CHAPTER 8

Solving Trigonometric Equations

n this chapter, students will solve rigonometric equations by graphing nd by algebraic methods. Most lgebraic methods will require the use f inverse trigonometric functions and ome algebraic methods will require he use of trigonometric identities. itudents will also use modeling to olve harmonic motion problems.

CHAPTER 8

Solving Trigonometric Equations

Round and round we go!

Trigonometric functions are used to analyze periodic phenomena, because simple harmonic motion models circular motion or any phenomenon that is "back and forth." Some examples of simple harmonic motion include a vibrating prong of a tuning fork, a buoy bobbing up and down in water, seismic and ocean waves, spring-mass systems, a piston in a running engine, a particle of air during the passage of a simple sound wave, or a turning Ferris wheel. See Exercise 1 of Section 8.4.

Chapter Outline

Interdependence of Sections

8.1

8.2 ⟶ 8.3 ⟶ 8.4

There are two kinds of trigonometric equations. *Identities,* which will be studied more in Chapter 9, are equations that are valid for all values of the variable for which the equation is defined, such as

$$\sin^2 x + \cos^2 x = 1 \qquad \text{and} \qquad \cot x = \frac{1}{\tan x}.$$

In this chapter, *conditional equations* will be studied. Conditional equations are valid only for certain values of the variable, such as

$$\sin x = 0, \qquad \cos x = \frac{1}{2}, \qquad \text{and} \qquad 3\sin^2 x - \sin x = 2.$$

If a trigonometric equation is conditional, solutions are found by using techniques similar to those used to solve algebraic equations. Graphs were used to solve some simple trigonometric equations in Chapter 7. This chapter will extend graphical solution techniques and introduce analytic solution methods.

Graphical solution methods are presented in 8.1. Inverse trigonometric functions are discussed in Section 8.2. Methods that use inverse functions, basic identities, and algebra to solve trigonometric equations are considered in Section 8.3. Skills from the Sections 8.1 through 8.3 are applied to problem-solving and real-world applications in Section 8.4.

NOTE In Chapter 7, the variable t was used for trigonometric functions to avoid confusion with the x's and y's that appear in their definitions. Now that you are comfortable with these functions, the letter x, or occasionally y, will be used for the variable. Unless otherwise stated, all trigonometric functions in this chapter are considered as functions of real numbers, rather than functions of angles in degree measure.

Math Background

Trigonometric identities are sometimes used to solve conditional equations. For example, in Section 8.3, **Example 9** (page 544), a Pythagorean identity is used in solving a conditional equation.

Teaching Notes

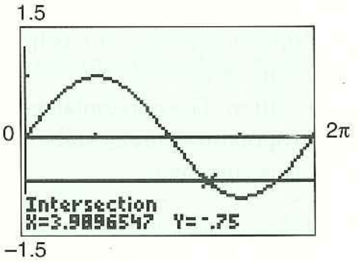
Students might need some explanation of the **NOTE** at the bottom of the page. Refer them to the first unit circle and its corresponding graph on page 474. Explain that they can now think of x instead of t as the radian measure of an angle in standard position. The horizontal axis on the corresponding graph can be labeled x, as is customary.

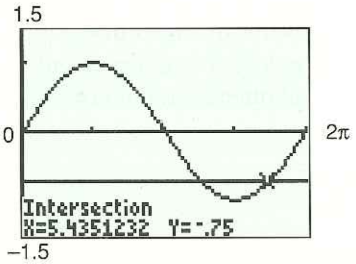
In this chapter, the trigonometric functions are functions of real numbers (which can be thought of as radian measures of angles unless otherwise noted). The independent variable will often be x, but not always.

Section 8.1 Graphical Solutions to Trigonometric Equations

Teaching Notes

After completing Example 2, consider having students solve $\sin x = -0.75$ by the intersection method: Let $Y_1 = \sin x$ and $Y_2 = -0.75$.





