Name _____ Date _____ Class _____

LESSON 6-7

Practice B

Investigating Graphs of Polynomial Functions

Identify the leading coefficient, degree, and end behavior.

1. $P(x) = 2x^5 - 6x^3 + x^2 - 2$

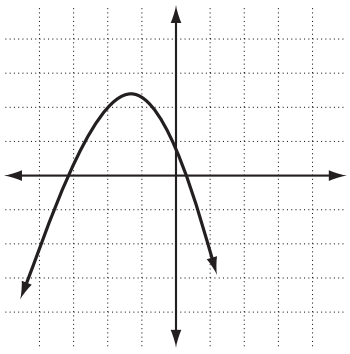
_____

2. $Q(x) = -4x^2 + x - 1$

_____

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

3.



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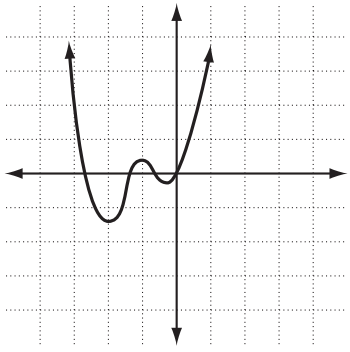
4.



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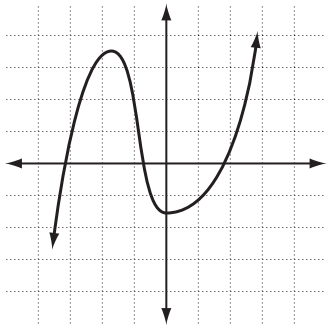
5.



_____

Graph the function $P(x) = x^3 + 6x^2 + 5x - 12$.

6. Identify the possible rational roots.

_____

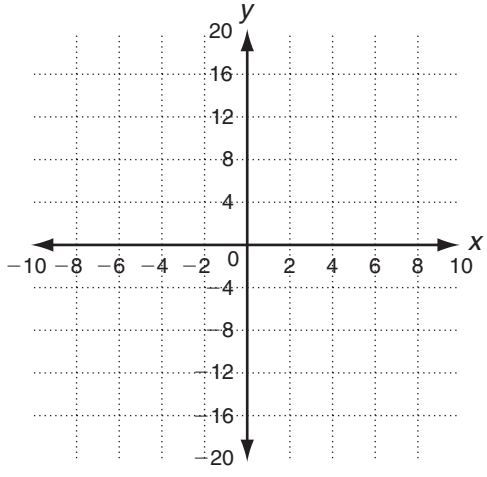
7. Identify the zeros.

_____

8. Describe the end behavior of the function.

_____

9. Sketch the graph of the function.



Solve.

10. The number, $N(y)$, of subscribers to a local magazine can be modeled by the function $N(y) = 0.1y^4 - 3y^3 + 10y^2 - 30y + 10{,}000$, where y is the number of years since the magazine was founded. Graph the polynomial on a graphing calculator and find the minimum number of subscribers and the year in which this occurs.

_____

LESSON 6-7

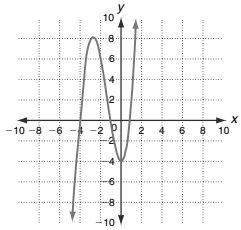
Practice A

Investigating Graphs of Polynomial Functions

Complete the table to identify the leading coefficient, degree, and end behavior of each polynomial function.

	Polynomial	Leading Coefficient	Degree	End Behavior
1.	$P(x) = x^2 + 3x + 6$	1	2	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to +\infty$
2.	$P(x) = -3x^3 + 2x - 5$	-3	3	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to -\infty$
3.	$P(x) = 2x^4 + 2x^3 + 3$	2	4	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to +\infty$
4.	$P(x) = -6x^5 + 3x^3 + 1$	-6	5	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to -\infty$

Graph the function $P(x) = x^3 + 4x^2 - x - 4$.



5. Identify possible rational roots.

   $\pm 1, \pm 2, \pm 4$

6. Test the roots using synthetic division to find a zero.

   $(x - 1)(x^2 + 5x + 4)$

7. Use your results from the synthetic division to factor the polynomial to find all zeros.

   $(x - 1)(x + 4)(x + 1)$

8. Find other points to use to draw the graph, such as the *y*-intercept and points between the zeros such as $P(-2)$ and $P(-3)$.

   y-intercept $= -4$; $P(-2) = 6$; $P(-3) = 8$

9. Identify the end behavior of the graph.

   As $x \to -\infty$, $P(x) \to -\infty$, as $x \to +\infty$, $P(x) \to +\infty$

10. Sketch the graph of the function.

LESSON 6-7

Practice B

Investigating Graphs of Polynomial Functions

Identify the leading coefficient, degree, and end behavior.

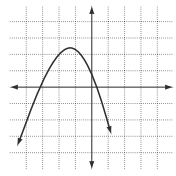
1. $P(x) = 2x^5 - 6x^3 + x^2 - 2$

   2; 5; as $x \to +\infty$, $P(x) \to +\infty$; and as $x \to -\infty$, $P(x) \to -\infty$

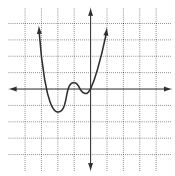
2. $Q(x) = -4x^2 + x - 1$

   -4; 2; as $x \to -\infty$, $Q(x) \to -\infty$; and as $x \to +\infty$, $Q(x) \to -\infty$

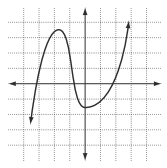
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

3. 

