Name _____ Date _____ Class _____

LESSON 6-4

Practice B

Factoring Polynomials

Determine whether the given binomial is a factor of the polynomial $P(x)$.

1. $(x - 4)$; $P(x) = x^2 + 8x - 48$

2. $(x + 5)$; $P(x) = 2x^2 - 6x - 1$

3. $(x - 6)$; $P(x) = -2x^2 + 15x - 18$

4. $(x + 3)$; $P(x) = 2x^2 - x + 7$

Factor each expression.

5. $2x^4 + 2x^3 - x^2 - x$

6. $4x^3 + x^2 - 8x - 2$

7. $5x^6 - 5x^4 + x^3 - x$

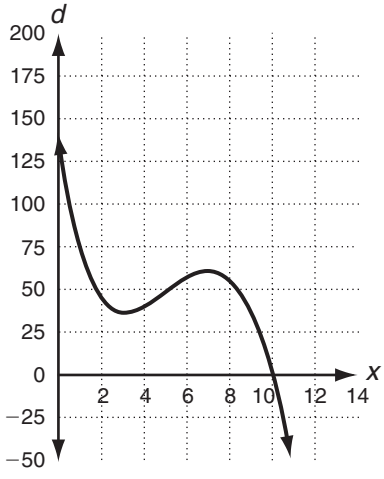
8. $2x^4 + 54x$

9. $64x^3 - 1$

10. $3x^4 + 24x$

Solve.

11. Since 2006, the water level in a certain pond has been modeled by the polynomial $d(x) = -x^3 + 16x^2 - 74x + 140$, where the depth d, is measured in feet over x years. Identify the year that the pond will dry up. Use the graph to factor $d(x)$.



LESSON 6-4

Practice A

Factoring Polynomials

Tell whether each statement is true or false.

Given: $P(x) = x^2 - 2x - 3$

1. $x - 1$ is a factor of $P(x)$. — False
2. $x + 1$ is a factor of $P(x)$. — True
3. $P(1) = 0$. — False
4. Synthetic substitution can be used to determine if $x + 1$ is a factor of $P(x)$. — True

Determine whether the given binomial is a factor of the polynomial $P(x)$.

5. $(x + 1)$; $P(x) = 5x^2 + 11x + 6$ — Yes
6. $(x - 3)$; $P(x) = 3x^4 + 3x^3 - 2x^2 - 2x$ — No

Factor each expression.

7. $2x^2 + 8x + 2x + 8$ — $2(x + 4)(x + 1)$
8. $x^3 + 2x^2 - x - 2$ — $(x + 2)(x + 1)(x - 1)$
9. $x^3 + x^2 + 7x + 7$ — $(x^2 + 7)(x + 1)$
10. $x^3 - 2x^2 - 4x + 8$ — $(x + 2)(x - 2)(x - 2)$
11. $g^3 + 8$ — $(g + 2)(g^2 - 2g + 4)$
12. $128m - 2m^4$ — $2m(4 - m)(16 + 4m + m^2)$

Solve.

13. June factored the polynomial $c^9 - d^{12}$ into $(c^3 + d^4)(c^6 - c^3d^4 + d^8)$. Is she correct? How do you know?

No; possible answer: the polynomial is the difference of two cubes; she used the formula for the sum of two cubes.

LESSON 6-4

Practice B

Factoring Polynomials

Determine whether the given binomial is a factor of the polynomial $P(x)$.

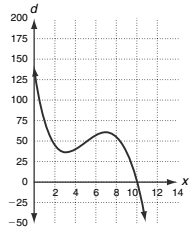
1. $(x - 4)$; $P(x) = x^2 + 8x - 48$ — Yes
2. $(x + 5)$; $P(x) = 2x^2 - 6x - 1$ — No
3. $(x - 6)$; $P(x) = -2x^2 + 15x - 18$ — Yes
4. $(x + 3)$; $P(x) = 2x^2 - x + 7$ — No

Factor each expression.

5. $2x^4 + 2x^3 - x^2 - x$ — $x(2x - 1)(x + 1)$
6. $4x^3 + x^2 - 8x - 2$ — $(4x + 1)(x^2 - 2)$
7. $5x^6 - 5x^4 + x^3 - x$ — $x(5x^3 + 1)(x^2 - 1)$
8. $2x^4 + 54x$ — $2x(x + 3)(x^2 - 3x + 9)$
9. $64x^3 - 1$ — $(4x - 1)(16x^2 + 4x + 1)$
10. $3x^4 + 24x$ — $3x(x + 2)(x^2 - 2x + 4)$

Solve.

