Name _____ Date _____ Class _____

LESSON 8-2

Reteach

Factoring by GCF

The Distributive Property states: $a(b + c) = ab + ac$

Factoring by GCF reverses the Distributive Property:

$ab + ac = a(b + c)$

Factor $12x^3 + 21x^2 + 15x$. Check your answer.

Step 1: Find the GCF of all the terms in the polynomial.

The factors of $12x^3$ are: 1, 2, **3**, 4, 6, 12, ***x***, *x*, *x*

The factors of $21x^2$ are: 1, **3**, 7, 21, ***x***, *x*

The factors of $15x$ are: 1, **3**, 5, 15, ***x***

} The GCF is **3*x***.

Step 2: Write terms as products using the GCF.

$12x^3 + 21x^2 + 15x$

$(3x)4x^2 + (3x)7x + (3x)5$

Step 3: Use the Distributive Property to factor out the GCF.

$3x(4x^2 + 7x + 5)$

***Check*:**

$3x(4x^2 + 7x + 5) = 12x^3 + 21x^2 + 15x$ ✓

Factor $5(x - 3) + 4x(x - 3)$.

Step 1: Find the GCF of all the terms in the polynomial.

The factors of $5(x - 3)$ are: 5, $(x - 3)$

The factors of $4x(x - 3)$ are: 4, *x*, $(x - 3)$

} The GCF is $(x - 3)$.

The terms are already written as products with the GCF.

Step 2: Use the Distributive Property to factor out the GCF.

$(x - 3)(5 + 4x)$

Factor each polynomial.

1. $20x^2 - 15x$ _____

2. $44a^2 + 11a$ _____

3. $24y - 36x$ _____

Factor each expression.

4. $5x(x + 7) + 2(x + 7)$ _____

5. $3a(a + 4) - 2(a + 4)$ _____

6. $4y(4y + 1) + (4y + 1)$ _____

Name _____ Date _____ Class _____

LESSON 8-2

Reteach

Factoring by GCF (continued)

When a polynomial has four terms, make two groups and factor out the GCF from each group.

Factor $8x^3 + 6x^2 + 20x + 15$.

Step 1: Group terms that have common factors.

$$(8x^3 + 6x^2) + (20x + 15)$$

Step 2: Identify and factor the GCF out of each group.

$(8x^3 + 6x^2) + (20x + 15)$

GCF is $\mathbf{2x^2}$. GCF is **5**.

$2x^2(4x + 3) + 5(4x + 3)$

Step 3: Factor out the common binomial factor.

$2x^2\mathbf{(4x + 3)} + 5\mathbf{(4x + 3)}$

GCF is $\mathbf{(4x + 3)}$. $(4x + 3)(2x^2 + 5)$

Check:

$(4x + 3)(2x^2 + 5)$

$4x(2x^2) + 4x(5) + 3(2x^2) + 3(5)$ *Use FOIL.*

$8x^3 + 20x + 6x^2 + 15$

$8x^3 + 6x^2 + 20x + 15$ *Rearrange terms.*

Factor each polynomial filling in the blanks.

7. $(18x^3 + 15x^2) + (24x + 20)$

GCF is ☐ GCF is ☐

☐$(6x + 5)$ + ☐$(6x + 5)$

(☐)$(6x + 5)$

8. $(10a^3 - 15a^2) + (12a - 18)$

GCF is ☐ GCF is ☐

☐$(2a - 3)$ + ☐$(2a - 3)$

(☐)$(2a - 3)$

Factor each polynomial by grouping.

9. $21x^3 + 12x^2 + 14x + 8$

10. $40x^3 - 50x^2 + 12x - 15$

LESSON 8-2

Practice A

Factoring by GCF

Factor each polynomial. Check your answer.

