Name _____ Date _____ Class _____

LESSON 7-1

Reteach

Integer Exponents

Remember that 2^3 means $2 \times 2 \times 2 = 8$. The base is 2, the exponent is positive 3. Exponents can also be 0 or negative.

	Zero Exponents	Negative Exponents	Negative Exponents in the Denominator
Definition	For any nonzero number x, $x^0 = 1$.	For any nonzero number x and any integer n, $x^{-n} = \frac{1}{x^n}$.	For any nonzero number x and any integer n, $\frac{1}{x^{-n}} = x^n$.
Examples	$6^0 = 1$ $\left(\frac{1}{2}\right)^0 = 1$	$5^{-3} = \frac{1}{5^3}$ $2^{-4} = \frac{1}{2^4}$	$\frac{1}{8^{-2}} = 8^2$ $\frac{1}{2^{-4}} = 2^4$
0^0 and 0^{-n} are undefined.			

Simplify 4^{-2}.

4^{-2}

$\frac{1}{4^2}$ *Write without negative exponents.*

$\frac{1}{4 \cdot 4}$ *Write in expanded form.*

$\frac{1}{16}$ *Simplify.*

Simplify $x^2y^{-3}z^0$.

$x^2y^{-3}z^0$

$\frac{x^2z^0}{y^3}$ *Write without negative exponents.*

$\frac{x^2(1)}{y^3}$ $z^0 = 1$.

$\frac{x^2}{y^3}$ *Simplify.*

Fill in the blanks to simplify each expression.

1. 2^{-5}

$2^{-5} = \frac{1}{2^{\square}}$

$\frac{1}{2^5} = \frac{1}{\square}$

$= \_\_\_\_\_$

2. 10^{-3}

$10^{-3} = \frac{1}{10}$

$\frac{1}{10^3} = \frac{1}{\square}$

$= \_\_\_\_\_$

3. $\frac{1}{5^{-4}}$

$\frac{1}{5^{-4}} = 5^{\square}$

$5^{\square} = \square$

$= \_\_\_\_\_$

Simplify.

4. $5y^{-4}$ _____

5. $\frac{8}{a^{-3}}$ _____

6. $9x^3y^{-2}$ _____

7. $\frac{x^3}{x^{-1}y}$ _____

8. $\frac{b^2}{a^{-1}b^3}$ _____

9. $5x^{-4}y^2$ _____

Name _____ Date _____ Class _____

LESSON 7-1

Reteach

Integer Exponents (continued)

Evaluate $a^{-3}b^4$ for $a = 5$ and $b = 2$.

$a^{-3}b^4$	
$(5^{-3})(2^4)$	*Substitute.*
$\frac{2^4}{5^3}$	*Write without negative exponents.*
$\frac{16}{125}$	*Simplify.*

When evaluating, it is important to determine whether the negative is raised to the power.

Evaluate $-x^{-2}$ for $x = 10$.

The negative is not raised to the power.

$-x^{-2}$	
-10^{-2}	*Substitute.*
$-\frac{1}{10^2}$	*Write without negative exponents.*
$-\frac{1}{10 \cdot 10}$	*Write in expanded form.*
$-\frac{1}{100}$	*Simplify.*

Evaluate $(-x)^{-2}$ for $x = 10$.

The negative is raised to the power.

$(-x)^{-2}$	
$(-10)^{-2}$	*Substitute.*
$\frac{1}{(-10)^2}$	*Write without negative exponents.*
$\frac{1}{(-10) \cdot (-10)}$	*Write in expanded form.*
$\frac{1}{100}$	*Simplify.*

Evaluate each expression for the given value(s) of the variable(s).

10. x^2y^0 for $x = -2$ and $y = 5$

_____

11. a^3b^3 for $a = 4$ and $b = 2$

_____

12. $\frac{z^3}{y^{-2}}$ for $z = 2$ and $y = 5$

_____

13. $-a^3b^{-4}$ for $a = 2$ and $b = -1$

_____

14. $\frac{n^{-2}}{m^{-4}}$ for $m = 6$ and $n = 2$

_____

15. $(-u)^2v^{-6}$ for $u = 2$ and $v = 2$

_____

LESSON 7-1

Practice A

Integer Exponents

Simplify.

1. $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

2. $2^{-4} = \frac{1}{2^4} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{16}$

3. $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-3 \cdot -3 \cdot -3} = \frac{1}{-27}$

4. $(-1)^{-5} = \frac{1}{(-1)^5} = \frac{1}{-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1} = \frac{1}{-1} = -1$

5. $-(7.2)^0$ **−1**

6. $(4)^{-3}$ **$\frac{1}{64}$**

Evaluate each expression for the given value(s) of the variable(s).

