7.3 Periodic Graphs and Amplitude

Objectives

- State the period and amplitude (if any) given the function rule or the graph of a sine, cosine, or tangent function
- Use the period and amplitude (if any) to sketch the graph of a sine, cosine, or tangent function

A surprisingly large number of physical phenomena can be described by functions like the following:

$$f(t) = 5\sin(3t + 4) \qquad \text{and} \qquad g(t) = -4\cos(0.5t + 1) + 3$$

In this section and in the next, the graphs of such functions will be analyzed. All of these functions are periodic and their graphs consist of a series of identical *waves*. A single *wave* of the graph is called a **cycle.** The length of each cycle is the period of the function.

sine cycle

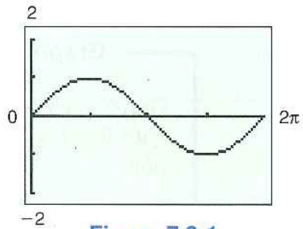


Figure 7.3-1

Every cycle for the sine function resembles the graph of $f(t) = \sin t$ from 0 to 2π, as shown in Figure 7.3-1.

- beginning at a point midway between its maximum and minimum value
- rising to its maximum value
- falling to its minimum value
- returning to the beginning point

Every cycle repeats the same pattern.

cosine cycle

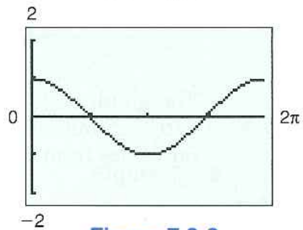


Figure 7.3-2

Similarly, every cycle for the cosine function resembles the graph of $g(t) = \cos t$ from 0 to 2π, as shown in Figure 7.3-2.

Section 7.3 Periodic Graphs and Amplitude

Real-World Application

A phenomenon that can be described by a sine function occurs every time you turn on a radio. In radio vocabulary, AM indicates Amplitude Modulation and FM indicates Frequency Modulation.

The current generated by an AM radio transmitter is given by a function of the form $f(t) = A\sin 2000\pi mt$, where A represents amplitude, m is the location on the broadcast dial, and t is measured in seconds. By modulating (changing) the amplitude, the sound information in the radio signal can be controlled.

In Exercise 65, you are asked to find the period of $f(t) = A\sin 2000\pi mt$ for a radio station at 900 on the AM broadcast dial.

Teaching Notes

Discuss the meanings of *wave, cycle,* and *period:*

- A wave is the same as a cycle. It is a portion of a graph that shows a complete pattern.
- A period is a number; it is the horizontal distance required for one cycle.

Also make sure students understand that only one wave (not two) is shown in Figure 7.3-1.

Math Background

Students might use the term *wavelength* in science. Explain that the math term for wavelength is period. In Figure 7.3-1, we say the *period* is 2π, while a physicist might say the *wavelength* is 2π.

In calculus, students will learn how to use a definite integral to represent an *arc length.* In Figure 7.3-1, if you were able to use a piece of string to exactly fit on the curve, and then straighten the string and measure it, you would have the arc length of the curve.

- beginning at its maximum value
- falling to its minimum value
- returning to the beginning point

Again, every cycle repeats the same pattern.

Period

Before proceeding to the discussion about functions that have different periods, it will be helpful to consider functions of the form

$$f(t) = \sin bt \qquad \text{and} \qquad g(t) = \cos bt$$

where b is a constant. The constant b changes the period of the sine or cosine function. Its effect on the graph is to increase or decrease the length of each cycle.

Graphing Exploration

Graph each function below, one at a time, in a viewing window with $0 \le t \le 2\pi$. Answer the questions that follow for each function.

$$f(t) = \cos 4t \qquad h(t) = \sin 5t$$

Determine the number of complete cycles between 0 and 2π.

Find the period, or length of one complete cycle. *Hint:* Use division.

The exploration above suggests the following rule.

Period of sin bt and cos bt

If $b > 0$, then the graph of either

$$f(t) = \sin bt \qquad \text{or} \qquad g(t) = \cos bt$$

makes b complete cycles between 0 and 2π, and each function has a period of $\frac{2\pi}{b}$.

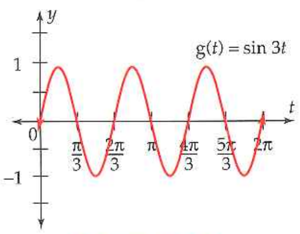


Figure 7.3-3

The graph of $h(t) = \sin t$ completes one cycle as t takes on values from 0 to 2π. Similarly, the graph of $g(t) = \sin 3t$ completes one cycle as $3t$ takes on values from 0 to 2π.

