7.4 Periodic Graphs and Phase Shifts

Objectives

- State the period, amplitude, vertical shift, and phase shift given the function rule or graph of a sine or cosine function
- Use graphs to determine whether an equation could possibly be an identity

In Section 7.3, you studied graphs of functions of the form

$$f(t) = a \sin bt \qquad \text{and} \qquad g(t) = a \cos bt$$

and learned how the constants a and b affect the amplitudes and periods of the functions. In this section, you will consider functions of the form

$$f(t) = a \sin(bt + c) + d \qquad \text{and} \qquad g(t) = a \cos(bt + c) + d$$

where a, b, c, and d are constants, and you will determine how these constants affect the graphs of the functions.

Vertical Shifts

Recall from Section 3.4 that adding a constant to the rule of a function shifts the graph vertically. Example 1 illustrates a vertical shift in combination with a reflection and a change in amplitude.

Example 1 Reflection, Vertical Stretch, and Vertical Shift

Describe the graph of $k(t) = -2 \cos t + 3$. Then graph k on the interval $[-2\pi, 2\pi]$.

Solution

The graph of $k(t) = -2 \cos t + 3$ is the graph of $g(t) = \cos t$ reflected across the horizontal axis, vertically stretched by a factor of 2, and shifted 3 units upward, as shown in Figure 7.4-1.

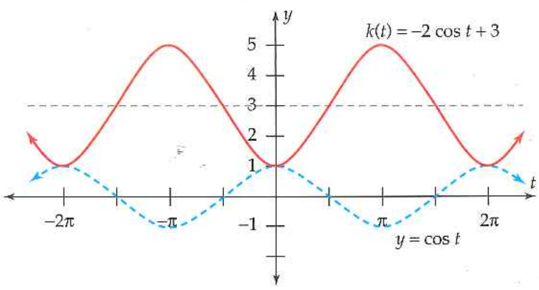


Figure 7.4-1

After the vertical shift, the graph of $k(t) = -2 \cos t + 3$ is vertically centered on the horizontal line $y = 3$.

Section 7.4 Periodic Graphs and Phase Shifts

Example Notes

For **Example 1**, ask students to identify the numbers or signs in the function $k(t) = -2 \cos t + 3$ that produce:

- a change in amplitude 2
- a reflection the negative sign before the 2
- a vertical shift 3

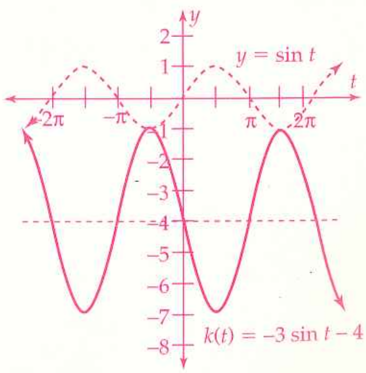
This Example contains the first discussion about a function that has amplitude other than 1 and is not vertically centered on the horizontal axis. The function k has amplitude 2 and is vertically centered on the line $y = 3$. Point out that the graph reaches a maximum of 2 units above the line $y = 3$ and a minimum of 2 units below the line $y = 3$.

ADDITIONAL EXAMPLES

Example 1

Describe the graph of $k(t) = -3 \sin t - 4$. Then graph k on the interval $[-2\pi, 2\pi]$.

The graph of $k(t) = -3 \sin t - 4$ is the graph of $g(t) = \sin t$ reflected across the horizontal axis, vertically stretched by a factor of 3, and shifted 4 units downward.



After the vertical shift, the graph of $k(t) = -3 \sin t - 4$ is vertically centered on the horizontal line $y = -4$.

Teaching Notes

Recall from Section 3.4 (page 175) that, if c is a positive number,

- The graph of $g(x) = f(x + c)$ is the graph of f shifted c units left.
- The graph of $g(x) = f(x - c)$ is the graph of f shifted c units right.

Point out that for periodic functions:

- A shift left is associated with a negative phase shift.
- A shift right is associated with a positive phase shift.

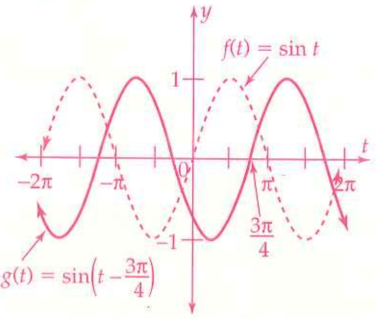
ADDITIONAL EXAMPLES

Example 2

Describe the graph of each function.

a. $g(t) = \sin\left(t - \frac{3\pi}{4}\right)$

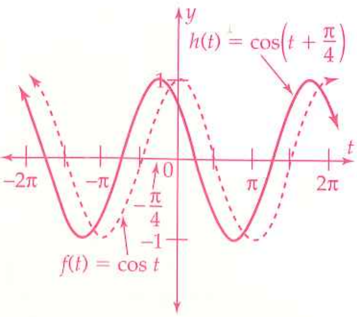
The graph of $g(t) = \sin\left(t - \frac{3\pi}{4}\right)$ is the graph of $f(t) = \sin t$ shifted $\frac{3\pi}{4}$ units to the right.



The cycle of f that begins at $t = 0$ becomes a cycle of g that begins at $t = \frac{3\pi}{4}$. Thus, g has a phase shift of $\frac{3\pi}{4}$.

b. $h(t) = \cos\left(t + \frac{\pi}{4}\right)$

The graph of $h(t) = \cos\left(t + \frac{\pi}{4}\right)$ is the graph of $f(t) = \cos t$ shifted $\frac{\pi}{4}$ units to the left.