   Even; negative

4. 

   Even; positive

5. 

   Odd; positive

Graph the function $P(x) = x^3 + 6x^2 + 5x - 12$.

6. Identify the possible rational roots.

   $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

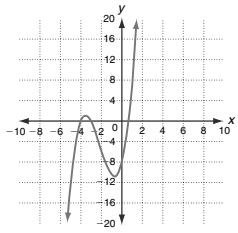
7. Identify the zeros.

   -4, -3, and 1

8. Describe the end behavior of the function.

   As $x \to +\infty$, $P(x) \to +\infty$, and as $x \to -\infty$, $P(x) \to -\infty$

9. Sketch the graph of the function.



Solve.

10. The number, $N(y)$, of subscribers to a local magazine can be modeled by the function $N(y) = 0.1y^4 - 3y^3 + 10y^2 - 30y + 10{,}000$, where y is the number of years since the magazine was founded. Graph the polynomial on a graphing calculator and find the minimum number of subscribers and the year in which this occurs.

    About 5400 in year 20

LESSON 6-7

Practice C

Investigating Graphs of Polynomial Functions

Identify the leading coefficient, degree, and end behavior.

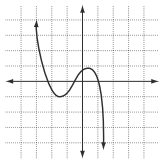
1. $R(x) = -6x^4 + 4x^3 - x^2 + 1$

   -6; 4; as $x \to -\infty$, $R(x) \to -\infty$; and as $x \to +\infty$, $R(x) \to -\infty$

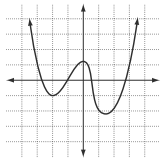
2. $Q(x) = 12 + 8x - 16x^3 - x^2$

   -16; 3; as $x \to -\infty$, $Q(x) \to +\infty$; and as $x \to +\infty$, $Q(x) \to -\infty$

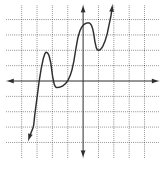
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

3. 

   Odd; negative

4. 

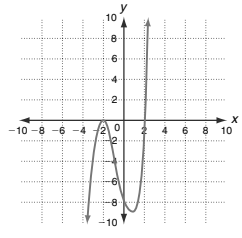
   Even; positive

5. 

   Odd; positive

Graph the function.

6. $P(x) = x^3 + 2x^2 - 4x - 8$



Graph each function on a calculator, and estimate the local maxima and minima.

7. $P(x) = -x^4 + 4x^3 - 2x^2 - x + 5$

   Minima: 4.5; maxima: 5.1 and 13.5

8. $P(x) = x^5 - x^4 - 5x^2$

   Minima: -8.68; maxima: 0

Solve.

9. An engineer needs a metal box to shield sensitive electronic devices from external electric fields. One side of the box should be open so that it can be placed over the components. The box can be made from a 3 m by 4 m sheet of metal by cutting squares from the corners and folding up the sides.

   a. What is the maximum volume of the box? 3.03 m^3

   b. What dimensions of the box result in the maximum volume? 1.9 m by 2.9 m by 0.55 m

LESSON 6-7

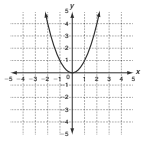
Review for Mastery

Investigating Graphs of Polynomial Functions

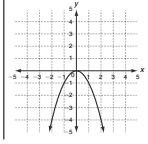
Examine the sign and the exponent of the leading term (term of greatest degree) of a polynomial $P(x)$ to determine the **end behavior** of the function.

Even degree functions: Exponent of leading term is even.

Positive leading coefficient
As $x \to -\infty$, $P(x) \to +\infty$.
As $x \to +\infty$, $P(x) \to +\infty$.



Negative leading coefficient
As $x \to -\infty$, $P(x) \to -\infty$.
As $x \to +\infty$, $P(x) \to -\infty$.



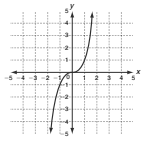
Read: As x approaches positive infinity, $P(x)$ approaches negative infinity.

Example: $P(x) = 3x^4 + 2x^3 - 5$ Leading term: $3x^4$
End behavior: As $x \to -\infty$, $P(x) \to +\infty$.
As $x \to +\infty$, $P(x) \to +\infty$.

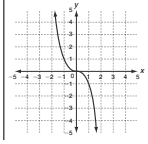
Sign: positive
Degree: 4, even

Odd degree functions: Exponent of leading term is odd.

Positive leading coefficient
As $x \to +\infty$, $P(x) \to +\infty$.
As $x \to -\infty$, $P(x) \to -\infty$.



Negative leading coefficient
As $x \to -\infty$, $P(x) \to +\infty$.
As $x \to +\infty$, $P(x) \to -\infty$.



Example: $P(x) = -2x^5 - 6x^2 + x$ Leading term: $-2x^5$
End behavior: As $x \to -\infty$, $P(x) \to +\infty$.
As $x \to +\infty$, $P(x) \to -\infty$.

Sign: negative
Degree: 5, odd

Identify the end behavior of each function.

1. $P(x) = 4x^3 + 8x^2 - 5$

   Leading term: $4x^3$

   Sign and degree: Positive, 3, odd

   End behavior: as $x \to -\infty$, $P(x) \to -\infty$

   as $x \to +\infty$, $P(x) \to +\infty$

2. $P(x) = -9x^6 + 2x^3 - x + 7$

   Leading term: $-9x^6$

   Sign and degree: negative, 6, even

   End behavior: as $x \to -\infty$, $P(x) \to -\infty$

   as $x \to +\infty$, $P(x) \to -\infty$