Name ______________________ Date __________ Class __________

LESSON 6-4

Reteach

Solving Special Systems

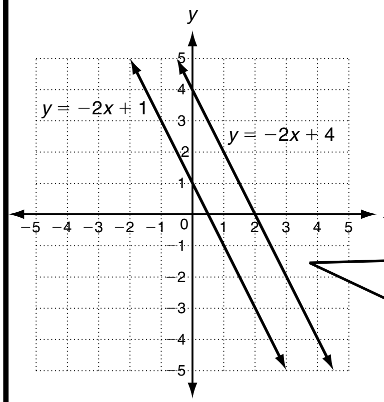
When solving equations in one variable, it is possible to have one solution, no solutions, or infinitely many solutions. The same results can occur when graphing systems of equations.

Solve $\begin{cases} 4x + 2y = 2 \\ 2x + y = 4 \end{cases}$.

Multiplying the second equation by -2 will eliminate the x-terms.

$$\begin{array}{ll} 4x + 2y = 2 & \\ -2(2x + y = 4) & \rightarrow \end{array} \quad \begin{array}{r} 4x + 2y = 2 \\ \underline{-4x - 2y = -8} \\ 0 + 0 = -6 \\ 0 = -6 \ ✗ \end{array}$$

The equation is a contradiction. **There is no solution.**



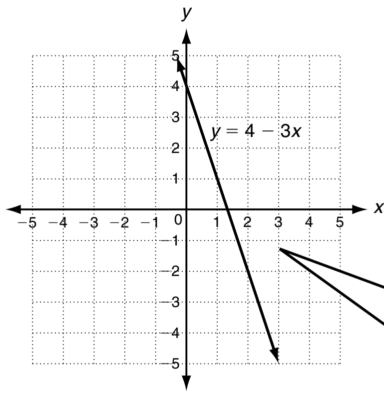
Graphing the system shows that these are parallel lines. They will never intersect, so there is no solution.

Solve $\begin{cases} y = 4 - 3x \\ 3x + y = 4 \end{cases}$.

Because the first equation is solved for a variable, use substitution.

$$3x + y = 4$$
$$3x + (4 - 3x) = 4 \quad \textit{Substitute } 4 - 3x \textit{ for } y$$
$$0 + 4 = 4$$
$$4 = 4 \ ✓$$

The equation is true for all values of x and y. **There are infinitely many solutions.**



The slopes and y-intercepts are the same. These are the same line.

Solve each system of linear equations algebraically.

1. $\begin{cases} y = 3x \\ 2y = 6x \end{cases}$ **2.** $\begin{cases} y = 2x + 5 \\ y - 2x = 1 \end{cases}$ **3.** $\begin{cases} 3x - 2y = 9 \\ -6x + 4y = 1 \end{cases}$

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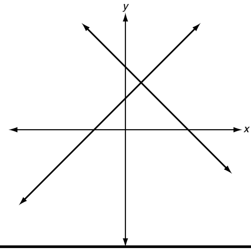
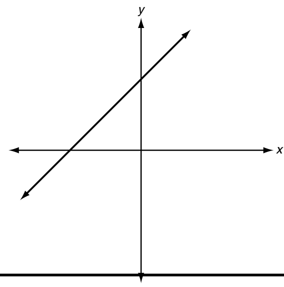
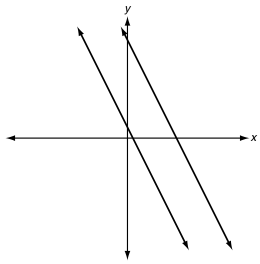
Name ______________________ Date __________ Class __________

LESSON **6-4**

Reteach

Solving Special Systems (continued)

A system of linear equations can be classified in three ways.

I. Consistent and independent one solution different slopes	Example: $\begin{cases} y = x + 3 \\ y = -x + 6 \end{cases}$	
II. Consistent and dependent infinitely many solutions same slope, same *y*-intercepts	Example $\begin{cases} y = 3x + 4 \\ y - 3x = 4 \end{cases}$	
III. Inconsistent no solutions same slope, different *y*-intercepts	Example $\begin{cases} y = 2x + 5 \\ y = 2x + 2 \end{cases}$	

Classify each system below by comparing the slopes and *y*-intercepts. Then give the number of solutions.

4. $\begin{cases} y = -3x - 2 \\ y = -3x - 4 \end{cases}$

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____

5. $\begin{cases} y = 2x + 5 \\ y = 5 + 2x \end{cases}$

____

____

6. $\begin{cases} y = -4x + 3 \\ y = 2x + 7 \end{cases}$

____

____

Classify each system and give the number of solutions. If there is one solution, provide it.