The cycle of f that begins at $t = 0$ becomes a cycle of h that begins at $t = -\frac{\pi}{4}$. Thus, h has a phase shift of $-\frac{\pi}{4}$.

Phase Shifts

Recall from Section 3.4 that when the independent variable t in the rule of a function is replaced by $t - c$ or $t + c$, where c is a constant, the graph is shifted horizontally. For periodic functions, the number c is the phase **shift** associated with the graph.

Example 2 Phase Shift

Describe the graph of each function.

a. $g(t) = \sin\left(t + \frac{\pi}{2}\right)$ **b.** $h(t) = \cos\left(t - \frac{2\pi}{3}\right)$

Solution

a. The graph of $g(t) = \sin\left(t + \frac{\pi}{2}\right)$ is the graph of $f(t) = \sin t$ shifted to the left $\frac{\pi}{2}$ units, as shown in Figure 7.4-2.

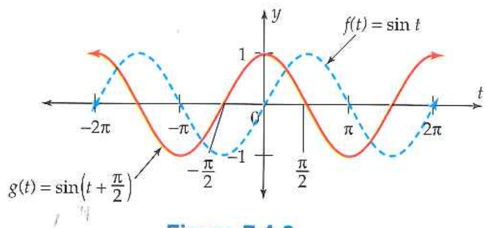


Figure 7.4-2

When the graph of $f(t) = \sin t$ is shifted to become the graph of $g(t) = \sin\left(t + \frac{\pi}{2}\right)$, the cycle of f that begins at $t = 0$ becomes a cycle of g that begins at $t = -\frac{\pi}{2}$. Thus, g has a phase shift of $-\frac{\pi}{2}$.

b. The graph of $h(t) = \cos\left(t - \frac{2\pi}{3}\right)$ is the graph of $f(t) = \cos t$ shifted to the right $\frac{2\pi}{3}$ units, as shown in Figure 7.4-3.

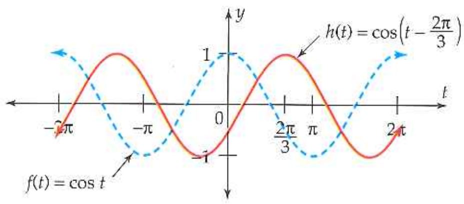


Figure 7.4-3

The cycle of $f(t) = \cos t$ that begins at $t = 0$ becomes a cycle of $h(t) = \cos\left(t - \frac{2\pi}{3}\right)$ that begins at $t = \frac{2\pi}{3}$. Thus, the function h has a phase shift of $\frac{2\pi}{3}$.

Combined Transformations

Now that you are familiar with the effects of various transformations on the sine and cosine functions, you are ready for some examples that simultaneously include changes in amplitude, period, and phase shift.

Example 3 Combined Transformations

State the amplitude, period, and phase shift of $f(t) = 3\sin(2t + 5)$.

Solution

Rewrite the function rule.

$$f(t) = 3\sin(2t + 5) = 3\sin\left[2\left(t + \frac{5}{2}\right)\right]$$

When the rule of f is written in this form, you can see that it is obtained from the rule of $k(t) = 3\sin 2t$ by replacing t with $t + \frac{5}{2}$. Therefore, the graph of f can be obtained by horizontally shifting the graph of k to the left $\frac{5}{2}$ units, as shown in Figure 7.4-4.

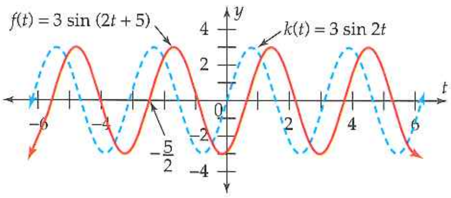


Figure 7.4-4

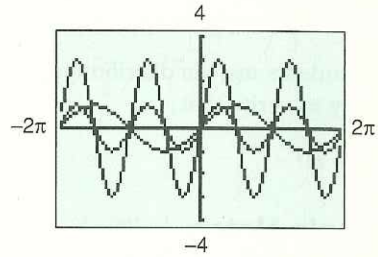
The cycle of k that begins at $t = 0$ becomes a cycle of f that begins at $t = -\frac{5}{2}$; so the function f has a phase shift of $-\frac{5}{2}$.

The amplitude of f is 3 and its period is $\frac{2\pi}{2} = \pi$.

Example Notes

For **Example 3,** students can review how the graph of $f(t) = 3\sin 2t$ is obtained by graphing the following functions on the same calculator screen: $Y_1 = \sin x$, $Y_2 = \sin(2x)$, and $Y_3 = 3\sin(2x)$

They will see the functions graphed in sequence, each function "building" on the previous function. The final result will be:



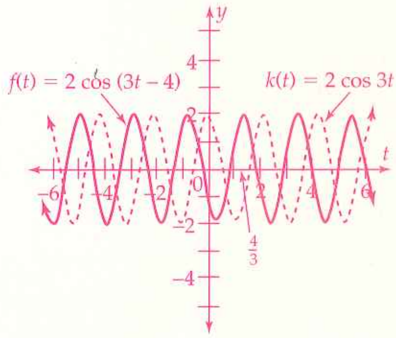
ADDITIONAL EXAMPLES

Example 3

State the amplitude, period, and phase shift of $f(t) = 2\cos(3t - 4)$.

Rewrite as $f(t) = 2\cos\left[3\left(t - \frac{4}{3}\right)\right]$.

The rule of f is obtained from the rule of $k(t) = 2\cos 3t$ by replacing t with $t - \frac{4}{3}$. So, the graph of f can be obtained by horizontally shifting the graph of k to the right $\frac{4}{3}$ units.