COMMON ERROR ALERT

When using the x-intercept method to solve an equation, sign errors are common. In Example 2, students might rewrite the equation as $\sin x - 0.75 = 0$. The graph of the incorrect function $f(x) = \sin x - 0.75$ might seem reasonable because it represents a *downward* shift of the graph of $y = \sin x$, and students see a *negative* number in the original equation. Caution students against such sign errors.

Real-World Application

The Basic Trigonometric Equation $\sin \alpha = \frac{1}{m}$ is used in Exercises 29–32 (page 529), where students will calculate the angle of elevation α of an airplane as a function of its Mach number m.

8.1 Graphical Solutions to Trigonometric Equations

Objectives

- Solve trigonometric equations graphically
- State the complete solution of a trigonometric equation

Any equation involving trigonometric functions can be solved graphically. To solve trigonometric equations graphically, the same methods of graphical solutions are used here as have been used previously to solve polynomial equations, except that trigonometric equations typically have an infinite number of solutions. These solutions are systematically determined by using the periodicity of the function.

Basic Trigonometric Equations

An equation that involves a single trigonometric function set equal to a number is called a **basic equation.** Some examples include the following.

$$\sin x = 0.39, \qquad \cos x = 0.5, \qquad \text{and} \qquad \tan x = -3$$

Examples 1 and 2 show how they can be solved graphically.

Example 1 The Intersection Method

Solve $\tan x = 2$.

Solution

The equation can be solved by graphing $Y_1 = \tan x$ and $Y_2 = 2$ on the same screen and finding intersection points. The x-coordinate of every such point is a number whose tangent is 2; or a solution of $\tan x = 2$. Figure 8.1-1 indicates that there are infinitely many intersection points, so the equation has an infinite number of solutions.

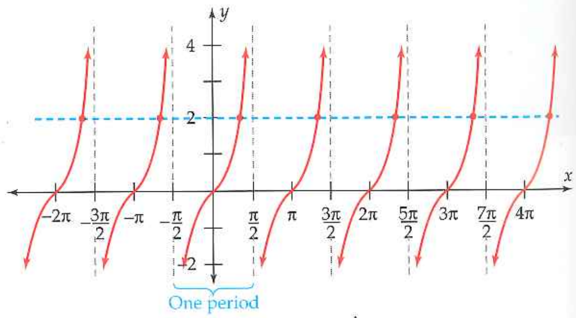


Figure 8.1-1

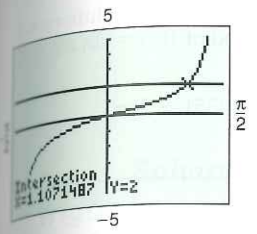


Figure 8.1-2

The function $f(x) = \tan x$ completes one cycle on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and there is one solution of $\tan x = 2$ in this interval. Using the intersection finder on a graphing calculator gives the approximate solution in this interval.

$$x = 1.1071$$

Because the graph of $f(x) = \tan x$ repeats its pattern to the left and to the right, the other solutions will differ from this first solution by multiples of π, the period of the tangent function. The other solutions are

$$1.1071 \pm \pi, \qquad 1.1071 \pm 2\pi, \qquad \text{and} \qquad 1.1071 \pm 3\pi,$$

and so on. All solutions can be expressed as

$$1.1071 + k\pi,$$

where k is any integer. ■

NOTE Solutions in this chapter are often rounded, but the full decimal expansion given by the calculator is used in all computations. The symbol = is used rather than ≈ even though these calculator solutions are approximations of the actual solutions.

Example 2 The x-Intercept Method

Solve $\sin x = -0.75$.

Solution

Rewrite the equation $\sin x = -0.75$ as

$$\sin x + 0.75 = 0.$$

Recall from Section 2.1 that the solutions of this equation are the x-intercepts of the graph of

$$f(x) = \sin x + 0.75.$$

The graph of f is shown in Figure 8.1-3. The function has a period of 2π and the viewing window can be modified to show one period of $f(x) = \sin x + 0.75$. Figures 8.1-4a and 8.1-4b show that there are two zeros of f on the interval $[0, 2\pi]$, so the equation has two solutions on that interval.

