Name _____ Date _____ Class _____

LESSON 6-6

Practice B

Fundamental Theorem of Algebra

Write the simplest polynomial function with the given roots.

1. 1, 4, and -3

2. $\frac{1}{2}$, 5, and -2

3. $2i$, $\sqrt{3}$, and 4

4. $\sqrt{2}$, -5, and $-3i$

Solve each equation by finding all roots.

5. $x^4 - 2x^3 - 14x^2 - 2x - 15 = 0$

6. $x^4 - 16 = 0$

7. $x^4 + 4x^3 + 4x^2 + 64x - 192 = 0$

8. $x^3 + 3x^2 + 9x + 27 = 0$

Solve.

9. An electrical circuit is designed such that its output voltage, V, measured in volts, can be either positive or negative. The voltage of the circuit passes through zero at $t = 1$, 2, and 7 seconds. Write the simplest polynomial describing the voltage $V(t)$.

LESSON 6-6 Practice A
Fundamental Theorem of Algebra

Identify the number of zeros for each function.

1. $P(x) = x^3 + 2x^2 - 12x + 1$ — 3
2. $P(x) = 2x^5 - 5x + 10$ — 5
3. $P(x) = 3x^4 + 2x$ — 4

Write the simplest polynomial function with the given zeros.

4. -1, 0, and 2
   - a. Write the factored expression. $P(x) = x(x + 1)(x - 2)$
   - b. Multiply the first two factors. $P(x) = (x^2 + x)(x - 2)$
   - c. Multiply the result by the remaining factor. $P(x) = x^3 - 2x^2 + x^2 - 2x$
   - d. Combine like terms. $P(x) = x^3 - x^2 - 2x$
5. -3, 1, and 5 — $P(x) = x^3 - 3x^2 - 13x + 15$
6. -4, -1, and 1 — $P(x) = x^3 + 4x^2 - x - 4$
7. $2i$
   - a. How many zeros does this function have? 2
   - b. Write the conjugate pair for the complex root. $2i, -2i$
   - c. Write the factored expression. $P(x) = (x + 2i)(x - 2i)$
   - d. Multiply the binomials. $P(x) = x^2 + 4$
8. -2 and $\sqrt{3}$ — $P(x) = x^3 + 2x^2 - 3x - 6$
9. 3 and $2 + i$ — $P(x) = x^3 - 7x^2 + 17x - 15$

Solve the equation by finding all roots.

10. $x^3 - 6x^2 - 2x + 12 = 0$ — $x = 6, \pm\sqrt{2}$

LESSON 6-6 Practice B
Fundamental Theorem of Algebra

Write the simplest polynomial function with the given roots.

1. 1, 4, and -3 — $P(x) = x^3 - 2x^2 - 11x + 12$
2. $\frac{1}{2}$, 5, and -2 — $P(x) = x^3 - \frac{7}{2}x^2 - \frac{17}{2}x + 5$
3. $2i$, $\sqrt{3}$, and 4 — $P(x) = x^5 - 4x^4 + x^3 - 4x^2 - 12x + 48$
4. $\sqrt{2}$, -5, and $-3i$ — $P(x) = x^5 + 5x^4 + 7x^3 + 35x^2 - 18x - 90$

Solve each equation by finding all roots.

5. $x^4 - 2x^3 - 14x^2 - 2x - 15 = 0$ — $x = i, -i, -3$, and 5
6. $x^4 - 16 = 0$ — $x = 2, -2, 2i$, and $-2i$
7. $x^4 + 4x^3 + 4x^2 + 64x - 192 = 0$ — $x = -4i, 4i, 2$, and -6
8. $x^3 + 3x^2 + 9x + 27 = 0$ — $x = -3i, 3i$, and -3

Solve.

9. An electrical circuit is designed such that its output voltage, V, measured in volts, can be either positive or negative. The voltage of the circuit passes through zero at $t = 1$, 2, and 7 seconds. Write the simplest polynomial describing the voltage $V(t)$.