7. x^{-2} for $x = 3$

$(3)^{-2} = \frac{1}{(3)^2} = \frac{1}{9}$

8. $m^0 n^{-3}$ for $m = 2$ and $n = 3$

$(2)^0(3)^{-3} = (1) \cdot \frac{1}{(3)^3} = \frac{1}{27}$

9. $5r^{-4}$ for $r = -2$

$5(-2)^{-4} = 5 \cdot \frac{1}{(-2)^4} = 5 \cdot \frac{1}{-2 \cdot -2 \cdot -2 \cdot -2} = 5 \cdot \frac{1}{16} = \frac{5}{16}$

Simplify.

10. $4x^{-3}$ **$\frac{4}{x^3}$**

11. $\frac{5}{b^{-2}}$ **$5b^2$**

12. $\frac{m^3 n^{-4}}{p^0}$ **$\frac{m^3}{n^4}$**

13. $\frac{k^{-4}}{2}$ **$\frac{1}{2k^4}$**

14. $\frac{f^4}{g^{-1}}$ **$f^4 g$**

15. $\frac{r^6 t^0}{s^{-2}}$ **$r^6 s^2$**

16. The weight of a silver charm is 2^{-2} grams. Evaluate this expression. **$\frac{1}{4}$ gram or 0.25 gram**

17. There are about 10^4 different species of birds on Earth. Just over 10^3 of them are threatened. Evaluate both expressions. **10,000; 1000**

LESSON 7-1

Practice B

Integer Exponents

Simplify.

1. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

2. $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

3. $(-5)^{-2}$ **$\frac{1}{25}$**

4. $-(4)^{-3}$ **$-\frac{1}{64}$**

5. -6^0 **−1**

6. $(7)^{-2}$ **$\frac{1}{49}$**

Evaluate each expression for the given value(s) of the variable(s).

7. d^{-3} for $d = -2$ **$-\frac{1}{8}$**

8. $a^5 b^{-6}$ for $a = 3$ and $b = 2$ **$\frac{243}{64}$**

9. $(b - 4)^{-2}$ for $b = 1$ **$\frac{1}{9}$**

10. $5z^{-x}$ for $z = -3$ and $x = 2$ **$\frac{5}{9}$**

11. $(5z)^{-x}$ for $z = -3$ and $x = 2$ **$\frac{1}{225}$**

12. $c^{-3}(16^{-2})$ for $c = 4$ **$\frac{1}{16{,}384}$**

Simplify.

13. t^{-4} **$\frac{1}{t^4}$**

14. $3r^{-5}$ **$\frac{3}{r^5}$**

15. $\frac{s^{-3}}{t^{-5}}$ **$\frac{t^5}{s^3}$**

16. $\frac{h^0}{3}$ **$\frac{1}{3}$**

17. $\frac{2x^{-3}y^{-2}}{z^4}$ **$\frac{2}{x^3 y^2 z^4}$**

18. $\frac{4fg^{-5}}{5h^{-3}}$ **$\frac{4fh^3}{5g^5}$**

19. $\frac{14a^{-4}}{20bc^{-1}}$ **$\frac{7c}{10a^4 b}$**

20. $\frac{a^4 c^2 e^0}{b^{-1} d^{-3}}$ **$a^4 bc^2 d^3$**

21. $\frac{-3g^{-2}hk^{-2}}{-6h^0}$ **$\frac{h}{2g^2 k^2}$**

22. A cooking website claims to contain 10^5 recipes. Evaluate this expression. **100,000**

23. A ball bearing has diameter 2^{-3} inches. Evaluate this expression. **$\frac{1}{8}$ inch or 0.125 inch**

LESSON 7-1

Practice C

Integer Exponents

Simplify.

1. 4^{-2} **$\frac{1}{16}$**

2. 6^0 **1**

3. -6^{-2} **$-\frac{1}{36}$**

4. $(-1)^{-5}$ **−1**

5. $(-3)^{-2}$ **$\frac{1}{9}$**

6. 5^{-3} **$\frac{1}{125}$**

7. -7^{-3} **$-\frac{1}{343}$**

8. $(-4)^{-5}$ **$-\frac{1}{1024}$**

9. $(-9)^0$ **1**

Evaluate each expression for the given value(s) of the variable(s).