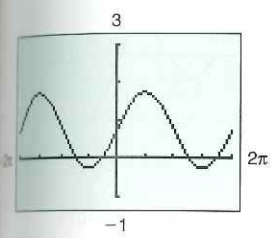


Figure 8.1-3

The calculator's zero finder calculates the zeros:

$$x = 3.9897 \qquad\qquad x = 5.4351$$

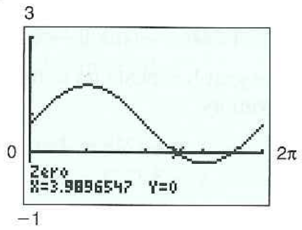


Figure 8.1-4a

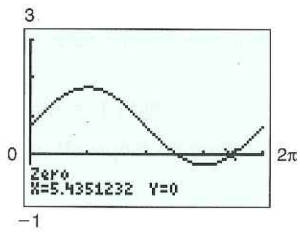


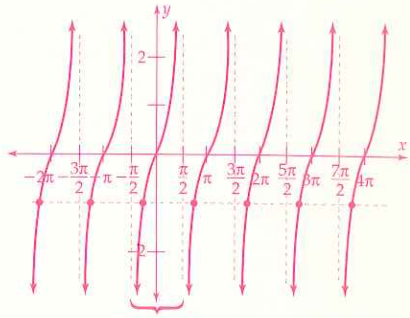
Figure 8.1-4b

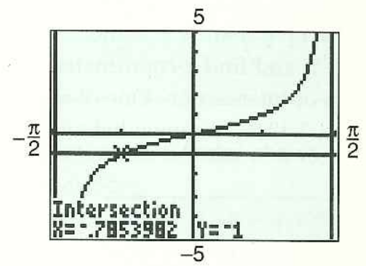
ADDITIONAL EXAMPLES

Example 1

Solve $\tan x = -1$.

The graphs of $Y_1 = \tan x$ and $Y_2 = -1$ intersect in an infinite number of points. The x-coordinates of the points of intersection are solutions of $\tan x = -1$.



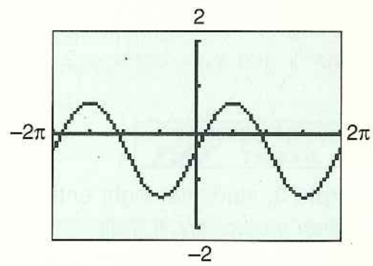


All solutions can be expressed as $-0.7854 + k\pi$, where k is any integer.

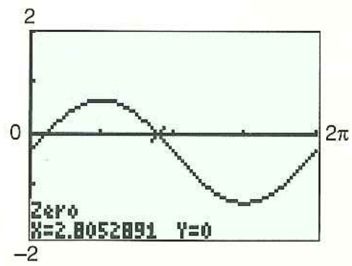
Example 2

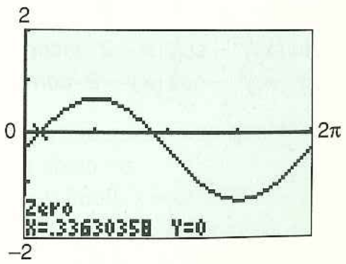
Solve $\sin x = 0.33$.

Rewrite the equation as $\sin x - 0.33 = 0$. Graph $f(x) = \sin x - 0.33$.



f has a period of 2π. Find the zeros of f on the interval $[0, 2\pi]$.