7. $\begin{cases} y = 2x + 8 \\ y - 4x = 8 \end{cases}$

____

____

8. $\begin{cases} y + 3x - 2 = 0 \\ 9x + 3y = 6 \end{cases}$

____

____

LESSON 6-4
Practice A
Solving Special Systems

Solve each system of linear equations. Tell whether the system has no solution or infinitely many solutions.

1. $\begin{cases} 2x + y = 1 \\ 2x + y = -3 \end{cases}$ no solution

2. $\begin{cases} y = 5x + 2 \\ y - 5x = 2 \end{cases}$ infinitely many solutions

3. $\begin{cases} y - 3x + 2 = 0 \\ 2 = -y + 3x \end{cases}$ infinitely many solutions

4. $\begin{cases} x + y = 4 \\ y - 4 = 1 - x \end{cases}$ no solution

Give the number of solutions to each system. Then classify the system as "consistent, independent", "consistent, dependent", or "inconsistent".

5. $\begin{cases} y = 2(x + 1) \\ y - 2x = 2 \end{cases}$ infinitely many solutions; consistent, dependent

6. $\begin{cases} y - 4x + 5 = 0 \\ 4x = y - 1 \end{cases}$ no solution; inconsistent

7. Marquis opens a savings account with \$60 and adds \$20 each month. His brother Jibran adds \$20 each month to the savings account that his grandmother opened with \$60. If the brothers continue to make deposits to their savings accounts at the same rate, when will they have the same amount of money? Explain.

They will always have the same amount of money. The graphs of these equations are the same line.

LESSON 6-4
Practice B
Solving Special Systems

Solve each system of linear equations.

1. $\begin{cases} y = 2x - 3 \\ y - 2x = -3 \end{cases}$ infinitely many solutions

2. $\begin{cases} 3x + y = 4 \\ -3x = y - 7 \end{cases}$ no solution

3. $\begin{cases} y = -4x + 1 \\ 4x = -y - 6 \end{cases}$ no solution

4. $\begin{cases} y - x + 3 = 0 \\ x = y + 3 \end{cases}$ infinitely many solutions

Classify each system. Give the number of solutions.

5. $\begin{cases} y = 3(x - 1) \\ -y + 3x = 3 \end{cases}$ consistent, dependent; infinitely many solutions

6. $\begin{cases} y - 2x = 5 \\ x = y - 3 \end{cases}$ consistent, independent; one solution

7. Sabina and Lou are reading the same book. Sabina reads 12 pages a day. She had read 36 pages when Lou started the book, and Lou reads at a pace of 15 pages per day. If their reading rates continue, will Sabina and Lou ever be reading the same page on the same day? Explain.

Yes. The graphs of the two equations have different slopes. They will intersect.

8. Brandon started jogging at 4 miles per hour. After he jogged 1 mile, his friend Anton started jogging along the same path at a pace of 4 miles per hour. If they continue to jog at the same rate, will Anton ever catch up with Brandon? Explain.

No. The graphs of the two equations are parallel lines. They will never intersect.

LESSON 6-4
Practice C
Solving Special Systems

Solve each system of linear equations.

1. $\begin{cases} y + 2x + 4 = 0 \\ 2x = -y - 4 \end{cases}$ infinitely many solutions

2. $\begin{cases} 5x + y = 8 \\ -5x = 8 + y \end{cases}$ no solution

3. $\begin{cases} 2x - y = 4 \\ 1 = y - 2x + 5 \end{cases}$ infinitely many solutions

4. $\begin{cases} y = -x - 6 \\ y - 2x = -3x + 6 \end{cases}$ no solution

Classify each system. Give the number of solutions.

5. $\begin{cases} y + 2(x - 3) = 0 \\ 2x = -y - 3 \end{cases}$ inconsistent; no solution

6. $\begin{cases} y + 3x = -1 \\ x = y + 3x - 1 \end{cases}$ consistent, independent; one solution

7. At a factory, Jin assembles 12 parts each minute. He has assembled 156 parts when Summer starts on the line, assembling at a pace of 15 parts per minute. If their assembly rates continue, will Summer ever catch up to Jin? Explain.

Yes. The graphs of the two equations have different slopes. They will intersect.

8. Kat is comparing monthly sales at her bookstore with those of her competitor, Gill. If the sales rates continue, will Kat's book sales ever catch up with her competitor's? Explain.

Kat's sales	112	118	124	130
Gill's sales	138	144	150	156

No. The graphs of the two equations are parallel lines. They will never intersect.

LESSON 6-4
Reteach
Solving Special Systems

When solving equations in one variable, it is possible to have one solution, no solutions, or infinitely many solutions. The same results can occur when graphing systems of equations.