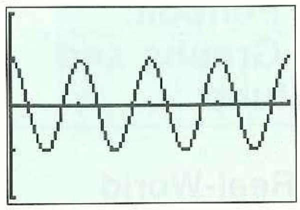
When $3t = 0$, t must be 0.

When $3t = 2\pi$, t must be $\frac{2\pi}{3}$.

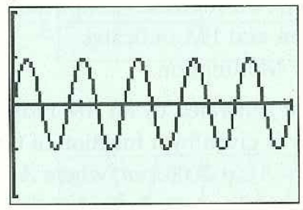
Therefore, the graph of $g(t) = \sin 3t$ completes one cycle as t takes on values from 0 to $\frac{2\pi}{3}$, as shown in Figure 7.3-3.

Teaching Notes

Solution to the **Graphing Exploration:**



$f(t) = \cos 4t$
number of cycles: 4
period: $\frac{2\pi}{4} = \frac{\pi}{2}$



$h(t) = \sin 5t$
number of cycles: 5
period: $\frac{2\pi}{5}$

Ask students to explain how they found the period of each function in the Graphing Exploration.
For $f(t) = \cos 4t$, divide 2π by 4.
For $h(t) = \sin 5t$, divide 2π by 5.

The discussion at the bottom of the page uses the function $g(t) = \sin 3t$ to illustrate the **Period of sin *bt*.** If students need help in understanding why the period of $\sin 3t$ is $\frac{2\pi}{3}$, set up the following table:

t	$3t$	$\sin 3t$
?	0	0
⋮	⋮	⋮
?	2π	0

Explain that $3t$ is "what we are taking the sine of," so a cycle beginning at 0 is complete when $3t = 2\pi$. Then ask how we can find the corresponding values of t. Divide each value of $3t$ by 3.
Show $0 \div 3 = 0$ and $2\pi \div 3 = \frac{2\pi}{3}$. This shows that a complete cycle occurs as t increases from 0 to $\frac{2\pi}{3}$.

Example 1 **Determining Period**

Determine the period of each function.

a. $k(t) = \cos 3t$

b. $f(t) = \sin \frac{t}{2}$

Solution

a. The function $k(t) = \cos 3t$ has a period of $\frac{2\pi}{b} = \frac{2\pi}{3}$, as shown in Figure 7.3-4.

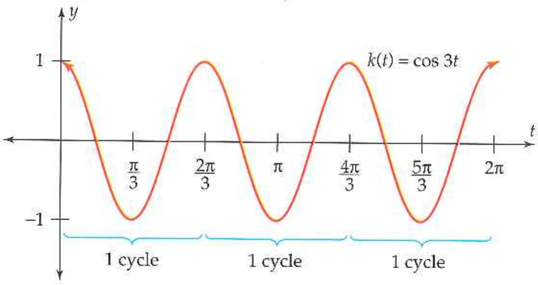


Figure 7.3-4

b. Rewrite $f(t) = \sin \frac{t}{2}$ as $f(t) = \sin\left(\frac{1}{2}t\right)$. The function $f(t) = \sin\left(\frac{1}{2}t\right)$ has a period of $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$, as shown in Figure 7.3-5.

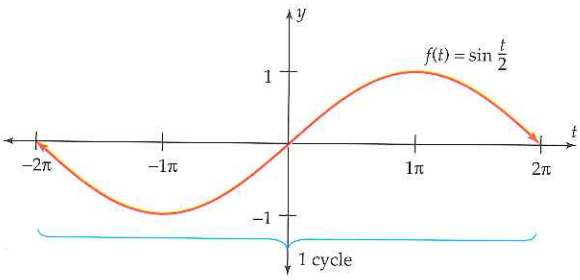


Figure 7.3-5

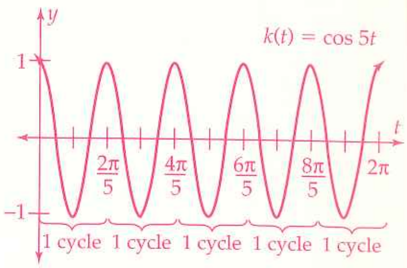
ADDITIONAL EXAMPLES

Example 1

Determine the period of each function.