The cycle of k that begins at $t = 0$ becomes a cycle of f that begins at $t = \frac{4}{3}$, so the function f has a phase shift of $\frac{4}{3}$.

amplitude: 2

period: $\frac{2\pi}{3}$

phase shift: $\frac{4}{3}$

Teaching Notes

All the previous results that were developed in Examples 1–3 are generalized in the **Combined Transformations** box. The fact that might be the most difficult to understand is that the phase shift $= -\frac{c}{b}$. Refer to the equation at the top of this page:

$$f(t) = a\sin(bt + c) + d$$
$$= a\sin\left[b\left(t + \frac{c}{b}\right)\right] + d$$

Have students use the distributive property to verify that

$$bt + c = b\left(t + \frac{c}{b}\right).$$

Example Notes

In **Example 4**, it is not necessary to rewrite the function in the form $f(t) = a\cos\left[b\left(t + \frac{c}{b}\right)\right] + d$ to find the phase shift. From the form $f(t) = a\cos(bt + c) + d$, the phase shift can be determined directly using $-\frac{c}{b}$.

ADDITIONAL EXAMPLES

Example 4

Describe the graph of $g(t) = 3\cos(2t - 1) + 4$.

Rewrite as

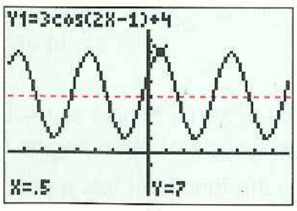
$g(t) = 3\cos[2t + (-1)] + 4$.

amplitude: $|a| = |3| = 3$

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift: $-\frac{c}{b} = -\frac{-1}{2} = \frac{1}{2}$

vertical shift: $d = 4$



The graph is vertically centered on the horizontal line $y = 4$. The waves reach a maximum of 3 units above and a minimum of 3 units below that horizontal line. The graph begins a cosine wave at $t = \frac{1}{2}$ and completes one cycle in π units.

The procedure that is used in Examples 1–3 can be used to analyze any function whose rule is of the form

$$f(t) = a\sin(bt + c) + d.$$

First rewrite the function rule as follows.

$$f(t) = a\sin(bt + c) + d = a\sin\left[b\left(t + \frac{c}{b}\right)\right] + d$$

Thus, the graph of f is obtained from the graph of $k(t) = a\sin bt$ by shifting it d units vertically and $-\frac{c}{b}$ units horizontally. The cycle of k that begins at $t = 0$ becomes the cycle of f that begins at $t = -\frac{c}{b}$, so f has phase shift $-\frac{c}{b}$. The amplitude of both f and k is $|a|$ and both have period $\frac{2\pi}{b}$. A similar analysis applies to the function $g(t) = a\cos(bt + c) + d$.

Combined Transformations

If $a \neq 0$ and $b > 0$, then each of the functions

$f(t) = a\sin(bt + c) + d$ and $g(t) = a\cos(bt + c) + d$

has the following characteristics:

amplitude $= |a|$ **period $= \frac{2\pi}{b}$**

phase shift $= -\frac{c}{b}$ **vertical shift $= d$**

Example 4 Combined Transformations

Describe the graph of $g(t) = 2\cos(3t - 4) - 1$.

Solution

Identify the amplitude, period, vertical shift and phase shift.

$$g(t) = 2\cos(3t - 4) - 1 = 2\cos[3t + (-4)] + (-1)$$

$$\text{amplitude } |a| = 2 \qquad \text{period} = \frac{2\pi}{b} = \frac{2\pi}{3}$$

$$\text{phase shift} = -\frac{c}{b} = -\left(\frac{-4}{3}\right) = \frac{4}{3} \qquad \text{vertical shift} = -1$$

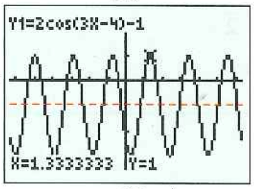


Figure 7.4-5

The graph of $g(t) = 2\cos(3t - 4) - 1$, shown in Figure 7.4-5, is vertically centered on the horizontal line $y = -1$. The waves reach a maximum of 2 units above that horizontal line and a minimum of 2 units below that horizontal line. The graph begins a cosine wave at $t = \frac{4}{3}$ and completes one cycle in $\frac{2\pi}{3}$ units.

Example 5 Combined Transformations

Identify the amplitude, period, vertical shift, and phase shift of $f(t) = -4\sin\left(\frac{t}{2} + 1\right) + 3$. Then graph at least one complete cycle of f.

Solution

The function rule $f(t)$ is of the form $a\sin(bt + c) + d$, with $a = -4$, $b = \frac{1}{2}$, $c = 1$, and $d = 3$.

$$\text{amplitude } |a| = |-4| = 4 \qquad \text{period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$\text{phase shift} = -\frac{c}{b} = -\frac{1}{\frac{1}{2}} = -2 \qquad \text{vertical shift} = 3$$

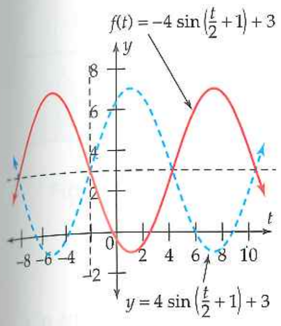


Figure 7.4-6

The waves of the graph are vertically centered on the horizontal line $y = 3$ reaching a maximum of 7 and a minimum of -1. The graph begins a sine wave at $t = -2$ and completes one cycle in $4\pi \approx 12.6$ units. The graph of f is the graph of $y = 4\sin\left(\frac{t}{2} + 1\right) + 3$ reflected across the horizontal line $y = 3$, as shown in Figure 7.4-6.