Solve $\begin{cases} \mathbf{4x + 2y = 2} \\ \mathbf{2x + y = 4} \end{cases}$.

Multiplying the second equation by -2 will eliminate the x-terms.

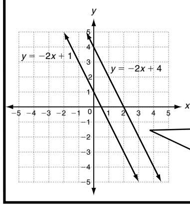
$4x + 2y = 2 \rightarrow 4x + 2y = 2$

$-2(2x + y = 4) \rightarrow -4x - 2y = -8$

$0 + 0 = -6$

$0 = -6$ ✗

The equation is a contradiction. **There is no solution.**



Graphing the system shows that these are parallel lines. They will never intersect, so there is no solution.

Solve $\begin{cases} \mathbf{y = 4 - 3x} \\ \mathbf{3x + y = 4} \end{cases}$.

Because the first equation is solved for a variable, use substitution.

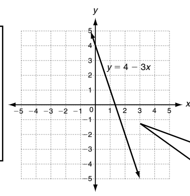
$3x + y = 4$

$3x + (4 - 3x) = 4$ *Substitute* $4 - 3x$ *for* y

$0 + 4 = 4$

$4 = 4$ ✓

The equation is true for all values of x and y. **There are infinitely many solutions.**



The slopes and y-intercepts are the same. These are the same line.

Solve each system of linear equations algebraically.

1. $\begin{cases} y = 3x \\ 2y = 6x \end{cases}$ infinitely many solutions

2. $\begin{cases} y = 2x + 5 \\ y - 2x = 1 \end{cases}$ no solution

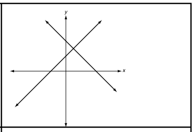
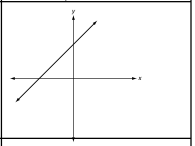
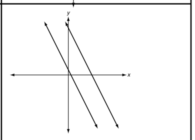
3. $\begin{cases} 3x - 2y = 9 \\ -6x + 4y = 1 \end{cases}$ no solution

LESSON 6-4

Reteach

Solving Special Systems (continued)

A system of linear equations can be classified in three ways.

I. Consistent and independent one solution different slopes	Example: $\begin{cases} y = x + 3 \\ y = -x + 6 \end{cases}$	
II. Consistent and dependent infinitely many solutions same slope, same *y*-intercepts	Example $\begin{cases} y = 3x + 4 \\ y - 3x = 4 \end{cases}$	
III. Inconsistent no solutions same slope, different *y*-intercepts	Example $\begin{cases} y = 2x + 5 \\ y = 2x + 2 \end{cases}$	

Classify each system below by comparing the slopes and *y*-intercepts. Then give the number of solutions.

4. $\begin{cases} y = -3x - 2 \\ y = -3x - 4 \end{cases}$
   inconsistent;
   no solutions

5. $\begin{cases} y = 2x + 5 \\ y = 5 + 2x \end{cases}$
   consistent and dependent;
   infinitely many solutions

6. $\begin{cases} y = -4x + 3 \\ y = 2x + 7 \end{cases}$
   consistent and independent;
   one solution

Classify each system and give the number of solutions. If there is one solution, provide it.

7. $\begin{cases} y = 2x + 8 \\ y - 4x = 8 \end{cases}$
   consistent and independent;
   one solution; (0, 8)

8. $\begin{cases} y + 3x - 2 = 0 \\ 9x + 3y = 6 \end{cases}$
   consistent and dependent;
   infinitely many solutions

LESSON 6-4

Challenge

Pick a Path

The puzzle below contains 23 linear equations. You goal is to find paths through the puzzle such that each pair of equations along your path forms a special type of system.

You must begin in the **Start** square, and end in the **Finish** square. Moves can be horizontal, vertical, or diagonal between adjacent squares. Each individual path cannot cross itself, but your answers to 1 and 2 might cross.

Start → ↓ ↘	1 $y = x + 2$	2 $3x = 3y - 6$	3 $3x + y = -1$	4 $8y = 8x + 16$
5 $y = 2x + 3$	6 $y = 3x - 1$	7 $y - x = 2$	8 $\frac{1}{3}x + \frac{1}{3}y = 1$	9 $3x = y + 1$
10 $2y = 6x + 6$	11 $2x - y = -3$	12 $2x + 2y = 4$	13 $2y - 4 = 2x$	14 $x - y = -2$
15 $\frac{1}{2}y = \frac{3}{2}x + \frac{1}{2}$	16 $3y = 2x + 1$	17 $3x - y = 4$	18 $\frac{1}{6}y = \frac{1}{3}x + \frac{1}{2}$	19 $\frac{1}{2}y = \frac{1}{2}x + 1$
20 $10x + 15y = 5$	21 $-12x + 4y = 8$	22 $\frac{1}{2}y = \frac{1}{3}x + \frac{1}{6}$	23 $x = \frac{1}{3}y + 1$	↘ ↓ → **Finish**

Find a path through the puzzle such that each pair of equations forms...