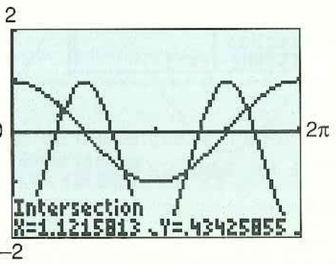
All solutions can be expressed as $0.3363 + 2k\pi$ or $2.8053 + 2k\pi$, where k is any integer.

eaching Notes

n the procedure **Solving rigonometric Equations Graphically,** ep 2 is to determine the period of f. o determine the period of f, it is a ood practice to graph f over a large nough interval so that you can see a epeating pattern. In **Example 3,** f is raphed in a viewing window from 2π to 4π.

oint out that this procedure applies o trigonometric equations that nvolve one trigonometric function, s in **Examples 1** and **2,** or more than ne trigonometric function, as in xamples **3** and **4.**

he equation in **Example 3** can also e solved using the intersection ethod. Rewrite the equation as $\sin^2 x - 2 = \cos x$. Graph the unctions $Y_1 = 3\sin^2 x - 2$ and $_2 = \cos x$, and find x-coordinates f points of intersection. One of the olutions, 1.1216, is shown below.



ou can ask students if they can think f another equation and pair of unctions they could use to solve the quation by the intersection method.

ossible answer: $3\sin^2 x = \cos x + 2$, $_1 = 3\sin^2 x$ and $Y_2 = \cos x + 2$

COMMON ERROR ALERT

n **Example 3,** students might enter he function incorrectly. If their alculator automatically inserts a left arenthesis after cos and sin, they *must* insert the matching right arenthesis.

$_1 = 3(\sin(x))^2 - \cos(x - 2$ *incorrect*

$_1 = 3(\sin(x))^2 - \cos(x) - 2$ *correct*

Because the graph repeats its pattern every 2π, the other solutions will differ from these two by multiples of 2π, the period of $f(x) = \sin x + 0.75$. Therefore, all solutions of $\sin x = -0.75$ are

$$x = 3.9897 + 2k\pi \quad \text{and} \quad x = 5.4351 + 2k\pi,$$

where k is any integer.

Other Trigonometric Equations

The procedures in Examples 1 and 2 can be used to solve any trigonometric equation graphically.

Technology Tip

Enter $3\sin^2 x$ on a calculator as

$3(\sin x)^2$

or $3(\sin(x))^2$.

Example 3 The x-intercept Method

Solve $3\sin^2 x - \cos x - 2 = 0$.

Solution

Both sine and cosine have period 2π, so the period of $f(x) = 3\sin^2 x - \cos x - 2$ is at most 2π. The graph of f, which is shown in two viewing windows in Figure 8.1-5, does not repeat its pattern over any interval of less than 2π, so you can conclude that f has a period of 2π.

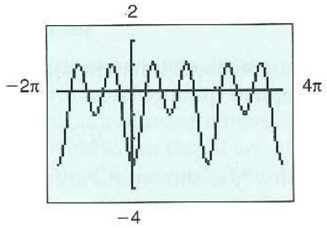


Figure 8.1-5a

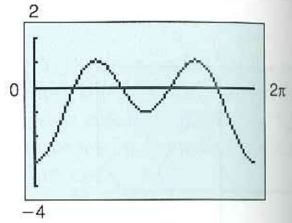


Figure 8.1-5b

The function f makes one complete period on the interval $[0, 2\pi)$, as shown in Figure 8.1-5b. The equation has four solutions between 0 and 2π, namely, the four x-intercepts of the graph in that interval. A graphical zero finder shows these four solutions.

$$x \approx 1.1216 \qquad x \approx 2.4459 \qquad x \approx 3.8373 \qquad x \approx 5.1616$$

Because the graph repeats its pattern every 2π, all solutions of the equation are given by

$$x \approx 1.1216 + 2k\pi, \qquad x \approx 2.4459 + 2k\pi,$$
$$x \approx 3.8373 + 2k\pi, \qquad x \approx 5.1616 + 2k\pi,$$

where k is any integer.

The solution methods in Examples 1 through 3 depend only on knowing the period of a function and all the solutions of the equation in one period. A similar procedure can be used to solve any trigonometric equation graphically.