Example 6 Identifying Graphs

Find a sine function and a cosine function whose graphs look like the graph shown in Figure 7.4-7.

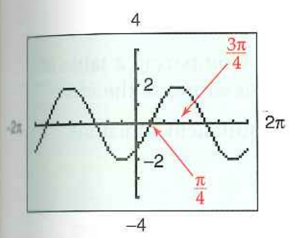


Figure 7.4-7

Solution

This graph appears to have an amplitude of 2 and to be centered vertically on the horizontal axis. The period appears to be 2π, so $b = 1$. Therefore, the graph looks like the graph of $f(t) = 2\sin t$ or $g(t) = 2\cos t$ shifted horizontally.

The graph of $f(t) = 2\sin t$ intercepts the x-axis at $t = 0$. The graph in Figure 7.4-7 intercepts the x-axis at $t = \frac{\pi}{4}$, so it looks like the graph of $f(t) = 2\sin t$ shifted $\frac{\pi}{4}$ units to the right. Therefore, this graph closely resembles the graph of $h(t) = 2\sin\left(t - \frac{\pi}{4}\right)$.

At $t = 0$, the graph of $g(t) = 2\cos t$ reaches its maximum of 2. The graph in Figure 7.4-7 reaches its maximum of 2 at $t = \frac{3\pi}{4}$, so it looks like the graph of $g(t) = 2\cos t$ shifted $\frac{3\pi}{4}$ units to the right. Therefore, this graph closely resembles the graph of $k(t) = 2\cos\left(t - \frac{3\pi}{4}\right)$.

ADDITIONAL EXAMPLES

Example 5

Identify the amplitude, period, vertical shift, and phase shift of $f(t) = -2\sin\left(\frac{t}{3} + 4\right) + 1$. Then graph at least one complete cycle of f.

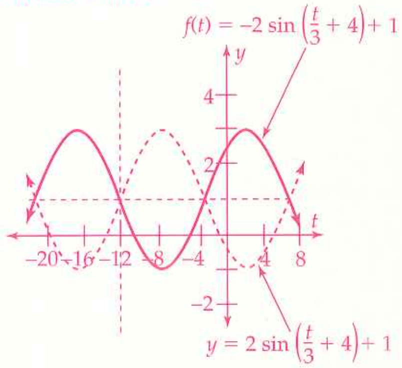
The function rule is of the form $a\sin(bt + c) + d$, with $a = -2$, $b = \frac{1}{3}$, $c = 4$, and $d = 1$.

amplitude: $|a| = |-2| = 2$

period: $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

phase shift: $-\frac{c}{b} = -\frac{4}{\frac{1}{3}} = -12$

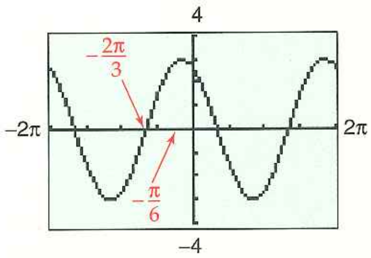
vertical shift: $d = 1$



The waves of the graph are vertically centered on the horizontal line $y = 1$, reaching a maximum of 3 and a minimum of -1. The graph begins a sine wave at $t = -12$ and completes one cycle in $6\pi \approx 18.8$ units. The graph of f is the graph of $y = 2\sin\left(\frac{t}{3} + 4\right) + 1$ reflected across the horizontal line $y = 1$.

Example 6

Find a sine function and a cosine function whose graphs look like the graph shown.



It appears that the graph has an amplitude of 3, is centered vertically on the horizontal axis, and has a period of 2π.

It looks like the graph of $f(t) = 3\sin t$ shifted $\frac{2\pi}{3}$ units to the left, so it could be the graph of $f(t) = 3\sin\left(t + \frac{2\pi}{3}\right)$.

It looks like the graph of $f(t) = 3\cos t$ shifted $\frac{\pi}{6}$ units to the left, so it could be the graph of $g(t) = 3\cos\left(t + \frac{\pi}{6}\right)$.

COMMON ERROR ALERT

n **Example 8a,** students might ncorrectly enter the function $(t) = \frac{1}{\frac{\tan t}{\cos t}}$ in their calculators they do not use parentheses as ecessary. Emphasize the correct se of parentheses:

ncorrect: $Y_1 = 1/\tan(x/\cos(x$

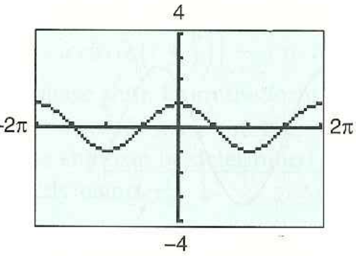
orrect: $Y_1 = 1/\tan(x)/\cos(x)$

ADDITIONAL EXAMPLES

Example 7

Vhich of the following equations could ossibly be an identity?

. $\sin\left(\frac{\pi}{2} - t\right) = \cos t$

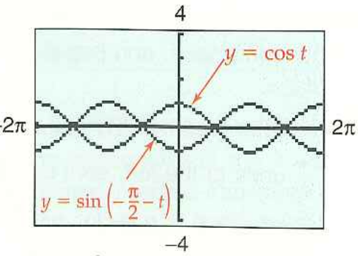


X	Y1	Y2
-3	-.99	-.99
-2	-.4161	-.4161
-1	.5403	.5403
0	1	1
1	.5403	.5403
2	-.4161	-.4161
3	-.99	-.99

X=0

The graphs of $f(t) = \sin\left(\frac{\pi}{2} - t\right)$ and $g(t) = \cos t$ appear to coincide, and the table values match, so $\sin\left(\frac{\pi}{2} - t\right) = \cos t$ is possibly an identity.