$V(t) = t^3 - 10t^2 + 23t - 14$

LESSON 6-6 Practice C
Fundamental Theorem of Algebra

Write the simplest polynomial function with the given roots.

1. $-\frac{3}{4}$, 6, and -1 — $P(x) = x^3 - \frac{17}{4}x^2 - \frac{39}{4}x - \frac{9}{2}$
2. $-5i$, 2, and 7 — $P(x) = x^4 - 9x^3 + 39x^2 - 225x + 350$
3. $-i$, -3, and -1 — $P(x) = x^4 + 4x^3 + 4x^2 + 4x + 3$
4. $2i$, 4, and $\sqrt{6}$ — $P(x) = x^5 - 4x^4 - 2x^3 + 8x^2 - 24x + 96$

Solve each equation by finding all roots.

5. $4x^4 - 8x^3 - 3x^2 - 18x - 27 = 0$ — $x = -\frac{3}{2}i, \frac{3}{2}i$, 3, and -1
6. $x^4 + 3x^3 - x^2 + 9x - 12 = 0$ — $x = i\sqrt{3}, -i\sqrt{3}$, 1, and -4
7. $x^4 - 3x^3 - 8x^2 + 22x - 24 = 0$ — $x = 1 + i, 1 - i, -3$, and 4
8. $x^3 + 6x^2 + 4x + 24 = 0$ — $x = 2i, -2i$, and -6

Solve.

9. For a scientific experiment, Tony needs a glass bell jar in the shape of a cylinder with a hemisphere on top. The height of the cylinder must be 3 inches longer than its radius and the volume must be 72π cubic inches. What should the radius of the cylinder be?

3 inches

LESSON 6-6 Review for Mastery
Fundamental Theorem of Algebra

If r is a root of a polynomial function, then $(x - r)$ is a factor of the polynomial, $P(x)$. So, you can use the roots to write the simplest form of a polynomial function.

Write the simplest polynomial function with roots -4, -2, and 3.

Step 1 Write the factors of the polynomial, $P(x) = 0$.
$(x + 4)(x + 2)(x - 3) = 0$

Root (a)	-4	-2	3
Factor ($x - a$)	$x + 4$	$x + 2$	$x - 3$

Step 2 Multiply the first two factors, $(x + 4)(x + 2)$.
$(x^2 + 6x + 8)(x - 3) = 0$

Step 3 Multiply $(x^2 + 6x + 8)(x - 3)$. Then simplify.
$x^3 - 3x^2 + 6x^2 - 18x + 8x - 24 = 0$
$x^3 + 3x^2 - 10x - 24 = 0$

The function is $P(x) = x^3 + 3x^2 - 10x - 24 = 0$.

Write the simplest polynomial function with the given roots.

1. -5, 1, and 2
   $(x + 5)(x - 1)(x - 2) = 0$
   $(x^2 + 4x - 5)(x - 2) = 0$
   $x^3 + 4x^2 - 5x - 2x^2 - 8x + 10$
   $x^3 + 2x^2 + 13x + 10$
2. -3, -1, and 0
   $x(x + 3)(x + 1) = 0$
   $(x^2 + 3x)(x + 1)$
   $x^3 + 3x^2 + x^2 + 3x$
   $x^3 + 4x^2 + 3x$
3. 1, 4, and 5
   $(x - 1)(x - 4)(x - 5)$
   $(x^2 - 5x + 4)(x - 5)$
   $x^3 - 10x^2 + 29x - 20$
4. -2, 3, and 6
   $(x + 2)(x - 3)(x - 6)$
   $(x^2 - x - 6)(x - 6)$
   $x^3 - 7x^2 + 36$
5. 2, 4, and 6
   $(x - 2)(x - 4)(x - 6)$
   $(x^2 - 6x + 8)(x - 6)$
   $x^3 - 12x^2 + 44x - 48$
6. -5, 0, and 5
   $(x + 5)(x)(x - 5)$
   $x(x^2 - 25)$
   $x^3 - 25x$