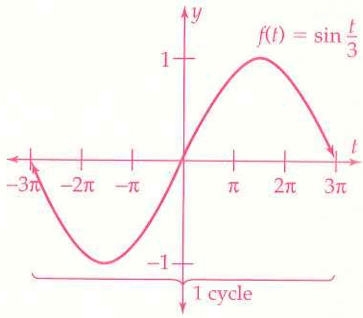
a. $k(t) = \cos 5t$

period: $\frac{2\pi}{5}$



b. $f(t) = \sin \frac{t}{3}$

period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$



Teaching Notes

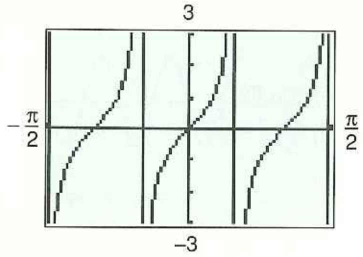
After completing **Example 1,** make sure students realize that, for $f(t) = \sin bt$ or $g(t) = \cos bt$:

- When $b > 1$, the function *repeats itself more frequently* than its parent function on any given interval (In **1a,** $k(t) = \cos 3t$ repeats itself more frequently than $y = \cos t$).
- When $b < 1$, the function *repeats itself less frequently* than its parent function on any given interval (In **1b,** $f(t) = \sin \frac{1}{2}t$ repeats itself less frequently than $y = \sin t$).

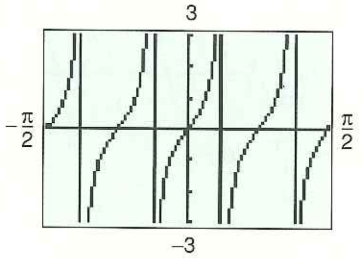
Note that changes in the period follow the rules for Horizontal Stretches and Compressions on page 179.

Teaching Notes

Solution to the **Graphing Exploration:**



$f(t) = \tan 3t$
number of cycles: 3
period: $\frac{\pi}{3}$



$g(t) = \tan 4t$
number of cycles: 4
period: $\frac{\pi}{4}$

Example Notes

For **Example 2**, refer students to the following phrase in the **Period of tan *bt*** box:
$f(t) = \tan bt$ makes b complete cycles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

For **2a**, $k(t) = \tan 2t$ makes 2 complete cycles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Therefore, as stated in the Solution, $k(t)$ makes 1 complete cycle between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$.

COMMON ERROR ALERT

Students might use 2π instead of π to find the period of a tangent function because 2π is used for sine and cosine functions). Remind them that:

The period of sin *bt* and cos *bt* is $\frac{2\pi}{b}$.

The period of tan *bt* is $\frac{\pi}{b}$.

CAUTION

A calculator may not produce an accurate graph of $f(t) = \sin bt$ or $g(t) = \cos bt$ for large values of b. For instance, the graph of

$$f(t) = \sin 50t$$

has 50 complete cycles between 0 and 2π, but that is not what your calculator will show. (try it!)

Graphing Exploration

Graph each function below, one at a time, in a viewing window with $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$. Answer the questions that follow for each function.

$$f(t) = \tan 3t \qquad g(t) = \tan 4t$$

Determine the number of complete cycles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Find the period, that is, the length of one complete cycle.

The exploration above suggests the following rule.

Period of tan *bt*

If $b > 0$, then the graph of

$$f(t) = \tan bt$$

makes b complete cycles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and the function has a period of $\frac{\pi}{b}$.

Example 2 Determining Period

Determine the period of each function.

a. $k(t) = \tan 2t$ **b.** $f(t) = \tan \frac{t}{3}$

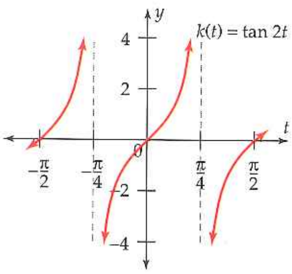


Figure 7.3-6

Solution

a. The function $k(t) = \tan 2t$ has a period of $\frac{\pi}{b} = \frac{\pi}{2}$. It completes one cycle between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$, as shown in Figure 7.3-6.

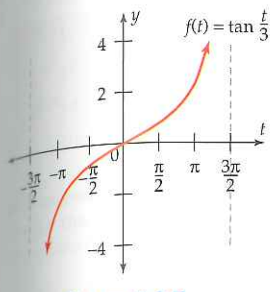


Figure 7.3-7

b. Rewrite as $f(t) = \tan \frac{t}{3}$ as $f(t) = \tan\left(\frac{1}{3}t\right)$. The function has a period of $\frac{\pi}{b} = \frac{\pi}{\frac{1}{3}} = 3\pi$, as shown in Figure 7.3-7.

Amplitude

Recall from Section 3.4 that multiplying the rule of a function by a positive constant has the effect of stretching or compressing its graph vertically.