. $\sin\left(-\frac{\pi}{2} - t\right) = \cos t$



The graphs do not coincide, so $\sin\left(-\frac{\pi}{2} - t\right) = \cos t$ is not an identity.

NOTE Identities are proved algebraically in Chapter 9.

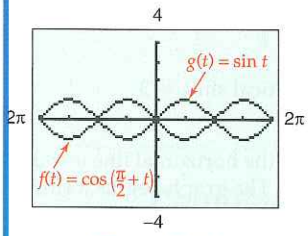


Figure 7.4-8

Graphs and Identities

Graphing calculators can be used to determine equations that could possibly be identities. A calculator cannot *prove* that such an equation is an identity, but it can provide evidence that it *might* be one. On the other hand, a calculator can prove that a particular equation is not an identity.

Example 7 **Possible Identities**

Which of the following equations could possibly be an identity?

a. $\cos\left(\frac{\pi}{2} + t\right) = \sin t$ **b.** $\cos\left(\frac{\pi}{2} - t\right) = \sin t$

Solution

a. If $\cos\left(\frac{\pi}{2} + t\right) = \sin t$ is an identity, then

$$f(t) = \cos\left(\frac{\pi}{2} + t\right) \text{ and } g(t) = \sin t$$

are equivalent functions and have the same graph. The graphs of f and g, shown in Figure 7.4-8 are obviously different. Therefore, $\cos\left(\frac{\pi}{2} + t\right) = \sin t$ is not an identity.

b. In Figure 7.4-9, the graphs of

$$f(t) = \cos\left(\frac{\pi}{2} - t\right) \text{ and } g(t) = \sin t$$

appear to coincide on the interval $[-2\pi, 2\pi]$. Comparing a table of values for f and g, shown in Figure 7.4-10, also supports the idea that $f(t) = \cos\left(\frac{\pi}{2} - t\right)$ and $g(t) = \sin t$ are equivalent functions.

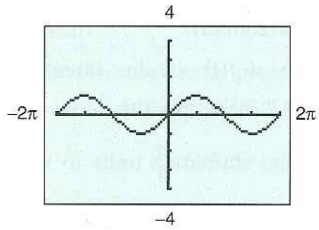


Figure 7.4-9

X	Y1	Y2
-3	-.1411	-.1411
-2	-.9093	-.9093
-1	-.8415	-.8415
0	0	0
1	.84147	.84147
2	.9093	.9093
3	.14112	.14112

X=0

Figure 7.4-10

This evidence strongly suggests that the equation $\cos\left(\frac{\pi}{2} - t\right) = \sin t$ is an identity, but does not prove it. Therefore, $\cos\left(\frac{\pi}{2} - t\right) = \sin t$ could possibly be an identity.

Example 8 Possible Identities

Which of the following equations could possibly be an identity?

a. $\frac{\cot t}{\cos t} = \sin t$ **b.** $\frac{\sin t}{\tan t} = \cos t$

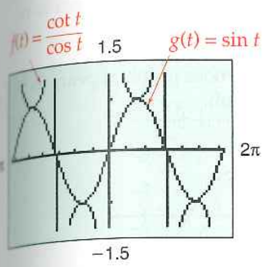


Figure 7.4-11

Solution

a. Rewrite f as $f(t) = \frac{\frac{1}{\tan t}}{\cos t}$ and compare its graph with the graph of $g(t) = \sin t$. The graphs of f and g, shown in Figure 7.4-11 are obviously different. Therefore, $\frac{\cot t}{\cos t} = \sin t$ is not an identity.

However, it does appear from the graph that $\frac{\cot t}{\cos t} = \csc t$ could possibly be an identity.

b. In Figure 7.4-12a, the graphs of $f(t) = \frac{\sin t}{\tan t}$ and $g(t) = \cos t$ appear to coincide. Comparing a table of values for f and g, shown in Figure 7.4-12b, also supports the idea that $f(t) = \frac{\sin t}{\tan t}$ and $g(t) = \cos t$ are equivalent functions for all values of t for which f is defined; that is, all values of t except those that make $\tan t = 0$.

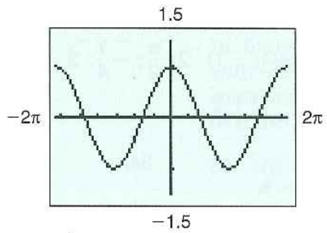


Figure 7.4-12a

X	Y1	Y2
-4	-.6536	-.6536
-3	-.99	-.99
-2	-.4161	-.4161
-1	.5403	.5403
0	ERROR	1
1	.5403	.5403
2	-.4161	-.4161

X=0

Figure 7.4-12b

Therefore, $\frac{\sin t}{\tan t} = \cos t$ could possibly be an identity for $\tan t \neq 0$.

CAUTION

Do not assume that two graphs that look the same on a calculator screen actually are the same. Depending on the viewing window, two graphs that are actually quite different may appear identical.

ADDITIONAL EXAMPLES

Example 8

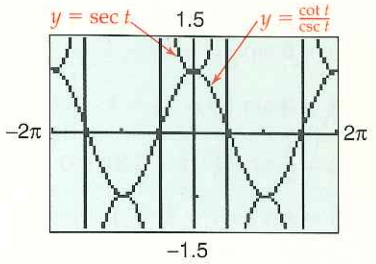
Which of the following equations could possibly be an identity?

a. $\frac{\cot t}{\csc t} = \sec t$

Rewrite the expressions:

$\frac{\cot t}{\csc t} = \frac{\frac{1}{\tan t}}{\frac{1}{\sin t}}$, $\sec t = \frac{1}{\cos t}$.