10. $x^{-4} y^3$ for $x = 2$ and $y = 3$ **$\frac{27}{16}$**

11. $5r^{-3} s^{-6}$ for $r = 3$ and $s = 0$ **$\frac{5}{27}$**

12. $(3 - m)^{-4}$ for $m = 6$ **$\frac{1}{81}$**

13. $-2a^{-1} b^{-3}$ for $a = 2$ and $b = 3$ **$-\frac{1}{27}$**

14. $(-2xy)^{-3}$ for $x = -2$ and $y = \frac{1}{2}$ **$\frac{1}{8}$**

15. $\left(\frac{4}{5}m\right)^{-3}$ for $m = 10$ **$\frac{1}{512}$**

Simplify.

16. x^{-3} **$\frac{1}{x^3}$**

17. z^0 **1**

18. t^{-9} **$\frac{1}{t^9}$**

19. $3n^{-2}$ **$\frac{3}{n^2}$**

20. $\frac{2}{3}x^{-4}$ **$\frac{2}{3x^4}$**

21. $-a^{-2}$ **$-\frac{1}{a^2}$**

22. $10r^{-3} s^4$ **$\frac{10s^4}{r^3}$**

23. $\frac{b^3}{c^{-2} d^3}$ **$\frac{b^3 c^2}{d^3}$**

24. $\frac{5x^{-2} y^{-3}}{z^0}$ **$\frac{5}{x^2 y^3}$**

25. $\frac{p^{-9} q^{-4}}{r^2 s^{-3}}$ **$\frac{s^3}{p^9 q^4 r^2}$**

26. $\frac{a^0 b^{-2}}{c^{-3} d}$ **$\frac{c^3}{b^2 d}$**

27. $\frac{g^3 h^{-2}}{k^{-1} j^{-5}}$ **$\frac{g^3 kj^5}{h^2}$**

28. A micrometer is an instrument that can measure the thickness of an object very accurately. One micrometer is accurate to within 10^{-4} inches. Evaluate this expression. **$\frac{1}{10{,}000}$ inch or 0.0001 inch**

29. An object is being measured by a micrometer. It has a thickness of 6^{-3} inches. Evaluate this expression. **$\frac{1}{216}$ inch or 0.00463 inch**

LESSON 7-1

Reteach

Integer Exponents

Remember that 2^3 means $2 \times 2 \times 2 = 8$. The base is 2, the exponent is positive 3. Exponents can also be 0 or negative.

	Zero Exponents	Negative Exponents	Negative Exponents in the Denominator
Definition	For any nonzero number x, $x^0 = 1$.	For any nonzero number x and any integer n, $x^{-n} = \frac{1}{x^n}$.	For any nonzero number x and any integer n, $\frac{1}{x^{-n}} = x^n$.
Examples	$6^0 = 1$ $\left(\frac{1}{2}\right)^0 = 1$	$5^{-3} = \frac{1}{5^3}$ $2^{-4} = \frac{1}{2^4}$	$\frac{1}{8^{-2}} = 8^2$ $\frac{1}{2^{-4}} = 2^4$
0^0 and 0^{-n} are undefined.			

Simplify 4^{-2}.

4^{-2}

$\frac{1}{4^2}$ *Write without negative exponents.*

$\frac{1}{4 \cdot 4}$ *Write in expanded form.*

$\frac{1}{16}$ *Simplify.*

Simplify $x^2 y^{-3} z^0$.

$x^2 y^{-3} z^0$

$\frac{x^2 z^0}{y^3}$ *Write without negative exponents.*

$\frac{x^2(1)}{y^3}$ $z^0 = 1$.

$\frac{x^2}{y^3}$ *Simplify.*

Fill in the blanks to simplify each expression.

1. 2^{-5}

$2^{-5} = \frac{1}{2^{\boxed{5}}}$

$\frac{1}{2^5} = \frac{1}{\boxed{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}} = \frac{1}{32}$

2. 10^{-3}

$10^{-3} = \frac{1}{10}$

$\frac{1}{10^3} = \frac{1}{\boxed{10 \cdot 10 \cdot 10}} = \frac{1}{1000}$

3. $\frac{1}{5^{-4}}$

$\frac{1}{5^{-4}} = 5^{\boxed{4}}$

$5^{\boxed{4}} = \boxed{5 \cdot 5 \cdot 5 \cdot 5} = 625$

Simplify.