1. $x^2 + 5x$

$x(\underline{x} + \underline{5})$

2. $5m^3 + 45$

$\underline{5}(\underline{m^3} + 9)$

3. $15y^3 + 20y^5 - 10$

$\underline{5}(3y^3 + 4\underline{y^5} - \underline{2})$

4. $10y^2 + 12y^3$

$2y^2(5 + 6y)$

5. $-12t^5 + 6t$

$6t(-2t^4 + 1)$

6. $6x^4 + 15x^3 + 3x^2$

$3x^2(2x^2 + 5x + 1)$

7. A golf ball is hit upward at a speed of 40 m/s. The expression $-5t^2 + 40t$ gives the approximate height of the ball after t seconds. Factor this expression.

$5t(-t + 8)$

8. The area of the Hillen family's television screen is $3x^2 + 24x$ in^2. Factor this polynomial to find expressions for the dimensions of their TV screen.

$3x$ and $x + 8$

Factor out the common binomial factor in each expression.

9. $4d(d + 2) + 9(d + 2)$

$(d + 2)(4d + 9)$

10. $12(x - 5) + 7x(x - 5)$

$(x - 5)(12 + 7x)$

Factor each polynomial by grouping.

11. $n^3 + 3n^2 + 4n + 12$

$(n^3 + \underline{3n^2}) + (4n + \underline{12})$

$n^2(n + \underline{3}) + 4(n + \underline{3})$

$(n + 3)(n^2 + 4)$

12. $2x^3 + 5x^2 + 2x + 5$

$(2x + 5)(x^2 + 1)$

Factor each polynomial by grouping and using opposites.

13. $2y^3 - 4y^2 + 6 - 3y$

$(\underline{2y^3} - 4y^2) + (\underline{6} - 3y)$

$2y^2(\underline{y} - 2) + 3(\underline{2} - y)$

$2y^2(\underline{y} - 2) + 3(-1)(\underline{y} - 2)$

$2y^2(\underline{y} - 2) - 3(y - \underline{2})$

$(y - 2)(2y^2 - 3)$

14. $4m^3 - 12m^2 + 15 - 5m$

$(m - 3)(4m^2 - 5)$

LESSON 8-2

Practice B

Factoring by GCF

Factor each polynomial. Check your answer.

1. $8c^2 + 7c$

$c(8c + 7)$

2. $3n^3 + 12n^2$

$3n^2(n + 4)$

3. $15x^5 - 18x$

$3x(5x^4 - 6)$

4. $-8s^4 + 20t^3 - 28$

$4(-2s^4 + 5t^3 - 7)$

5. $6n^6 + 18n^4 - 24n$

$6n(n^5 + 3n^3 - 4)$

6. $-5m^4 - 5m^3 + 5m^2$

$5m^2(-m^2 - m + 1)$

7. A ball is hit vertically into the air using a paddle at a speed of 32 ft/sec. The expression $-16t^2 + 32t$ gives the ball's height after t seconds. Factor this expression.

$16t(-t + 2)$

8. The area of Margo's laptop computer screen is $12x^2 + 3x$ in^2. Factor this polynomial to find expressions for the dimensions of her computer screen.

$3x$ and $4x + 1$

Factor each expression.

9. $3m(m + 5) + 4(m + 5)$

$(m + 5)(3m + 4)$

10. $16b(b - 3) + (b - 3)$

$(b - 3)(16b + 1)$

Factor each polynomial by grouping.

11. $2x^3 + 8x^2 + 3x + 12$

$(x + 4)(2x^2 + 3)$

12. $4n^3 + 3n^2 + 4n + 3$

$(4n + 3)(n^2 + 1)$

13. $10d^2 - 6d + 35d - 21$

$(5d - 3)(2d + 7)$

14. $12n^3 - 15n^2 - 8n + 10$

$(4n - 5)(3n^2 - 2)$

15. $5b^4 - 15b^3 + 3 - b$

$(b - 3)(5b^3 - 1)$

16. $t^3 - 5t^2 + 10 - 2t$

$(t^2 - 2)(t - 5)$

LESSON 8-2

Practice C

Factoring by GCF

Factor each polynomial. Check your answer.