1. ...a consistent and dependent system.
   Start — 1 — 2 — 7 — 13 — 14 — 19 — Finish
2. ...an inconsistent system.
   Start — 6 — 10 — 15 — 21 — 17 — 23 — Finish

LESSON 6-4

Problem Solving

Solving Special Systems

Write the correct answer.

1. Tyra and Charmian are training for a bike race. Tyra has logged 256 miles so far and rides 48 miles per week. Charmian has logged 125 miles so far and rides 48 miles per week. If these rates continue, will Tyra's distance ever equal Charmian's distance? Explain.
   No; the graphs are parallel lines so there is no solution.

2. Metroplexpress and Local Express are courier companies. Metroplexpress charges \$15 to pick up a package and \$0.50 per mile. Local Express charges \$10 to pick up a package and \$0.55 per mile. Classify this system and find its solution, if any.
   consistent and independent;
   100 mi and \$65

3. The Singhs start savings accounts for their twin boys. The accounts earn 5% annual interest. The initial deposit in each account is \$200. Classify this system and find its solution, if any.
   consistent and dependent;
   infinitely many solutions

4. Frank earns \$8 per hour. Madison earns \$7.50 per hour. Frank started working after Madison had already earned \$300. If these rates continue, will Frank's earnings ever equal Madison's earnings? If so, when?
   Yes;
   at 600 hours.

Select the best answer.

5. A studio apartment at The Oaks costs \$400 per month plus a \$350 deposit. A studio apartment at Crossroads costs \$400 per month plus a \$300 deposit. How many solutions does this system have?
   - (A) no solutions
   - B 1 solution
   - C 2 solutions
   - D an infinite number of solutions

6. Jane and Gary are both landscape designers. Jane charges \$75 for a consultation plus \$25 per hour. Gary charges \$50 for a consultation plus \$30 per hour. For how many hours will Jane's charges equal Gary's charges?
   - F never
   - G after 2 hours
   - (H) after 5 hours
   - J always

7. A tank filled with 75 liters of water loses 0.5 liter of water per hour. A tank filled with 50 liters of water loses 0.1 liter of water per hour. How would this system be classified?
   - A inconsistent
   - B dependent
   - (C) consistent and independent
   - D consistent and dependent

8. Simon is 3 years older than Renata. Five years ago, Renata was half as old as Simon is now. How old are Simon and Renata now?
   - F Simon is 13 and Renata is 10.
   - G Simon is 15 and Renata is 10.
   - H Simon is 16 and Renata is 8.
   - (J) Simon is 16 and Renata is 13.

LESSON 6-4

Reading Strategies

Use a Table

The table below can help you answer questions about linear systems.

Classification	Number of Solutions	Similarities and Differences in $y = mx + b$	Description of Graphed Lines	Result of Solving with Algebra
Consistent, Independent	1	different slopes (m)	intersecting	values for x and y ex. $x = 3$, $y = -5$
Consistent, Dependent	infinitely many	same slope (m) same *y*-int. (b)	coincident	identity statement ex. $4 = 4$
Inconsistent	0	same slope (m) different *y*-int. (b)	parallel	false statement ex. $-2 = 3$

1. Mary Kate solved a system by elimination as shown below. Classify the system.
   $\begin{cases} 2x + 3y = 5 \\ 2x + 3y = 7 \end{cases} \Rightarrow \begin{array}{r} 2x + 3y = 5 \\ -(2x + 3y = 7) \\ \hline 0 = -2 \end{array}$
   inconsistent
2. Raul solved a system of equations by substitution. He ended up with an identity statement. How many solutions does his system have?
   infinitely many
3. How many solutions does a system have if, when graphed, the lines are the same line?
   infinitely many
4. Two equations in a system have the same slope. Which classification can NOT describe the system?
   consistent, independent
5. The graph of a system consists of two intersecting lines. How many solutions does the system have?
   one

Classify each system, give the number of solutions, and describe its graph.

6. $\begin{cases} y = -3x \\ y = -3x + 2 \end{cases}$
   inconsistent;
   0;
   parallel lines

7. $\begin{cases} y = -x + 4 \\ x + y = 4 \end{cases}$
   consistent, dependent;
   infinitely many;
   coincident lines

8. $\begin{cases} y = 2x - 1 \\ x = y \end{cases}$
   consistent, independent;
   1;
   intersecting lines