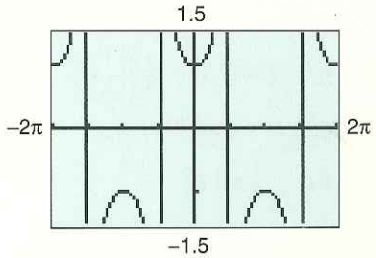
Compare graphs:



$\frac{\cot t}{\csc t} = \sec t$ is not an identity because the graphs do not coincide.

b. $\frac{\tan t}{\sin t} = \sec t$

Rewrite $\sec t$ as $\frac{1}{\cos t}$. Compare graphs:



X	Y1	Y2
-3	-1.01	-1.01
-2	-2.403	-2.403
-1	1.8508	1.8508
0	ERROR	1
1	1.8508	1.8508
2	-2.403	-2.403
3	-1.01	-1.01

X=0

The graphs appear to coincide, and table values match for t-values that make both expressions defined, so $\frac{\tan t}{\sin t} = \sec t$ is possibly an identity for all values of t such that $\frac{\tan t}{\sin t}$ and $\sec t$ are both defined, (that is, all values of t such that $\sin t \neq 0$ and $\cos t \neq 0$).

Teaching Notes

In Additional Example 8b, both $\frac{\tan t}{\sin t}$ and $\sec t$ are undefined for certain values of t.

Exercises 7.4 ANSWERS

. $f(t) = 3 \sin 8\left(t - \frac{\pi}{5}\right)$

. $f(t) = \sin \pi(t - 3) + 4$

. $f(t) = \frac{2}{3} \sin \frac{2}{3}\left(t + \frac{2\pi}{3}\right) - 2$

. $f(t) = 8 \sin 4\pi\left(t - \frac{2}{3}\right) + 4$

. $f(t) = 0.5 \sin 0.8\pi(t - 1.5) - 0.6$

. $f(t) = \sin \frac{2\pi}{5}(t - 0) + 3$

. $f(t) = 6 \sin \frac{6}{5}(t - 0) - 1$

. $f(t) = 2 \sin \frac{1}{4}(t - 1) + 1$

. $f(t) = \frac{5}{2} \sin \frac{10\pi}{9}(t - 0.2) + 0$

. $f(t) = \sin 2\pi(t + 1) - 1$

. a. $f(t) = 12 \sin\left(10t - \frac{\pi}{2}\right)$
 b. $g(t) = -12 \cos 10t$

. a. $f(t) = 18 \sin(\pi t)$
 b. $g(t) = 18 \cos\left(\pi t - \frac{\pi}{2}\right)$

. a. $f(t) = -\sin 2t$
 b. $g(t) = -\cos\left(2t - \frac{\pi}{2}\right)$

. a. $f(t) = \sin\left(4t - \frac{\pi}{2}\right)$
 b. $g(t) = -\cos(4t\pi)$

. a. $f(t) = \frac{1}{2} \sin 8t$
 b. $g(t) = \frac{1}{2} \cos\left(8t - \frac{\pi}{2}\right)$

. a. $f(t) = -3.5 \sin\left(3t - \frac{\pi}{2}\right)$
 b. $g(t) = 3.5 \cos(3t)$

. a. $f(t) = -4 \sin \frac{2}{3}t + 1$
 b. $f(t) = -4 \cos\left(\frac{2}{3}t - \frac{\pi}{2}\right) + 1$

. a. $f(t) = 3 \sin\left(4t - \frac{\pi}{2}\right) - 5$
 b. $g(t) = -3 \cos 4t - 5$

. a. $f(t) = -2 \sin\left(t - \frac{3\pi}{4}\right) + 6$
 b. $f(t) = 2 \cos\left(t - \frac{\pi}{4}\right) + 6$

. a. $f(t) = -2 \sin\left(t + \frac{\pi}{4}\right) - 5$
 b. $g(t) = 2 \cos\left(t + \frac{3\pi}{4}\right) - 5$

Exercises 7.4

In Exercises 1–20, state the amplitude, period, phase shift, and vertical shift of the function.