1. $8x^4 - 12x^2$

$4x^2(2x^2 - 3)$

2. $-12ab^3 + 20b$

$4b(-3ab^2 + 5)$

3. $16m^2 - 2n^3 + 30m$

$2(8m^2 - n^3 + 15m)$

4. $27j^4 - 72j^3 + 9j$

$9j(3j^3 - 8j^2 + 1)$

5. $-5x^5 + 35x^4 - 30x^3$

$-5x^3(x^2 - 7x + 6)$

6. $16x^6y + 16x^2y^4 + 32x^3y^2$

$16x^2y(x^4 + y^3 + 2xy)$

7. The expression used for finding the surface area of a cylinder is $2\pi r^2 + 2\pi rh$. Factor this expression.

$2\pi r(r + h)$

8. The area of a hallway rug is $\frac{3}{2}x^2 + \frac{1}{2}x$ ft^2. Factor this polynomial to find expressions for the dimensions of the rug.

$\frac{1}{2}x$ and $3x + 1$

Factor each expression.

9. $10(k - 2) + 7k(k - 2)$

$(k - 2)(10 + 7k)$

10. $9m^2(m + 7) + 5(m + 7)$

$(m + 7)(9m^2 + 5)$

Factor each polynomial by grouping.

11. $2t^3 + 6t^2 + t + 3$

$(t + 3)(2t^2 + 1)$

12. $3n^4 + 2n^3 - 15n - 10$

$(3n + 2)(n^3 - 5)$

13. $12a^2 + 30a - 14a - 35$

$(6a - 7)(2a + 5)$

14. $-28n^2 - 14 + 10n^5 + 5n^3$

$(2n^2 + 1)(5n^3 - 14)$

15. $3b^4 - 24b^3 + 8 - b$

$(3b^3 - 1)(b - 8)$

16. $3x^3 - 12x^2 + 20 - 5x$

$(x - 4)(3x^2 - 5)$

LESSON 8-2

Reteach

Factoring by GCF

The Distributive Property states: $a(b + c) = ab + ac$

Factoring by GCF reverses the Distributive Property:

$ab + ac = a(b + c)$

Factor $12x^3 + 21x^2 + 15x$. Check your answer.

Step 1: Find the GCF of all the terms in the polynomial.

The factors of $12x^3$ are: 1, 2, **3**, 4, 6, 12, ***x***, *x*, *x*

The factors of $21x^2$ are: 1, **3**, 7, 21, ***x***, *x*

The factors of $15x$ are: 1, **3**, 5, 15, ***x***

} The GCF is **3*x***.

Step 2: Write terms as products using the GCF.

$12x^3 + 21x^2 + 15x$

$(\mathbf{3x})4x^2 + (\mathbf{3x})7x + (\mathbf{3x})5$

Step 3: Use the Distributive Property to factor out the GCF.

$3x(4x^2 + 7x + 5)$

Check:

$3x(4x^2 + 7x + 5) = 12x^3 + 21x^2 + 15x$ ✓

Factor $5(x - 3) + 4x(x - 3)$.

Step 1: Find the GCF of all the terms in the polynomial.

The factors of $5(x - 3)$ are: 5, $(\boldsymbol{x - 3})$

The factors of $4x(x - 3)$ are: 4, *x*, $(\boldsymbol{x - 3})$

} The GCF is $(\boldsymbol{x - 3})$.

The terms are already written as products with the GCF.

Step 2: Use the Distributive Property to factor out the GCF.

$(x - 3)(5 + 4x)$

Factor each polynomial.

1. $20x^2 - 15x$

$5x(4x - 3)$

2. $44a^2 + 11a$

$11a(4a + 1)$

3. $24y - 36x$

$12(2y - 3x)$

Factor each expression.

