad \text{if} \quad 0 \le u \le \pi$$

$$\cos(\cos^{-1} v) = v \quad \text{if} \quad -1 \le v \le 1$$

Inverse Tangent Function

For each real number v,

$\tan^{-1} v$ is the unique number u in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is v; that is,

$$\tan^{-1} v = u \quad \text{exactly when} \quad \tan u = v.$$

Properties of Inverse Tangent

$$\tan^{-1}(\tan u) = u \quad \text{if} \quad -\frac{\pi}{2} < u < \frac{\pi}{2}$$

$$\tan(\tan^{-1} v) = v \text{ for every real number } v.$$