Solving Trigonometric Equations Graphically

1. **Write the equation in the form $f(x) = 0$.**
2. **Determine the period p of f.**
3. **Graph f over an interval of length p.**
4. **Use a calculator's zero finder to determine the x-intercepts of the graph in this interval.**
5. **For each x-intercept u, all of the numbers**

$$u + kp \text{ where } k \text{ is any integer}$$

are solutions of the equation.

In Example 1, for example, p was π. In Examples 2 and 3, p was 2π.

Example 4 Solving Any Trigonometric Equation

Solve $\tan x = 3 \sin 2x$.

Solution

First rewrite the equation

$$\tan x - 3 \sin 2x = 0$$

Next, determine the period of $f(x) = \tan x - 3 \sin 2x$. Recall from Section 7.3 that $y = 3 \sin 2x$ has a period of $\frac{2\pi}{2} = \pi$, which is also the period of $y = \tan x$. Therefore, the period of f is π. Figure 8.1-6 shows the graph of f on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, an interval of length π.

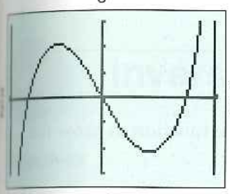


Figure 8.1-6

Even without the graph, it can be easily verified that there is an x-intercept at 0.

$$f(0) = \tan 0 - 3 \sin(2 \cdot 0) = 0$$

Using the calculator's zero finder gives the other x-intercepts of the graph of f on this interval.

$$x = -1.1503 \quad \text{and} \quad x = 1.1503$$

Because f has a period of π, all solutions of the equation $\tan x = 3 \sin 2x$ are

$$x = -1.1503 + k\pi, \quad x = 0 + k\pi, \quad \text{and} \quad x = 1.1503 + k\pi,$$

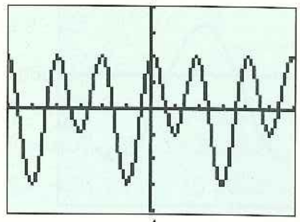
where k is any integer.

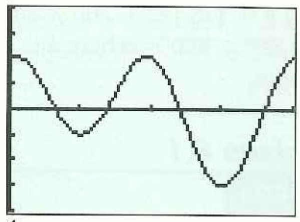
ADDITIONAL EXAMPLES

Example 3

Solve $4 \cos^2 x + \sin x - 2 = 0$.

The graph of $f(x) = 4\cos^2 x + \sin x - 2$, shown below in two viewing windows, has a period of 2π.





A graphical zero finder shows these four solutions:

$x \approx 1.0030$ $\quad$ $x \approx 2.1386$

$x \approx 3.7765$ $\quad$ $x \approx 5.6483$

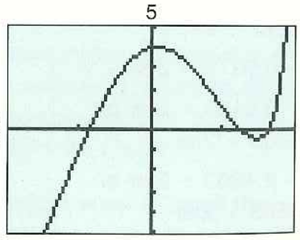
All solutions are given by $1.0030 + 2k\pi$, $2.1386 + 2k\pi$, $3.7765 + 2k\pi$, and $5.6483 + 2k\pi$, where k is any integer.

Example 4

Solve $\tan x = -4 \cos 2x$.

Rewrite the equation as $\tan x + 4 \cos 2x = 0$. Determine the period of $f(x) = \tan x + 4 \cos 2x$.

The period of $4 \cos 2x$ is $\frac{2\pi}{2} = \pi$, and the period of $\tan x$ is π. So, the period of f is π. Graph f over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



A graphical zero finder shows these three solutions:

$x \approx -0.6830$, $x \approx 0.9736$, $x \approx 1.2801$

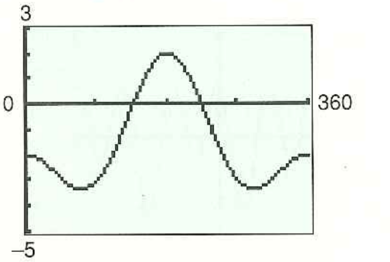
All solutions of the equation are given by $-0.6830 + k\pi$, $0.9736 + k\pi$, and $1.2801 + k\pi$, where k is any integer.