Example 3 Vertical and Horizontal Stretches or Compressions

Graph each function.

a. $g(t) = 7 \cos 3t$ **b.** $h(t) = \frac{1}{3} \sin \frac{t}{2}$

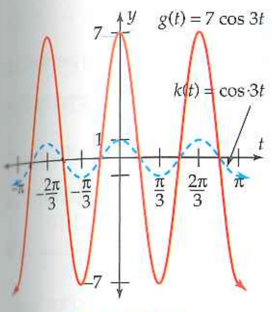


Figure 7.3-8

Solution

a. The function $g(t) = 7 \cos 3t$ is the function $k(t) = \cos 3t$ multiplied by 7. Consequently, the graph of g is the graph of k (see Example 1**a**) stretched vertically by a factor of 7.

As Figure 7.3-8 shows, stretching the graph affects only the height of the waves in the graph, not the period of the function. So the period of g is the same as that of $k(t) = \cos 3t$, namely $\frac{2\pi}{b} = \frac{2\pi}{3}$.

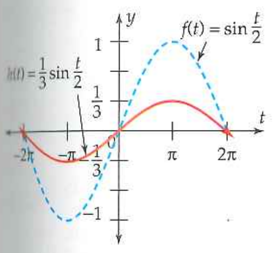


Figure 7.3-9

b. The function $h(t) = \frac{1}{3} \sin \frac{t}{2}$ is the function $f(t) = \sin \frac{t}{2}$ multiplied by $\frac{1}{3}$. Consequently, the graph of h is the graph of f (see Example 1**b**) vertically compressed by a factor of $\frac{1}{3}$. The period of h is the same as the period of f, namely $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

As the graphs in Example 3 illustrate, vertically stretching or compressing the graph affects only the height, not the period of the function.

The graph of $g(t) = 7 \cos 3t$ in Example 3 reaches a maximum value of 7 units above the horizontal axis and a minimum value of 7 units below the horizontal axis. In general, the graph of $f(t) = a \sin bt$ or $g(t) = a \cos bt$ reaches a distance of $|a|$ units above and below the horizontal axis, and is said to have an **amplitude** of $|a|$. The graph of $g(t) = 7 \cos 3t$ has an amplitude of 7.

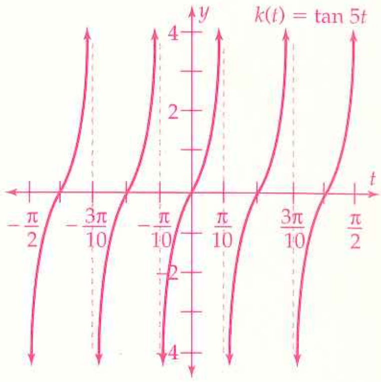
ADDITIONAL EXAMPLES

Example 2

Determine the period of each function.

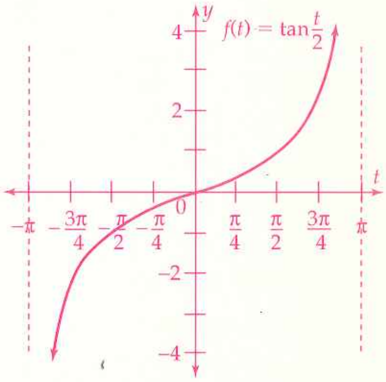
a. $k(t) = \tan 5t$ period: $\frac{\pi}{5}$

The function completes one cycle between $-\frac{\pi}{10}$ and $\frac{\pi}{10}$.



b. $f(t) = \tan \frac{t}{2}$ period: $\frac{\pi}{\frac{1}{2}} = 2\pi$

The function completes one cycle between $-\pi$ and π.

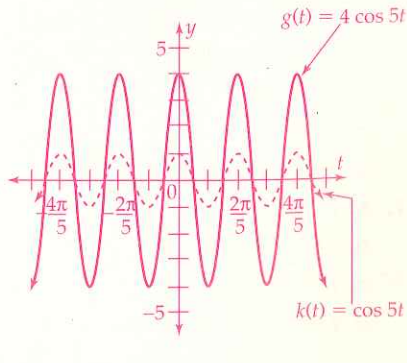


Example 3

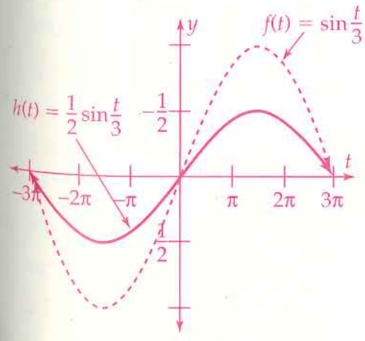
Graph each function.

a. $g(t) = 4 \cos 5t$

The graph of g is the graph of $k(t) = \cos 5t$ (see Additional Example 1a) stretched vertically by a factor of 4.



b. $h(t) = \frac{1}{2} \sin \frac{t}{3}$



The graph of h is the graph of $f(t) = \sin \frac{t}{3}$ (see Additional Example 1b) vertically compressed by a factor of $\frac{1}{2}$.

