

7-7 Study Guide and Intervention

The Law of Cosines

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

| | |
|-----------------------|--|
| Law of Cosines | Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$ |
|-----------------------|--|

Example 1 In $\triangle ABC$, find c .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48^\circ$$

$$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48^\circ}$$

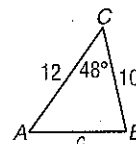
$$c \approx 9.1$$

Law of Cosines

$$a = 12, b = 10, m\angle C = 48$$

Take the square root of each side.

Use a calculator.



Example 2 In $\triangle ABC$, find $m\angle A$.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$$

$$49 = 25 + 64 - 80 \cos A$$

$$-40 = -80 \cos A$$

$$\frac{1}{2} = \cos A$$

$$\cos^{-1} \frac{1}{2} = A$$

$$60^\circ = A$$

Law of Cosines

$$a = 7, b = 5, c = 8$$

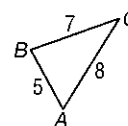
Multiply.

Subtract 89 from each side.

Divide each side by -80 .

Use the inverse cosine.

Use a calculator.



Exercises

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

- If $b = 14$, $c = 12$, and $m\angle A = 62$, find a .
- If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$.
- If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$.
- If $a = 20$, $c = 25$, and $m\angle B = 82$, find b .
- If $b = 18$, $c = 28$, and $m\angle A = 59$, find a .
- If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$.

7-7 Study Guide and Intervention *(continued)***The Law of Cosines**

Use the Law of Cosines to Solve Problems You can use the **Law of Cosines** to solve some problems involving triangles.

| | |
|-----------------------|--|
| Law of Cosines | Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$ |
|-----------------------|--|

Example

Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.

Use the Law of Cosines to find the value of a .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Cosines

$$a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88^\circ$$

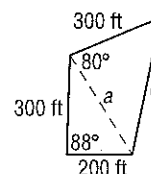
$$b = 300, c = 200, m\angle A = 88$$

$$a = \sqrt{130,000 - 120,000 \cos 88^\circ}$$

Take the square root of each side.

$$\approx 354.7$$

Use a calculator.



Use the Law of Cosines again to find the value of c .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Cosines

$$c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80^\circ$$

$$a = 354.7, b = 300, m\angle C = 80$$

$$c = \sqrt{215,812.09 - 212,820 \cos 80^\circ}$$

Take the square root of each side.

$$\approx 422.9$$

Use a calculator.

The perimeter of the land is $300 + 200 + 422.9 + 200$ or about 1223 feet.

Exercises

Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.
2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.
3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0° , and 40 miles from the other location with an angle of elevation of 21.8° . How far apart are the two locations?
4. A ranger tower at point A is directly north of a ranger tower at point B . A fire at point C is observed from both towers. The distance from the fire to tower A is 60 miles, and the distance from the fire to tower B is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.

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The Law of Cosines

Use the Law of Cosines to Solve Problems You can use the Law of Cosines to solve some problems involving triangles.

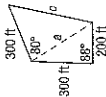
Law of Cosines Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Example 1 Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.



Law of Cosines
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88^\circ$
 $a = \sqrt{130,000 - 120,000 \cos 88^\circ}$
 ≈ 354.7

Law of Cosines
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $b = 300$, $c = 200$, $m\angle A = 88^\circ$
 Take the square root of each side.
 Use a calculator.
 $c^2 = a^2 + b^2 - 2ab \cos C$
 $c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80^\circ$
 $c = \sqrt{215,812.09 - 212,820 \cos 80^\circ}$
 ≈ 422.9

The perimeter of the land is $300 + 200 + 422.9 + 200$ or about 1223 feet.

Example 2 Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.
 75° ; 48° ; 57°
2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.
 11.7 ; 16.7
3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0° , and 40 miles from the other location with an angle of elevation of 21.8° . How far apart are the two locations?
 50.5 mi
4. A ranger tower at point A is directly north of a ranger tower at point B. A fire at point C is observed from both towers. The distance from the fire to tower A is 60 miles, and the distance from the fire to tower B is 50 miles. If $m\angle ACB = 62^\circ$, find the distance between the towers.
 57.3 mi

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$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Example 1 In $\triangle ABC$, find c .



Law of Cosines
 $a = 12$, $b = 10$, $m\angle C = 48^\circ$
 Take the square root of each side.
 Use a calculator.
 $c = \sqrt{12^2 + 10^2 - 2(12)(10) \cos 48^\circ}$
 $c \approx 9.1$

Example 2 In $\triangle ABC$, find $m\angle A$.



Law of Cosines
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$
 $49 = 25 + 64 - 80 \cos A$
 $-40 = -80 \cos A$
 $\frac{1}{2} = \cos A$
 Divide each side by -80 .
 $\cos^{-1} \frac{1}{2} = A$
 $60^\circ = A$

Example 3

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62^\circ$, find a . 13.5
2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$. 51
3. If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$. 42
4. If $a = 20$, $c = 25$, and $m\angle B = 82^\circ$, find b . 29.8
5. If $b = 18$, $c = 28$, and $m\angle A = 59^\circ$, find a . 24.3
6. If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$. 51

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