1. $h(t) = \cos(t + 1)$ — $1;\ 2\pi;\ -1;\ 0$
2. $m(t) = 7\cos(t + 3)$ — $7;\ 2\pi;\ -3;\ 0$
3. $f(t) = -5 \sin 2t$ — $5;\ \pi;\ 0;\ 0$
4. $k(t) = \cos\left(\frac{2\pi t}{3}\right)$ — $1;\ 3;\ 0;\ 0$
5. $k(t) = \sin(t - \pi) - 4$ — $1;\ 2\pi;\ \pi;\ -4$
6. $g(t) = 3\sin(2t - \pi)$ — $3;\ \pi;\ \frac{\pi}{2};\ 0$
7. $p(t) = 6\cos(3\pi t + 1)$ — $6;\ \frac{2}{3};\ -\frac{1}{3\pi};\ 0$
8. $f(t) = 4.5 \sin(12t - 6) - 5$ — $4.5;\ \frac{\pi}{6};\ \frac{1}{2};\ -5$
9. $h(t) = -4\cos\left(3t - \frac{\pi}{6}\right) + 1$ — $4;\ \frac{2\pi}{3};\ \frac{\pi}{18};\ 1$
10. $p(t) = -5\sin\left(\frac{t}{4} + 3\right) - 1$ — $5;\ 8\pi;\ -12;\ -1$
11. $q(t) = -7\sin\left(7t + \frac{1}{7}\right)$ — $7;\ \frac{2\pi}{7};\ -\frac{1}{49};\ 0$
12. $h(t) = 16\sin\left(\frac{2t}{3} - 4\right)$ — $16;\ 3\pi;\ 6;\ 0$
13. $d(t) = -3\sin\left(2t - \frac{5\pi}{4}\right)$ — $3;\ \pi;\ \frac{5\pi}{8};\ 0$
14. $c(t) = -\cos\left(\frac{3t}{2} - \frac{\pi}{3}\right) - 5$ — $1;\ \frac{4\pi}{3};\ \frac{2\pi}{9};\ -5$
15. $g(t) = 97\cos(14t + 5)$ — $97;\ \frac{\pi}{7};\ -\frac{5}{14};\ 0$
16. $f(t) = 3 - 2\cos(4t + 1)$ — $2;\ \frac{\pi}{2};\ -\frac{1}{4};\ 3$
17. $s(t) = 7 - \cos 2\pi t$ — $1;\ 1;\ 0;\ 7$
18. $m(t) = 4\cos(t - 5) + 2$ — $4;\ 2\pi;\ 5;\ 2$
19. $k(t) = 3\cos\left(\frac{\pi t}{3} - 1\right) + 5$ — $3;\ 6;\ \frac{3}{\pi};\ 5$
20. $h(t) = -4 - \sin\left(\frac{t}{3} + \frac{\pi}{4}\right)$ — $1;\ 6\pi;\ -\frac{3\pi}{4};\ -4$

In Exercises 21–30, state the rule of a sine function with the given amplitude, period, phase shift, and vertical shift, respectively.

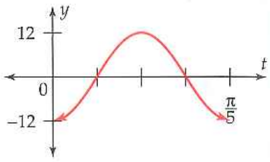
21. $3,\ \frac{\pi}{4},\ \frac{\pi}{5},\ 0$
22. $1, 2, 3, 4$
23. $\frac{2}{3},\ 3\pi,\ -\frac{2\pi}{3},\ -2$
24. $8,\ \frac{1}{2},\ \frac{2}{3},\ 4$
25. $0.5,\ 2.5,\ 1.5,\ -0.6$
26. $1, 5, 0, 3$
27. $6,\ \frac{5\pi}{3},\ 0,\ -1$
28. $2,\ 8\pi,\ 1,\ 1$
29. $\frac{5}{2},\ 1.8,\ 0.2,\ 0$
30. $1,\ 1,\ -1,\ -1$

In Exercises 31–40,

a. State the rule of a function of the form $f(t) = a\sin(bt + c) + d$ whose graph appears to be identical to the given graph.

b. State the rule of a function of the form $f(t) = a\cos(bt + c) + d$ whose graph appears to be identical to the given graph.

31.



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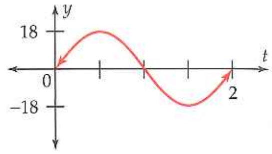

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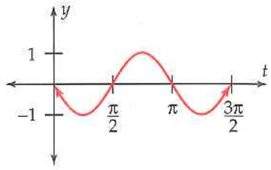

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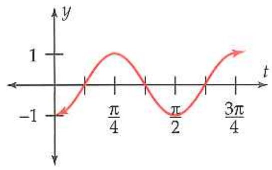

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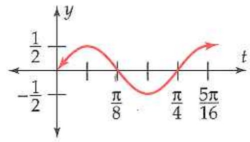

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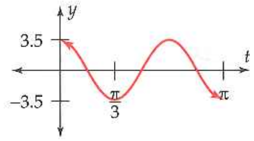

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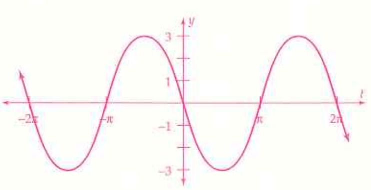

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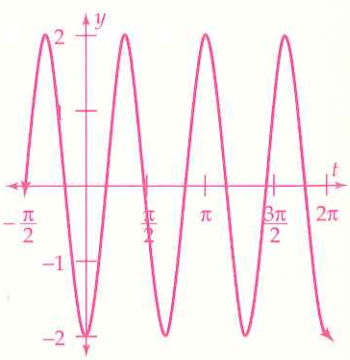


37.

38.

39.

40.

In Ex
cycle

41. k

43. p

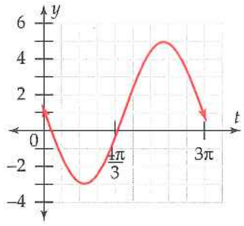
45. h

52.

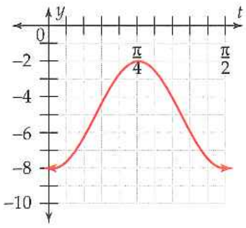
53.

58-

37.



38.



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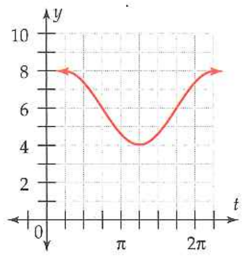

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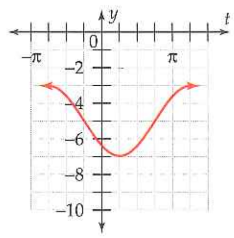


In Exercises 41–48, sketch the graph of at least one cycle of the function.