ADDITIONAL EXAMPLES

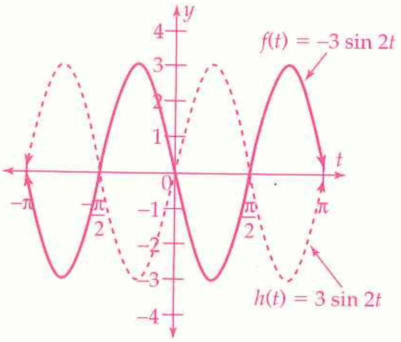
Example 4

Determine the amplitude and period of $f(t) = -3 \sin 2t$. Then graph f on the interval $[-\pi, \pi]$.

amplitude: $|-3| = 3$

period: $\frac{2\pi}{2} = \pi$

$f(t) = -3 \sin 2t$ is the graph of $h(t) = 3 \sin 2t$ reflected across the x-axis.



Amplitude and Period

If $a \neq 0$ and $b > 0$, then each of the functions

$$f(t) = a \sin bt \quad \text{or} \quad g(t) = a \cos bt$$

has an amplitude of $|a|$ and a period of $\frac{2\pi}{b}$.

Example 4 **Determining Amplitude and Period**

Determine the amplitude and period of $f(t) = -2 \sin 4t$. Then graph f on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

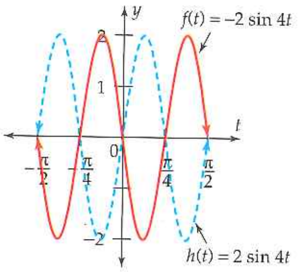


Figure 7.3-10

Solution

The amplitude of $f(t) = -2 \sin 4t$ is $|a| = |-2| = 2$, and the period of $f(t) = -2 \sin 4t$ is $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$. So the graph of f consists of cycles that are $\frac{\pi}{2}$ long and rise and fall between the heights of -2 and 2. To graph this function, be sure to notice that its graph is the reflection of $h(t) = 2 \sin 4t$ across the horizontal axis, as shown in Figure 7.3-10. ■

Although the graph of any function can be vertically stretched or compressed, amplitude only applies to bounded periodic functions.

Exercises 7.3

In Exercises 1–15, state the amplitude (if any) and period of each function.

1. $f(t) = -\cos t$
2. $f(t) = \sin 2t$
3. $f(t) = \cos 3t$
4. $f(t) = 2.5 \tan t$
5. $f(t) = 4 \cos t$
6. $f(t) = -3 \sin t$
7. $f(t) = 5 \tan 2t$
8. $f(t) = 1.2 \cos 0.5t$
9. $f(t) = -0.3 \sin \frac{t}{3}$
10. $f(t) = -\tan 0.4t$
11. $f(t) = \frac{1}{2} \sin 3t$
12. $f(t) = -\frac{1}{2} \tan 3t$
13. $f(t) = 5 \cos 1.7t$
14. $f(t) = 2 \sin \frac{2\pi t}{3}$
15. $f(t) = \frac{1}{3} \tan \frac{\pi t}{4}$

16. a. What is the period of $f(t) = \sin 2\pi t$?
    b. For what values of t (with $0 \le t \le 1$) is $f(t) = 0$?
    c. For what values of t (with $0 \le t \le 1$) is $f(t) = 1$?
    d. For what values of t (with $0 \le t \le 1$) is $f(t) = -1$?
17. a. What is the period of $f(t) = \cos \pi t$?
    b. For what values of t (with $0 \le t \le 2$) is $f(t) = 0$?
    c. For what values of t (with $0 \le t \le 2$) is $f(t) = 1$?
    d. For what values of t (with $0 \le t \le 2$) is $f(t) = -1$?
18. a. What is the period of $f(t) = \tan \pi t$?
    b. For what values of t $\left(\text{with } -\frac{1}{2} \le t \le \frac{1}{2}\right)$ is $f(t) = 0$?