ADDITIONAL EXAMPLES

Example 5

Solve $3\cos^2\theta - 2\cos\theta - 3 = 0$.

The period of the function $f(\theta) = 3\cos^2\theta - 2\cos\theta - 3$ is 360° and the graph is shown below.



All solutions of the equation are given by $\theta \approx 136.12° + 360°k$ and $\theta \approx 223.88° + 360°k$, where k is any integer.

Exercises 8.1

ANSWERS

1–14. For graphs see pp. 1069–1070.

1. $x = 0.5275 + k\pi$ or $1.6868 + k\pi$
2. $x = 0.7124 + \frac{2\pi}{3}k$ or $1.8452 + \frac{2\pi}{3}k$
3. $x = 0.4959 + 2k\pi$ or $1.2538 + 2k\pi$ or $1.5708 + 2k\pi$ or $1.8877 + 2k\pi$ or $2.6457 + 2k\pi$ or $4.7124 + 2k\pi$
4. $x = 0.3289 + k\pi$ or $2.8127 + k\pi$
5. $x = 0.1671 + 2k\pi$ or $1.8256 + 2k\pi$ or $2.8867 + 2k\pi$ or $4.5453 + 2k\pi$
6. $x = 4.7123 + 2k\pi$
7. $x = 1.2161 + 2k\pi$ or $5.0671 + 2k\pi$
8. $x = 1.0106 + 2k\pi$ or $2.1310 + 2k\pi$
9. $x = 2.4620 + 2k\pi$ or $3.8212 + 2k\pi$
10. $x = 0.9273 + 2k\pi$
11. $x = 0.5166 + 2k\pi$ or $5.6766 + 2k\pi$
12. $x = 2.4263 + 2k\pi$ or $3.8569 + 2k\pi$
13. a. The graph of $f(x) = \sin x$ on the interval from 0 to 2π shows that $\sin x = 1$ only when $x = \frac{\pi}{2}$. Since $\sin x$ has period 2π, all other solutions are obtained by adding or subtracting integer multiples of 2π from $\frac{\pi}{2}$, that is,
$\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}, \frac{\pi}{2} + 2(2\pi) = \frac{9\pi}{2}$,
$\frac{\pi}{2} + 3(2\pi) = \frac{13\pi}{2}$, etc., and
$\frac{\pi}{2} - 2\pi = -\frac{3\pi}{2}, \frac{\pi}{2} - 2(2\pi) = -\frac{7\pi}{2}$,
$\frac{\pi}{2} - 3(2\pi) = -\frac{11\pi}{2}$, etc.
   b. Similarly, the graph shows that $\sin x = -1$ only when $x = \frac{3\pi}{2}$, so that all solutions are obtained by adding or subtracting integer multiples of 2π from $\frac{3\pi}{2}$:
$\frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}, \frac{3\pi}{2} + 2(2\pi) = \frac{11\pi}{2}$,
$\frac{3\pi}{2} + 3(2\pi) = \frac{15\pi}{2}$, etc.,
and $\frac{3\pi}{2} - 2\pi = -\frac{\pi}{2}, \frac{3\pi}{2} - 2(2\pi) = -\frac{5\pi}{2}$
$\frac{3\pi}{2} - 3(2\pi) = -\frac{9\pi}{2}$, etc.

Trigonometric Equations in Degree Measure

Some real-world applications of trigonometric equations require solutio to be expressed as angles in degree measure. The graphical solution pro cedure is the same, except that you must set the mode of your calculat to "degree."

Example 5 Trigonometric Equations in Degree Measure

Solve $2\sin^2\theta - 3\sin\theta - 3 = 0$.