4. $5y^{-4}$ **$\frac{5}{y^4}$**

5. $\frac{8}{a^{-3}}$ **$8a^3$**

6. $9x^3 y^{-2}$ **$\frac{9x^3}{y^2}$**

7. $\frac{x^3}{x^{-1} y}$ **$\frac{x^4}{y}$**

8. $\frac{b^2}{a^{-1} b^3}$ **$\frac{a}{b}$**

9. $5x^{-4} y^2$ **$\frac{5y^2}{x^4}$**

LESSON 7-1

Reteach

Integer Exponents (continued)

Evaluate $a^{-3}b^4$ for $a = 5$ and $b = 2$.

$a^{-3}b^4$	
$(5^{-3})(2^4)$	*Substitute.*
$\frac{2^4}{5^3}$	*Write without negative exponents.*
$\frac{16}{125}$	*Simplify.*

When evaluating, it is important to determine whether the negative is raised to the power.

Evaluate $-x^{-2}$ for $x = 10$.

The negative is not raised to the power.

$-x^{-2}$	
-10^{-2}	*Substitute.*
$-\frac{1}{10^2}$	*Write without negative exponents.*
$-\frac{1}{10 \cdot 10}$	*Write in expanded form.*
$-\frac{1}{100}$	*Simplify.*

Evaluate $(-x)^{-2}$ for $x = 10$.

The negative is raised to the power.

$(-x)^{-2}$	
$(-10)^{-2}$	*Substitute.*
$\frac{1}{(-10)^2}$	*Write without negative exponents.*
$\frac{1}{(-10) \cdot (-10)}$	*Write in expanded form.*
$\frac{1}{100}$	*Simplify.*

Evaluate each expression for the given value(s) of the variable(s).

10. x^2y^0 for $x = -2$ and $y = 5$

4

11. a^3b^3 for $a = 4$ and $b = 2$

512

12. $\frac{z^3}{y^{-2}}$ for $z = 2$ and $y = 5$

200

13. $-a^3b^{-4}$ for $a = 2$ and $b = -1$

-8

14. $\frac{n^{-2}}{m^{-4}}$ for $m = 6$ and $n = 2$

324

15. $(-u)^2v^{-6}$ for $u = 2$ and $v = 2$

$\frac{1}{16}$

LESSON 7-1

Challenge

Exploring Patterns in the Units Digit of x^n

When you write out the first several powers of x^n, where x and n are positive integers, you can discover interesting patterns in the units digits of x^n.

	x^1	x^2	x^3	x^4	x^5	x^6
$x = 2$	$2^1 = \mathbf{2}$	$2^2 = 2(2) = \mathbf{4}$	$2^3 = 2(4) = \mathbf{8}$	$2^4 = 2(8) = \mathbf{16}$	$2^5 = 2(16) = \mathbf{32}$	$2^6 = 2(32) = \mathbf{64}$

Notice that 2^1 and 2^5 have the same units digit and that 2^2 and 2^6 have the same units digit. In the exercises that follow, you can discover other number patterns involving the units digits of x^n.

In Exercises 1–10, find the first nine powers of each value of x. Using the units digit of each result, complete the table. You may find a calculator useful.

		x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
1.	$x = 1$	1	1	1	1	1	1	1	1	1
2.	$x = 2$	2	4	8	6	2	4	8	6	2
3.	$x = 3$	3	9	7	1	3	9	7	1	3
4.	$x = 4$	4	6	4	6	4	6	4	6	4
5.	$x = 5$	5	5	5	5	5	5	5	5	5
6.	$x = 6$	6	6	6	6	6	6	6	6	6
7.	$x = 7$	7	9	3	1	7	9	3	1	7
8.	$x = 8$	8	4	2	6	8	4	2	6	8
9.	$x = 9$	9	1	9	1	9	1	9	1	9
10.	$x = 10$	0	0	0	0	0	0	0	0	0

Refer to the table that you completed in Exercises 1–10. Describe the pattern in the units digits of x^n.

11. 1^n For all n, 1^n has 1 as its units digit.

12. 2^n The pattern is 2, 4, 8, and 6, for $n = 1, 2, 3$, and 4 and then repeats.

13. 3^n The pattern is 3, 9, 7, and 1, for $n = 1, 2, 3$, and 4 and then repeats.

14. 5^n For all $n > 0$, 5^n has 5 as its units digit.

15. Write a rule that determines the units digit of 7^n as a function of n.

If you divide n by 4, then the units digit is 7, 9, 3, or 1, depending on whether the remainder is 1, 2, 3, or 0, respectively.

