NAME ______________________ DATE __________ PERIOD _____

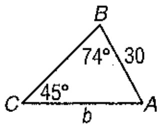
7-6 Study Guide and Intervention

The Law of Sines

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

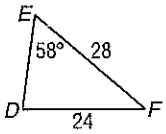
Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **In $\triangle ABC$, find b.**



$\frac{\sin C}{c} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$	$m\angle C = 45$, $c = 30$, $m\angle B = 74$
$b \sin 45° = 30 \sin 74°$	Cross multiply.
$b = \frac{30 \sin 74°}{\sin 45°}$	Divide each side by sin 45°.
$b \approx 40.8$	Use a calculator.

Example 2 **In $\triangle DEF$, find $m\angle D$.**



$\frac{\sin D}{d} = \frac{\sin E}{e}$	Law of Sines
$\frac{\sin D}{28} = \frac{\sin 58°}{24}$	$d = 28$, $m\angle E = 58$, $e = 24$
$24 \sin D = 28 \sin 58°$	Cross multiply.
$\sin D = \frac{28 \sin 58°}{24}$	Divide each side by 24.
$D = \sin^{-1} \frac{28 \sin 58°}{24}$	Use the inverse sine.
$D \approx 81.6°$	Use a calculator.

Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.

2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.

3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.

4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b.

5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a.

6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.

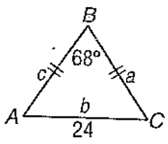
7-6 Study Guide and Intervention *(continued)*

The Law of Sines

Use the Law of Sines to Solve Problems You can use the **Law of Sines** to solve some problems that involve triangles.

Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Example **Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.**



The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ Law of Sines

$\frac{\sin 68°}{24} = \frac{\sin 56°}{a}$ $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ Cross multiply.

$a = \frac{24 \sin 56°}{\sin 68°}$ Divide each side by sin 68°.

≈ 21.5 Use a calculator.

The triangle is isosceles, so $c = 21.5$.

The perimeter is $24 + 21.5 + 21.5$ or about 67 centimeters.

Exercises

Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?

3. A civil engineer wants to determine the distances from points A and B to an inaccessible point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are 82.0 feet apart, find the distance from C to each point.

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?

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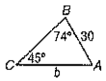

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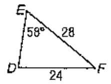

$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

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$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

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Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.
   18.4

2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.
   37

3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.
   62

4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b.
   7.7

5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a.
   12.2

6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.
   35

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7-6 Study Guide and Intervention *(continued)*

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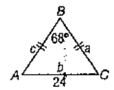

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The perimeter is $24 + 21.5 + 21.5$ or about 67 centimeters.

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Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.
   192.9 ft

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?
   25.5 mi from A; 23.2 mi from B

3. A civil engineer wants to determine the distances from points A and B to an inaccessible point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are 82.0 feet apart, find the distance from C to each point.
   86.3 ft to point B; 73.9 ft to point A

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?
   Tower B is 11 km closer than Tower A.