41. $k(t) = -3\sin t$

42. $y(t) = -2\cos 3t$

43. $p(t) = -\frac{1}{2}\sin 2t$

44. $q(t) = \frac{2}{3}\cos\frac{3}{2}t$

45. $h(t) = 3\sin\left(2t + \frac{\pi}{2}\right)$

46. $p(t) = 3\cos(3t - \pi)$

47. $f(t) = -\sin(2t - 3) + 1$

48. $g(t) = 5\cos\left(t - \frac{\pi}{3}\right) + 2$

In Exercises 49–52, graph the function over the interval $(0, 2\pi)$, and determine the location of all local maxima and minima.

49. $f(t) = \frac{1}{2}\sin\left(t - \frac{\pi}{3}\right)$

50. $g(t) = 2\sin\left(\frac{2t}{3} - \frac{\pi}{9}\right)$

51. $f(t) = -2\sin(3t - \pi)$

52. $h(t) = \frac{1}{2}\cos\left(\frac{\pi}{2}t - \frac{\pi}{8}\right) + 1$

53. Describe the graph of $f(t) = \sin^2 t + \cos^2 t$.

In Exercises 54–57, use graphs to determine whether the equation could possibly be an identity or is definitely not an identity.

54. $\dfrac{\cos t}{\cos\left(t - \frac{\pi}{2}\right)} = \cot t$
possibly an identity

55. $\dfrac{\sin t}{1 - \cos t} = \cot t$
not an identity

56. $\dfrac{\sec t + \csc t}{1 + \tan t} = \csc t$
possibly an identity

57. $\tan t = \cot\left(\frac{\pi}{2} - t\right)$
possibly an identity

In Exercises 58–61, graph f in a viewing window with $-2\pi \le t \le 2\pi$. Use the trace feature to determine constants a, b, and c such that the graph of f appears to coincide with the graph of $g(t) = a\sin(bt + c)$.

58. $f(t) = -3\sin t + 2\cos t$
$A \approx -3.6056$, $b = 1$, $c \approx -0.588$

59. $f(t) = 3\sin(4t + 2) + 2\cos(4t - 1)$
$A \approx 3.8332$, $b = 4$, $c \approx 1.4572$

60. $f(t) = 2\sin(3t - 5) - 3\cos(3t + 2)$
$A \approx 4.5699$, $b = 3$, $c \approx 0.765$

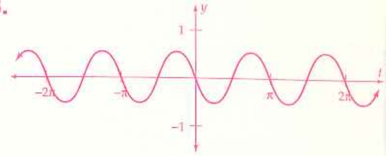
61. $f(t) = 2\sin t + 5\cos t$
$A \approx 5.3852$, $b = 1$, $c \approx 1.1903$

In Exercises 62–63, explain why there could not possibly be constants a, b, and c such that the graph of $g(t) = a\sin(bt + c)$ coincides with the graph of f.

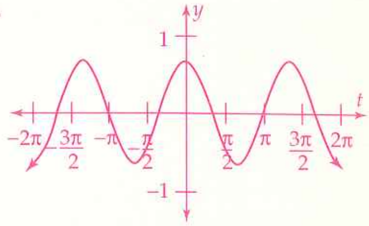
62. $f(t) = 2\sin(3t - 1) + 3\cos(4t + 1)$

63. $f(t) = \sin 2t + \cos 3t$

43.



44.



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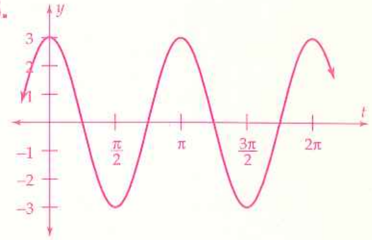
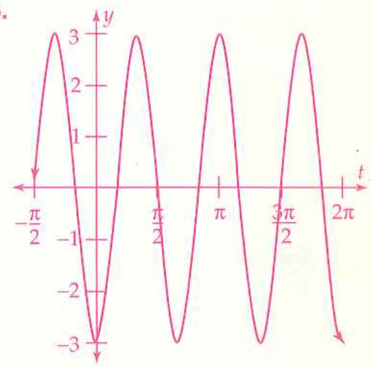


46.



47–48. *See p. 1069.*

49–52. *For graphs, see p. 1069.*

49. Local maximum at $t = \frac{5\pi}{6} \approx 2.6180$; local minimum at $t = \frac{11\pi}{6} \approx 5.7596$

50. Local maximum at $t = \frac{11\pi}{12} \approx 2.8798$; local minimum at $t = \frac{29\pi}{12} \approx 7.5922$

51. Local maxima at $t = \frac{\pi}{6} \approx 0.5236$, $t = \frac{5\pi}{6} \approx 2.6180$, $t = \frac{3\pi}{2} \approx 4.7124$; local minima at $t = \frac{\pi}{2} \approx 1.5708$, $t = \frac{7\pi}{6} \approx 3.6652$, $t = \frac{11\pi}{6} \approx 5.7596$

52. local maxima at $x = \frac{1}{4}$, $x = \frac{17}{4}$; local minima at $x = \frac{9}{4}$, $x = \frac{25}{4}$

53. The graph of $f(t) = \sin^2 t + \cos^2 t$ is the same as the graph of $f(x) = 1$, a horizontal line intercepting the y-axis at 1.

58–61. *For graphs, see p. 1069.*

62. Although the graph is periodic, it is not a sine wave and cannot be constructed from a sine curve through translations, stretches, or contractions.

63. All waves in the graph of g are of equal height, which is not the case with the graph of f. It cannot be constructed from a sine curve through translations, stretches, or contractions.