LESSON 7-1

Problem Solving

Integer Exponents

Write the correct answer.

1. At the 2005 World Exposition in Aichi, Japan, tiny mu-chips were embedded in the admissions tickets to prevent counterfeiting. The mu-chip was developed by Hitachi in 2003. Its area is $4^2(10)^{-2}$ square millimeters. Simplify this expression.

$\frac{4}{25}$ or 0.16 mm^2

2. Despite their name, Northern Yellow Bats are commonly found in warm, humid areas in the southeast United States. An adult has a wingspan of about 14 inches and weighs between $3(2)^{-3}$ and $3(2)^{-2}$ ounces. Simplify these expressions.

$\frac{3}{8}$ and $\frac{3}{4}$ oz

3. Saira is using the formula for the area of a circle to determine the value of π. She is using the expression Ar^{-2} where $A = 50.265$ and $r = 4$. Use a calculator to evaluate Saira's expression to find her approximation of the value of π to the nearest thousandth.

3.142

4. The volume of a freshwater tank can be expressed in terms of x, y, and z. Expressed in these terms, the volume of the tank is $x^3y^{-2}z$ liters. Determine the volume of the tank if $x = 4$, $y = 3$, and $z = 6$.

$42\frac{2}{3}$ liters

Alison has an interest in entomology, the study of insects. Her collection of insects from around the world includes the four specimens shown in the table below. Select the best answer.

Insect	Mass
Emperor Scorpion	2^{-5} kg
African Goliath Beetle	11^{-1} kg
Giant Weta	2^{-4} kg
Madagascar Hissing Cockroach	5^{-3} kg

5. Cockroaches have been found on every continent, including Antarctica. What is the mass of Alison's Madagascar Hissing Cockroach expressed as a quotient?

A $-\frac{1}{125}$ kg **C** $\frac{1}{15}$ kg

(B) $\frac{1}{125}$ kg **D** 125 kg

6. Many Giant Wetas are so heavy that they cannot jump. Which expression is another way to show the mass of the specimen in Alison's collection?

F $-(2)^4$ kg **(H)** $\frac{1}{2 \cdot 2 \cdot 2 \cdot 2}$ kg

G $\left(\frac{1}{2}\right)^{-4}$ kg **J** $4\frac{1}{2}$ kg

7. Scorpions are closely related to spiders and horseshoe crabs. What is the mass of Alison's Emperor Scorpion expressed as a quotient?

A $\frac{1}{32}$ kg **(C)** $\frac{1}{32}$ kg

B $\frac{1}{25}$ kg **D** 32 kg

LESSON 7-1

Reading Strategies

Using Patterns

Studying the patterns that are found in expressions with exponents can help you remember the rules for evaluating expressions with integer exponents.

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
$3^3 = 3 \cdot 3 \cdot 3 = 27$
$3^2 = 3 \cdot 3 = 9$
$3^1 = 3$

Positive exponents: The answer is the base multiplied by itself the number of times identified by the exponent.

$3^0 = 1$

Zero exponent: The answer is always 1 (if the base is not 0; $0^0 = 0$).

$3^{-1} = \frac{1}{3}$
$3^{-2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$
$3^{-3} = \frac{1}{3 \cdot 3 \cdot 3} = \frac{1}{27}$
$3^{-4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$

Negative exponents: The answer is the *reciprocal* of the same expression with a positive exponent.

Note that the rules are the same when the base is a variable:

$b^3 = b \cdot b \cdot b$ $\quad g^0 = 1$ $\quad k^{-5} = \frac{1}{k^5}$ $\quad \frac{1}{m^{-3}} = m^3$

Answer each question.

1. What is the base of the expression 6^{-4}? 6

2. What number can go in the box to make a true statement: $5^{\square} = 1$? 0

3. Write the expression $\frac{1}{8^3}$ with a negative exponent. 8^{-3}

4. What is the *reciprocal* of b^7? $\frac{1}{b^7}$

Simplify each expression.

5. 2^5 32

6. 2^{-5} $\frac{1}{32}$

7. 7^0 1

8. 10^{-6} $\frac{1}{1{,}000{,}000}$

9. $(-4)^3$ -64

10. $(-4)^{-3}$ $-\frac{1}{64}$

11. t^{-4} $\frac{1}{t^4}$

12. c^2d^{-3} $\frac{c^2}{d^3}$

13. $8x^{-5}$ $\frac{8}{x^5}$

14. $12r^0$ 12