4. $5x(x + 7) + 2(x + 7)$

$(x + 7)(5x + 2)$

5. $3a(a + 4) - 2(a + 4)$

$(a + 4)(3a - 2)$

6. $4y(4y + 1) + (4y + 1)$

$(4y + 1)^2$

LESSON 8-2

Reteach

Factoring by GCF (continued)

When a polynomial has four terms, make two groups and factor out the GCF from each group.

Factor $8x^3 + 6x^2 + 20x + 15$.

Step 1: Group terms that have common factors.

$$(8x^3 + 6x^2) + (20x + 15)$$

Step 2: Identify and factor the GCF out of each group.

$(8x^3 + 6x^2) + (20x + 15)$

GCF is $2x^2$. GCF is 5.

$2x^2(4x + 3) + 5(4x + 3)$

Step 3: Factor out the common binomial factor.

$2x^2(4x + 3) + 5(4x + 3)$

GCF is $(4x + 3)$.

$(4x + 3)(2x^2 + 5)$

Check:

$(4x + 3)(2x^2 + 5)$

$4x(2x^2) + 4x(5) + 3(2x^2) + 3(5)$ *Use FOIL.*

$8x^3 + 20x + 6x^2 + 15$

$8x^3 + 6x^2 + 20x + 15$ *Rearrange terms.*

Factor each polynomial filling in the blanks.

7. $(18x^3 + 15x^2) + (24x + 20)$

GCF is $3x^2$ GCF is 4

$3x^2(6x + 5) + 4(6x + 5)$

$(3x^2 + 4)(6x + 5)$

8. $(10a^3 - 15a^2) + (12a - 18)$

GCF is $5a^2$ GCF is 6

$5a^2(2a - 3) + 6(2a - 3)$

$(5a^2 + 6)(2a - 3)$

Factor each polynomial by grouping.

9. $21x^3 + 12x^2 + 14x + 8$

$(3x^2 + 2)(7x + 4)$

10. $40x^3 - 50x^2 + 12x - 15$

$(10x^2 + 3)(4x - 5)$

LESSON 8-2

Challenge

More Factoring by Grouping

Factoring by grouping requires 4 or more terms. When a polynomial has 3 terms, it is possible to divide the middle term into two terms and factor by grouping. When deciding on which two terms to use, think of terms that will have a common factor with the first and last terms. If that pair doesn't work, change the order of the pairs, or try another pair.

$x^2 + 10x + 21$
$x^2 + 3x + 7x + 21$
$(x^2 + 3x) + (7x + 21)$
$x(x + 3) + 7(x + 3)$
$(x + 3)(x + 7)$

Divide the *b* term into two separate terms. Then factor by grouping.

1. $x^2 + 8x + 12$

$(x + 6)(x + 2)$

2. $x^2 - 13x + 42$

$(x - 6)(x - 7)$

3. $2x^2 + 13x + 21$

$(2x + 7)(x + 3)$

4. $10x^2 + 19x + 6$

$(5x + 2)(2x + 3)$

5. $5x^2 + 19x + 12$

$(5x + 4)(x + 3)$

6. $4x^2 - 16x + 15$

$(2x - 5)(2x - 3)$

When a polynomial has more than 4 terms, it is sometimes possible to factor by grouping.

$(x^3 + 4x^2) + (x^2 + 4x) + (3x + 12)$
$x^2(x + 4) + x(x + 4) + 3(x + 4)$
$(x + 4)(x^2 + x + 3)$

Factor by grouping.

7. $(x^3 + x^2) + (x^2 + x) + (x + 1)$

$(x + 1)(x^2 + x + 1)$

8. $(3x^3 - 6x^2) + (4x^2 - 8x) + (5x - 10)$

$(x - 2)(3x^2 + 4x + 5)$

Factor by grouping in groups of three. You may need to rearrange the terms.