Exercises 7.3

ANSWERS

1. amplitude: 1; period: 2π
2. amplitude: 1; period: π
3. amplitude: 1; period: $\frac{2\pi}{3}$
4. amplitude: none; period: π
5. amplitude: 4; period: 2π
6. amplitude: 3; period: 2π
7. amplitude: none; period: $\frac{\pi}{2}$
8. amplitude: 1.2; period: 4π
9. amplitude: 0.3; period: 6π
10. amplitude: none; period: $\frac{5\pi}{2}$
11. amplitude: $\frac{1}{2}$; period: $\frac{2\pi}{3}$
12. amplitude: none; period: $\frac{\pi}{3}$
13. amplitude: 5; period: $\frac{20\pi}{17}$
14. amplitude: 2; period: 3
15. amplitude: none; period: 4
16. a. 1
    b. $t = 0, \frac{1}{2}$, or 1
    c. $t = \frac{1}{4}$
    d. $t = \frac{3}{4}$
17. a. 2
    b. $t = \frac{1}{2}$ or $\frac{3}{2}$
    c. $t = 0$ or 2
    d. $t = 1$
18. a. 1
    b. $t = 0$
    c. $t = \frac{1}{4}$
    d. $t = -\frac{1}{4}$
19. g is the graph of f horizontally compressed by a factor of $\frac{1}{5}$; amplitude: 1; period: $\frac{2\pi}{5}$
20. g is the graph of f horizontally compressed by a factor of $\frac{1}{3}$; amplitude: none; period: $\frac{\pi}{3}$
21. g is the graph of f horizontally compressed by a factor of $\frac{1}{8}$; amplitude: 1; period: $\frac{\pi}{4}$

c. For what values of t $\left(\text{with } -\frac{1}{2} \le t \le \frac{1}{2}\right)$ is $f(t) = 1$?

d. For what values of t $\left(\text{with } -\frac{1}{2} \le t \le \frac{1}{2}\right)$ is $f(t) = -1$?

In Exercises 19–38, describe the transformations that change the graph of f into the graph of g. State the amplitude (if any) and the period of g.

19. $f(t) = \sin t;\ g(t) = \sin 5t$

20. $f(t) = \tan t;\ g(t) = \tan 3t$

21. $f(t) = \cos t;\ g(t) = \cos 8t$

22. $f(t) = \cos t;\ g(t) = \cos(-t)$

23. $f(t) = \tan t;\ g(t) = \tan(-t)$

24. $f(t) = \sin t;\ g(t) = \sin(-t)$

25. $f(t) = \sin t;\ g(t) = \sin 1.6t$

26. $f(t) = \cos t;\ g(t) = \cos 2.6t$

27. $f(t) = \sin t;\ g(t) = 3 \sin t$

28. $f(t) = \cos t;\ g(t) = \frac{1}{2} \cos t$

29. $f(t) = \tan t;\ g(t) = \frac{1}{3} \tan t$

30. $f(t) = \sin t;\ g(t) = 4 \sin \frac{t}{2}$

31. $f(t) = \sin t;\ g(t) = 5 \sin 2t$

32. $f(t) = \tan t;\ g(t) = -2 \tan \frac{t}{2}$

33. $f(t) = \tan t;\ g(t) = -2 \tan 0.2t$

34. $f(t) = \cos t;\ g(t) = 3 \cos 6t$

35. $f(t) = \cos t;\ g(t) = \frac{2}{5} \cos 8t$

36. $f(t) = \sin t;\ g(t) = -2 \sin \frac{3\pi t}{5}$

37. $f(t) = \tan t;\ g(t) = \frac{1}{3} \tan \pi t$

38. $f(t) = \cos t;\ g(t) = \frac{5}{3} \cos \frac{\pi t}{3}$

In Exercises 39–44, sketch at least one cycle of the graph of each function.

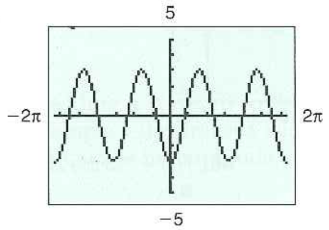
39. $f(t) = 4 \cos \frac{t}{2}$ **40.** $f(t) = \frac{2}{3} \sin 2t$

41. $f(t) = 2 \tan 3t$ **42.** $f(t) = -0.8 \cos \pi t$

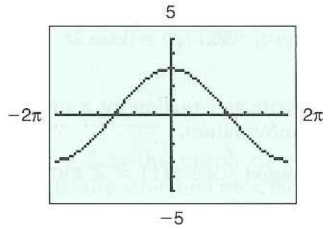
43. $f(t) = 3.5 \sin 2\pi t$ **44.** $f(t) = \tan \frac{\pi t}{2}$

In Exercises 45–50, match a graph to a function. Only one graph is possible for each function.