Solution

The period of the function $f(\theta) = 2\sin^2\theta - 3\sin\theta - 3$ is 360°, and Figure 8.1-7 shows the graph of f on the interval [0°, 360°).

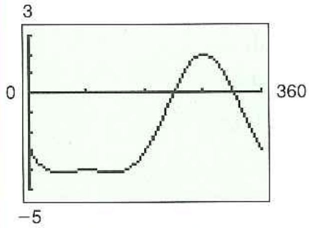


Figure 8.1-7

A graphical zero finder determines the approximate x-intercepts.

$$\theta \approx 223.33° \quad \text{and} \quad \theta \approx 316.67°$$

Using the fact that the period of f is 360°, all solutions of the equation are

$$\theta \approx 223.33° + 360°k \quad \text{and} \quad \theta \approx 316.67° + 360°k,$$

where k is any integer.

Exercises 8.1

In Exercises 1–12, solve the equation graphically.

1. $4\sin 2x - 3\cos 2x = 2$
2. $5\sin 3x + 6\cos 3x = 1$
3. $3\sin^3 2x = 2\cos x$
4. $\sin^2 2x - 3\cos 2x + 2 = 0$
5. $\tan x + 5\sin x = 1$
6. $2\cos^2 x + \sin x + 1 = 0$
7. $\cos^3 x - 3\cos x + 1 = 0$
8. $\tan x = 3\cos x$
9. $\cos^4 x - 3\cos^3 x + \cos x = 1$
10. $\sec x + \tan x = 3$
11. $\sin^3 x + 2\sin^2 x - 3\cos x + 2 = 0$
12. $\csc^2 x + \sec x = 1$
13. Use the graph of the sine function to show the following.
    a. The solutions of $\sin x = 1$ are
    $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ and
    $x = \frac{-3\pi}{2}, \frac{-7\pi}{2}, \frac{-11\pi}{2}, \ldots$.
    b. The solutions of $\sin x = -1$ are
    $x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$ and
    $x = \frac{-\pi}{2}, \frac{-5\pi}{2}, \frac{-9\pi}{2}, \ldots$.
14. Use the graph of the cosine function to show the following.
    a. The solutions of $\cos x = 1$ are $x = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$.
    b. The solutions of $\cos x = -1$ are $x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$.

In Exercises 15–18, approximate all solutions of the given equation in $(0, 2\pi)$.

15. $\sin x = 0.119$ **16.** $\cos x = 0.958$

17. $\tan x = 5$ **18.** $\tan x = 17.65$

In Exercises 19–28, find all angles θ with $0° \le \theta < 360°$ that are solutions of the given equation.

19. $\tan \theta = 7.95$ **20.** $\tan \theta = 69.4$

21. $\cos \theta = -0.42$ **22.** $\cot \theta = -2.4$

23. $2 \sin^2 \theta + 3 \sin \theta + 1 = 0$

24. $4 \cos^2\theta + 4 \cos \theta - 3 = 0$

25. $\tan^2\theta - 3 = 0$ **26.** $2 \sin^2\theta = 1$

27. $4 \cos^2\theta + 4 \cos \theta + 1 = 0$
$\theta = 120°, 240°$

28. $\sin^2\theta - 3 \sin \theta = 10$
no solution

At the instant you hear a sonic boom from an airplane overhead, your angle of elevation α to the plane is given by the equation

$$\sin \alpha = \frac{1}{m}$$

where m is the Mach number for the speed of the plane (Mach 1 is the speed of sound, Mach 2.5 is 2.5 times the speed of sound, etc.). In Exercises 29–32, find the angle of elevation (in degrees) for the given Mach number. Remember that an angle of elevation must be between 0° and 90°.