9. $x^6 + 5x^5 + 7x^4 + 3x^2 + 15x + 21$

$(x^4 + 3)(x^2 + 5x + 7)$

10. $6x^{10} + 24x^9 + 48x^8 + 7x^6 + 28x^5 + 56x^4 + 3x^2 + 12x + 24$

$(6x^8 + 7x^4 + 3)(x^2 + 4x + 8)$

LESSON 8-2

Problem Solving

Factoring by GCF

Write the correct answer.

1. The area of a rug, which is shaped like a rectangle, is $4x^2 + 4x$ square feet. Factor this polynomial to find expressions for the dimensions of the rug.

$4x$ ft; $(x + 1)$ ft

2. The number of customers visiting a local museum since the year 2000 can be modeled by the expression $-3x^2 - 27x + 825$, where x is the number of years since 2000. Factor this polynomial.

$-3(x^2 + 9x - 275)$

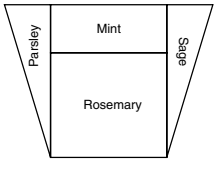
3. The perimeter of a rhombus is $12x + 28$ feet. Factor this expression. Then find the length of one side if $x = 8$. (*Hint:* A rhombus is a parallelogram with four congruent sides.)

$4(3x + 7)$; 31 feet

4. The foundation for a new high school building is rectangular in shape, and the area is $5x^3 + 4x^2 - 10x - 8$ square meters. Factor by grouping to find expressions for the dimensions of the building.

$(5x + 4)$ m; $(x^2 - 2)$ m

The diagram shows four sections of an herb garden. Use the figure to answer questions 5–8. Select the best answer.



5. The section where rosemary grows is square and has an area of $4x^2$ square feet. What is the length of one side?

A x feet **C** $2x$ feet

B x^2 feet **D** $4x$ feet

6. Rosemary and mint cover $6x^2 - 2x$ square feet. Assuming the length is adjacent to rosemary, what is the width of the mint section?

F $2x$ feet **H** $2x - 2$ feet

G $x - 1$ feet **J** $3x - 1$ feet

7. The parsley and sage sections each have an area of $\frac{1}{2}(3x^2 - 6x - x + 2)$ square feet. Factor $3x^2 - 6x - x + 2$. What are the base and height of each triangular section?

A $2x - 3$ feet; $x + 1$ feet

B $2x - 3$ feet; $x^2 + 1$ feet

C $3x - 1$ feet; $x - 2$ feet

D $3x - 1$ feet; $x^2 - 2$ feet

8. Assuming the side adjacent to mint and rosemary is the base, what is the height of each triangle on which parsley and sage grow?

F $x - 2$ feet

G $x + 1$ feet

H $x^2 + 1$ feet

J $2x$ feet

LESSON 8-2

Reading Strategies

Follow a Procedure

To factor a polynomial by using the GCF, think of using the Distributive Property in reverse. More specifically, follow the procedure shown here.

$15b^5 + 18b^3 - 6b^2$

Step 1: Find the GCF of the coefficients.

15: 1, ③ 5, 15

18: 1, 2, ③ 6, 9, 18

6: 1, 2, ③ 6

Step 2: Identify the lowest degree of the variable.

b^5

b^3

(b^2)

Step 3: Write the results of Steps 1 and 2 as a product.

$3b^2$

Step 4: Use the Distributive Property to factor out this GCF.

$3b^2(5b^3 + 6b - 2)$

Step 5: Check using multiplication.

$3b^2(5b^3 + 6b - 2) = 15b^5 + 18b^3 - 6b^2$ ✓

Factor each polynomial using the procedure shown above.

1. $4x^3 + 12x^2$

$4x^2(x + 3)$

2. $30t^4 - 18t$

$6t(5t^3 - 3)$

3. $9p^2 + p$

$p(9p + 1)$

4. $28r^4 - 20r^2 - 8$

$4(7r^4 - 5r^2 - 2)$

5. $30p^8 + 45p^5$

$15p^5(2p^3 + 3)$

6. $6m^8 - 16m^3 + 6m^2$

$2m^2(3m^6 - 8m + 3)$