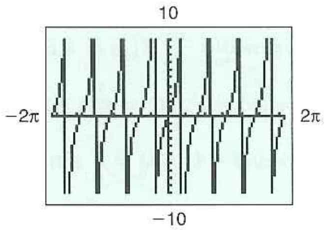
a.



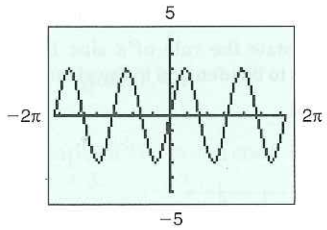
b.



c.



d.



22. g is the graph of f reflected across the y-axis; amplitude: 1; period: 2π

23. g is the graph of f reflected across the y-axis; amplitude: none; period: π

24. g is the graph of f reflected across the y-axis; amplitude: 1; period: 2π

25. g is the graph of f horizontally compressed by a factor of $\frac{5}{8}$; amplitude: 1; period: $\frac{5\pi}{4}$

26. g is the graph of f horizontally compressed by a factor of $\frac{5}{13}$; amplitude: 1; period: $\frac{10\pi}{13}$

27. g is the graph of f vertically stretched by a factor of 3; amplitude: 3; period: 2π

28. g is the graph of f vertically compressed by a factor of $\frac{1}{2}$; amplitude: $\frac{1}{2}$; period: 2π

29. g is the graph of f vertically compressed by a factor of $\frac{1}{3}$; amplitude: none; period: π

30. g is the graph of f vertically stretched by a factor of 4 and horizontally stretched by a factor of 2; amplitude: 4; period: 4π

31. g is the graph of f vertically stretched by a factor of 5 and horizontally compressed by a factor of $\frac{1}{2}$; amplitude: 5; period: π

32. g is the graph of f reflected across the x-axis, vertically stretched by a factor of 2, and horizontally stretched by a factor of 2; amplitude: none; period: 2π

33. g is the graph of f reflected across the x-axis, vertically stretched by a factor of 2, and horizontally stretched by a factor of 5; amplitude: none; period: 5π

34. g is the graph of f vertically stretched by a factor of 3 and horizontally compressed by a factor of $\frac{1}{6}$; amplitude: 3; period: $\frac{\pi}{3}$

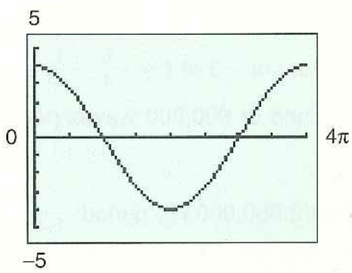
35. g is the graph of f vertically compressed by a factor of $\frac{2}{5}$ and horizontally compressed by a factor of $\frac{1}{8}$; amplitude: $\frac{2}{5}$; period: $\frac{\pi}{4}$

36. g is the graph of f reflected across the x-axis, vertically stretched by a factor of 2, and horizontally compressed by a factor of $\frac{5\pi}{3}$; amplitude: 2; period: $3\frac{1}{3}$

37. g is the graph of f vertically compressed by a factor of $\frac{1}{3}$ and horizontally compressed by a factor of $\frac{1}{\pi}$; amplitude: none; period: 1

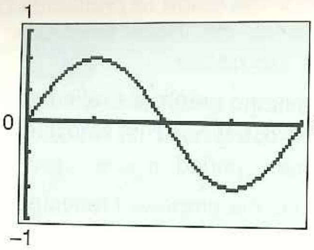
38. g is the graph of f vertically stretched by a factor of $\frac{5}{3}$ and horizontally compressed by a factor of $\frac{3}{\pi}$; amplitude: $\frac{5}{3}$; period: 6

39.

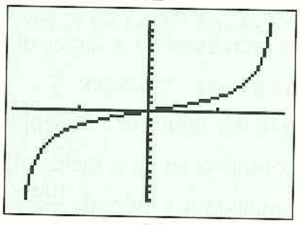


40–44. *See p. 500.*

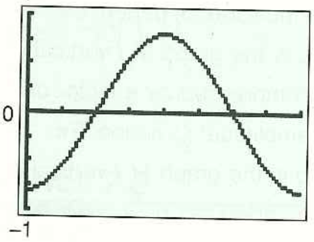
40.



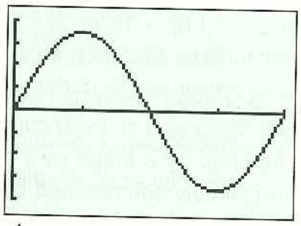
41.



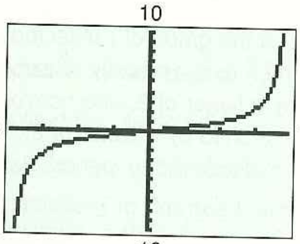
42.