29. $m = 1.1$
$\alpha \approx 65.38°$

30. $m = 1.6$
$\alpha \approx 38.68°$

31. $m = 2$
$\alpha = 30°$

32. $m = 2.4$
$\alpha \approx 24.62°$

33. *Critical Thinking* Under what conditions (on the constant) does a basic equation involving the sine and cosine function have no solutions?

34. *Critical Thinking* Under what conditions (on the constant) does a basic equation involving the secant and cosecant function have no solutions?

8.2 Inverse Trigonometric Functions

Objectives

- Define the domain and range of the inverse trigonometric functions
- Use inverse trigonometric function notation

Many trigonometric equations can be solved without graphing. Nongraphical solution methods make use of the *inverse trigonometric functions* that are introduced in this section.

Recall from Section 3.6 that a function cannot have an inverse function unless its graph has the following property.

No horizontal line intersects the graph more than once.

You have seen that the graphs of trigonometric functions do not have this property. However, restricting their domains can modify the trigonometric functions so that they do have inverse functions.

NOTE Other ways of restricting the domains of trigonometric functions are possible. Those presented here for sine, cosine, and tangent are the ones universally agreed upon by mathematicians.

Inverse Sine Function

The *restricted sine function* is $f(x) = \sin x$, when its domain is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Its graph in Figure 8.2-1 shows that for each number v in the interval $[-1, 1]$, there is exactly one number u in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin u = v$.

15. $x \approx 0.1193$ or 3.0223

16. $x \approx 0.2909$ or $x = 5.9923$

17. $x \approx 1.3734$ or 4.5150

18. $x \approx 1.5142$ or $x = 4.6558$

19. $\theta \approx 82.83°, 262.83°$

20. $\theta \approx 89.17°, 269.17°$

21. $\theta \approx 114.83°, 245.17°$

22. $\theta \approx 337.38°, 157.38°$

23. $\theta = 210°, 270°, 330°$

24. $\theta = 60°, 300°$

25. $\theta = 60°, 120°, 240°, 300°$

26. $\theta = 45°, 135°, 225°, 315°$

33. $\sin x = k$ and $\cos x = k$ have no solutions when $k > 1$ or $k < -1$

34. $\sec x = k$ and $\csc x = k$ have no solutions when $-1 < k < 1$

Section 8.2 Inverse Trigonometric Functions

Teaching Notes

Review these facts about inverse functions from page 210:
Let f be a function. The following statements are equivalent.

- The inverse of f is a function.
- f is one-to-one.
- The graph of f passes the Horizontal Line Test.

The inverse function, if it exists, is written as f^{-1}, where if $y = f(x)$, then $x = f^{-1}(y)$.

Point out that trigonometric functions are not one-to-one because different inputs can result in the same output. To illustrate this fact for the sine function, note that $\sin(\pi) = \sin(3\pi) = 0$.

You might want to recall the definition of a one-to-one function (page 208):

A function f is one-to-one if $f(a) = f(b)$ implies that $a = b$.

14. a. The graph of $f(x) = \cos x$ on the interval from 0 to 2π shows that $\cos x = 1$ only when $x = 2\pi$ and 0. Since $\cos x$ has period 2π, all other solutions are obtained by adding or subtracting integer multiples of 2π from 2π, that is, $2\pi + 2\pi = 4\pi$, $2\pi + 2(2\pi) = 6\pi$, $2\pi + 3(2\pi) = 8\pi$, etc., and $2\pi - 2\pi = 0$, $2\pi - 2(2\pi) = -2\pi$, $2\pi - 3(2\pi) = -4\pi$, etc.

b. Similarly, the graph shows that $\cos x = -1$ only when $x = \pi$, so that all solutions are obtained by adding or subtracting integer multiples of 2π from π: $\pi + 2\pi = 3\pi$, $\pi + 2(2\pi) = 5\pi$, $\pi + 3(2\pi) = 7\pi$, etc., and $\pi - 2\pi = -\pi$, $\pi - 2(2\pi) = -3\pi$, $\pi - 3(2\pi) = -5\pi$, etc.