11. Since 2006, the water level in a certain pond has been modeled by the polynomial $d(x) = -x^3 + 16x^2 - 74x + 140$, where the depth d, is measured in feet over x years. Identify the year that the pond will dry up. Use the graph to factor $d(x)$.



2016; $-(x - 10)(x^2 - 6x + 14)$

LESSON 6-4

Practice C

Factoring Polynomials

Use the Factor Theorem to verify that each linear binomial is a factor of the given polynomial. Then use synthetic division to write the polynomial as a product.

1. $(x + 5)$; $P(x) = 2x^2 + 6x - 20$ — $(x + 5)(2x - 4)$
2. $(x - 1)$; $P(x) = x^4 - 6x^3 + 4x^2 + 1$ — $(x - 1)(x^3 - 5x^2 - x - 1)$
3. $(x + 2)$; $P(x) = 3x^3 + 12x^2 + 17x + 10$ — $(x + 2)(3x^2 + 6x + 5)$
4. $(x - 8)$; $P(x) = x^4 - 8x^3 - 4x^2 + 33x - 8$ — $(x - 8)(x^3 - 4x + 1)$

Factor each expression.

5. $16x^3 - 12x^2 + 20x - 15$ — $(4x - 3)(4x^2 + 5)$
6. $3x^6 + 54x^4 + 243x^2$ — $3x^2(x^2 + 9)(x^2 + 9)$
7. $x^6 - 10x^5 + 25x^4$ — $x^4(x - 5)^2$
8. $6x^3 + 12x^2 + 4x + 8$ — $2(3x^2 + 2)(x + 2)$
9. $250x^4 + 54x$ — $2x(5x + 3)(25x^2 - 15x + 9)$
10. $-3x^5 + 24x^2$ — $-3x^2(x - 2)(x^2 + 2x + 4)$

Solve.

11. The voltage generated by an electrical circuit changes over time according to the polynomial $V(t) = t^3 - 4t^2 - 25t + 100$, where V is in volts and t is in seconds. Factor the polynomial to find the times when the voltage is equal to zero.

$(t - 4)(t - 5)(t + 5)$; the voltage is equal to zero at 4 s and 5 s.

LESSON 6-4

Review for Mastery

Factoring Polynomials

Sometimes you can use grouping to factor a third degree polynomial. To **factor by grouping** means to group terms with common factors. Then factor the common factors. Continue to factor until the expression can no longer be factored.

Factor: $x^3 + 4x^2 - 9x - 36$.

Start by grouping terms to factor out the greatest possible power of x.

x^2 is a factor of x^3 and $4x^2$.

$x^3 + 4x^2 - 9x - 36$

$(x^3 + 4x^2) + (-9x - 36)$ — -9 is a factor of -9 and -36.

$x^2(x + 4) - 9(x + 4)$ — $(x + 4)$ is a common factor.

$(x + 4)(x^2 - 9)$ — $(x^2 - 9)$ is the difference of squares.

$(x + 4)(x + 3)(x - 3)$ — Recall that $(a^2 - b^2) = (a + b)(a - b)$. So $(x^2 - 9) = (x + 3)(x - 3)$.

Factor each expression.

1. $x^3 - 3x^2 - 4x + 12$

$(x^3 - 3x^2) + (-4x + 12)$

$x^2(x - 3) - 4(x - 3)$

$(x - 3)(x^2 - 4)$

$(x - 3)(x + 2)(x - 2)$

2. $x^3 + 6x^2 - x - 6$

$(x^3 + 6x^2) + (-x - 6)$

$x^2(x + 6) - 1(x + 6)$

$(x + 6)(x^2 - 1)$

$(x + 6)(x + 1)(x - 1)$

3. $x^3 + x^2 - 9x - 9$

$(x^3 + x^2) + (-9x - 9)$

$x^2(x + 1) - 9(x + 1)$

$(x + 1)(x^2 - 9)$

$(x + 1)(x + 3)(x - 3)$

4. $x^3 + 2x^2 - 16x - 32$

$(x^3 + 2x^2) + (-16x - 32)$

$x^2(x + 2) - 16(x + 2)$

$(x + 2)(x^2 - 16)$

$(x + 2)(x + 4)(x - 4)$