43.



44.



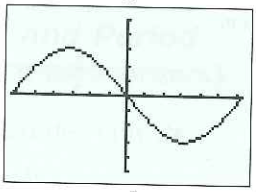
60. local maximum of 1 at $t = \frac{\pi}{4}$; local minimum of −1 at $t = \frac{3\pi}{4}$

61. local maximum of 1 at $t = 0$ and $t = \frac{2\pi}{3}$; local minimum of −1 at $t = \frac{\pi}{3}$

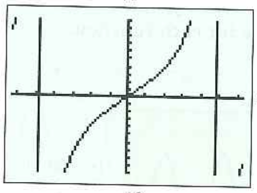
62. local maximum of 1 at $t = 0$; local minimum of −1 at $t = -2\pi$

63. local maximum of $\frac{\sqrt{3}}{2}$ at $t = \pi$; local minimum of −1 at $t = -\frac{3\pi}{2}$

e.



f.



45. $f(t) = 3 \sin 2t$ **d**

46. $f(t) = -3 \cos 2t$ **a**

47. $f(t) = 3 \cos \frac{t}{2}$ **b**

48. $f(t) = -3 \sin \frac{t}{2}$ **e**

49. $f(t) = 5 \tan \frac{t}{3}$ **f**

50. $f(t) = 3 \tan 2t$ **c**

In Exercises 51–56, write an equation for a sine function with the given information.

51. amplitude = 2, period = 4π $f(t) = 2 \sin \frac{t}{2}$

52. amplitude = $\frac{1}{2}$, period = $\frac{\pi}{2}$ $f(t) = \frac{1}{2} \sin 4t$

53. amplitude = 1.8, period = $\frac{3\pi}{2}$ $f(t) = 1.8 \sin \frac{4t}{3}$

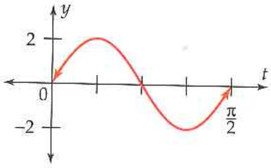
54. amplitude = 1, period = 2 $f(t) = \sin \pi t$

55. amplitude = $\frac{3}{2}$, period = 4 $f(t) = \frac{3}{2} \sin \frac{\pi t}{2}$

56. amplitude = 6, period = $\frac{1}{2}$ $f(t) = 6 \sin 4\pi t$

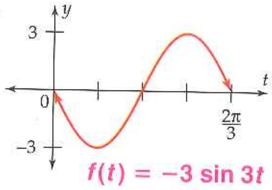
In Exercises 57–59, state the rule of a sine function whose graph appears to be identical to the given graph.

57.



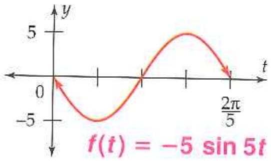
$f(t) = 2 \sin 4t$

58.



$f(t) = -3 \sin 3t$

59.



$f(t) = -5 \sin 5t$

In Exercises 60–64, state all local minima and maxima of the function on the given interval.

60. $f(t) = \sin 2t; 0 \le t \le \pi$

61. $f(t) = \cos 3t; 0 \le t \le \pi$

62. $f(t) = \cos \frac{t}{2}; -2\pi \le t \le \pi$

63. $f(t) = \sin \frac{t}{3}; -2\pi \le t \le \pi$

64. $f(t) = 3 \sin 2\pi t; -1.5 \le t \le 1.5$

65. The current generated by an AM radio transmitter is given by a function of the form $f(t) = A \sin 2000\pi m t$, where $550 \le m \le 1600$ is the location on the broadcast dial and t is measured in seconds. For example, a station at 900 on the AM dial has a function of the form

$$f(t) = A \sin 2000\pi(900)\, t = A \sin 1{,}800{,}000\pi t$$

Sound information is added to this signal by modulating A, that is, by changing the amplitude of the waves being transmitted. AM means amplitude modulation. For a station at 900 on the dial, what is the period of function f?

66. Find the function f, its period, and its frequency for a radio station at 1440 on the dial. (See Exercise 65.)

64. local maximum of 3 at $t = -\frac{3}{4}, \frac{1}{4}$, and $\frac{5}{4}$; local minimum of −3 at $t = -\frac{5}{4}, -\frac{1}{4}$, and $\frac{3}{4}$

65. $\frac{1}{900{,}000}$ second or 900,000 waves per second

66. $f(t) = A \sin(2{,}880{,}000\pi t)$; period: $\frac{1}{1{,}440{,}000}$ seconds; frequency: 1,440,000 waves per second