Name ______________________________ Date __________ Class __________

LESSON
6-3

Reteach

Solving Systems by Elimination

Elimination can be used to solve a system of equations by adding terms vertically. This will cause one of the variables to be eliminated. It may be necessary to multiply one or both equations by some number to use this method.

I. Elimination may require no change to either equation.

$$\begin{cases} 3x + y = 6 \\ 5x - y = 10 \end{cases}$$

Adding vertically will eliminate *y*.

$$\begin{array}{r} 3x + y = 6 \\ \underline{5x - y = 10} \\ 8x + 0 = 16 \end{array}$$

II. Elimination may require multiplying one equation by an appropriate number.

$$\begin{cases} 2x + 5y = 9 \\ x - 3y = 10 \end{cases}$$

Multiply bottom equation by −2.

$$\begin{array}{r} 2x + 5y = 9 \\ \underline{-2(x - 3y) = -2(10)} \end{array} \qquad \begin{array}{r} 2x + 5y = 9 \\ \underline{-2x + 6y = -20} \\ 0 + 11y = -11 \end{array}$$

III. Elimination may require multiplying both equations by different numbers.

$$\begin{cases} 5x + 3y = 2 \\ 4x + 2y = 10 \end{cases}$$

Multiply the top by −2 and the bottom by 3.

$$\begin{array}{r} -2(5x + 3y = 2) \\ \underline{3(4x + 2y = 10)} \end{array} \qquad \begin{array}{r} -10x + -6y = -4 \\ \underline{12x + 6y = 30} \\ 2x + 0 = 26 \end{array}$$

Solve each system by elimination.

1. $\begin{cases} 2x - y = 20 \\ 3x + 2y = -19 \end{cases}$

____

2. $\begin{cases} 3x + 2y = 10 \\ 3x - 2y = 14 \end{cases}$

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3. $\begin{cases} x + y = 12 \\ 2x + y = 6 \end{cases}$

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4. $\begin{cases} 3x - y = 2 \\ -8x + 2y = 4 \end{cases}$

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LESSON **6-3**

Reteach

Solving Systems by Elimination (continued)

A system of equations can be solved by graphing, substitution, or elimination.

- Use graphing if both equations are solved for *y*, or if you want an estimate of the solution.
- Use substitution if either equation is solved for a variable, or has a variable with a coefficient of 1 or −1.
- Use elimination if both equations have the same variable with the same or opposite coefficients.

It may be necessary to manipulate your equations to get them in any of the three forms above.

Solve $\begin{cases} \mathbf{y = 3 - x} \\ \mathbf{2x - y = 6} \end{cases}$.

One equation is solved for a variable. Use substitution.

$\boxed{3 - x}$

$$2x - y = 6$$
$$2x - (3 - x) = 6$$
$$3x - 3 = 6$$
$$\underline{+3} \quad \underline{+3}$$
$$3x = 9$$
$$x = 3$$

Substitute x + 2 for y.

Substitute $x = 3$ into one of the original equations to find the value of *y*.

$y = 3 - x$

$y = 3 - 3$

$y = 0$ The solution is $(3, 0)$.

Solve $\begin{cases} \mathbf{-2x - y = -5} \\ \mathbf{3x + y = -1} \end{cases}$.

The equations have the same variable with opposite coefficients. Use elimination.

$$-2x - y = -5$$
$$\underline{3x + y = -1}$$
$$x + 0 = -6$$
$$x = -6$$

Substitute $x = -6$ into one of the original equations to find the value of *y*.

$$3x + y = -1$$
$$3(-6) + y = -1$$
$$-18 + y = -1$$
$$\underline{+18} \qquad \underline{+18}$$
$$y = 17$$

The solution is $(-6, 17)$.

Solve each system by any method.

5. $\begin{cases} y = x + 3 \\ -2x + y = -4 \end{cases}$

6. $\begin{cases} 4x + y = 10 \\ -2x - y = 4 \end{cases}$

7. $\begin{cases} 2x + y = 8 \\ 3x + 5y = 5 \end{cases}$

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LESSON 6-3

Practice A

Solving Systems by Elimination

Fill in the blanks to solve each system by elimination.

1. $\begin{cases} x + 3y = 14 \\ 2x - 3y = -8 \end{cases}$

Add the equations:

$x + 3y = 14$

$\underline{+2x - 3y = 28}$

$3x + \underline{0y} = 6$

$\underline{3x} = 6$

$\div \underline{3} \quad \div \underline{3}$

$x = \underline{2}$

Substitute $\underline{2}$ for x in one of the equations:

$x + 3y = 14$

$\underline{2} + 3y = 14$

$-\underline{2} \quad -\underline{2}$

$3y = \underline{12}$

$\div 3 \quad \div 3$

$y = \underline{4}$

Solution: $(\underline{2}, \underline{4})$

2. $\begin{cases} 2x + 2y = 4 \\ 3x + 2y = 7 \end{cases}$

Subtract the equations:

$2x + 2y = 4$

$-(\underline{3x + 2y = 7})$

or

$2x + 2y = 4$

$\underline{-3x - \underline{2} = \underline{-7}}$

$-x + \underline{0y} = \underline{-3}$

$-x = \underline{-3}$

$\div \underline{-1} \quad \div \underline{-1}$

$x = \underline{3}$

Substitute $\underline{3}$ for x in one of the equations:

$3x + 2y = 7$

$3(\underline{3}) + 2y = 7$

$\underline{9} + 2y = 7$

$-\underline{9} \quad -\underline{9}$

$2y = \underline{-2}$

$\div \underline{2} \quad \div \underline{2}$

$y = \underline{-1}$

Solution: $(\underline{3}, \underline{-1})$

3. $\begin{cases} 3x + 4y = 26 \\ x - 2y = -8 \end{cases}$

Multiply the second equation by 2. Then, add the equations:

$\begin{cases} 3x + 4y = 26 \\ 2(x - 2y = -8) \end{cases}$

$3x + 4y = 26$

$\underline{+ \underline{2}x - \underline{4}y = \underline{-16}}$

$\underline{5}x + 0 = \underline{10}$

$\underline{5}x = \underline{10}$

$\div \underline{5} \quad \div \underline{5}$

$x = \underline{2}$

Substitute $\underline{2}$ for x in one of the equations:

$x - 2y = -8$

$\underline{2} - 2y = -8$

$-\underline{2} \quad -\underline{2}$

$-2y = \underline{-10}$

$\div \underline{-2} \quad \div \underline{-2}$

$y = \underline{5}$

Solution: $(\underline{2}, \underline{5})$

Solve each system by elimination.

4. $\begin{cases} 3x - 2y = 1 \\ 2x + 2y = 14 \end{cases}$ $\underline{(3, 4)}$

5. $\begin{cases} x + y = 4 \\ 3x + y = 16 \end{cases}$ $\underline{(6, -2)}$

6. $\begin{cases} 3x + 2y = -26 \\ 2x - 6y = -10 \end{cases}$ $\underline{(-8, -1)}$

7. The sum of two numbers is −1. When twice the first number and four times the second number are added, the sum is −10. What are the two numbers? $\underline{\text{3 and } -4}$

LESSON 6-3

Practice B

Solving Systems by Elimination

Follow the steps to solve each system by elimination.

1. $\begin{cases} 2x - 3y = 14 \\ 2x + y = -10 \end{cases}$

Subtract the second equation:

$2x - 3y = 14$

$\underline{-(2x + y = -10)}$

$-4y = 24$

Solve the resulting equation:

$y = \underline{-6}$

Use your answer to find the value of x:

$x = \underline{-2}$

Solution: $(\underline{-2}, \underline{-6})$

2. $\begin{cases} 3x + y = 17 \\ 4x + 2y = 20 \end{cases}$

Multiply the first equation by −2. Then, add the equations:

$\underline{-6}x - \underline{2}y = \underline{-34}$

$\underline{+ 4x + 2y = 20}$

$-2x = -14$

Solve the resulting equation:

$x = \underline{7}$

Use your answer to find the value of y:

$y = \underline{-4}$

Solution: $(\underline{7}, \underline{-4})$

Solve each system by elimination. Check your answer.

3. $\begin{cases} x + 3y = -7 \\ -x + 2y = -8 \end{cases}$ $\underline{(2, -3)}$

4. $\begin{cases} 3x + y = -26 \\ 2x - y = -19 \end{cases}$ $\underline{(-9, 1)}$

5. $\begin{cases} x + 3y = -14 \\ 2x - 4y = 32 \end{cases}$ $\underline{(4, -6)}$

6. $\begin{cases} 4x - y = -5 \\ -2x + 3y = 10 \end{cases}$ $\underline{\left(-\frac{1}{2}, 3\right)}$

7. $\begin{cases} y - 3x = 11 \\ 2y - x = 2 \end{cases}$ $\underline{(-4, -1)}$

8. $\begin{cases} -10x + y = 0 \\ 5x + 3y = -7 \end{cases}$ $\underline{\left(-\frac{1}{5}, -2\right)}$

Solve.

9. Brianna's family spent $134 on 2 adult tickets and 3 youth tickets at an amusement park. Max's family spent $146 on 3 adult tickets and 2 youth tickets. What is the price of a youth ticket? $\underline{\$22}$

10. Carl bought 19 apples of 2 different varieties to make a pie. The total cost of the apples was $5.10. Granny Smith apples cost $0.25 each and Gala apples cost $0.30 each. How many of each type of apple did Carl buy? $\underline{\text{7 Gala apples; 12 Granny Smith apples}}$

LESSON 6-3

Practice C

Solving Systems by Elimination

Solve each system by elimination.

1. $\begin{cases} x + y = 2 \\ 2x - y = 7 \end{cases}$ $\underline{(3, -1)}$

2. $\begin{cases} 3x - 2y = -2 \\ 3x + y = 10 \end{cases}$ $\underline{(2, 4)}$

3. $\begin{cases} x + y = -7 \\ x - y = 5 \end{cases}$ $\underline{(-1, -6)}$

4. $\begin{cases} -3x - 4y = -2 \\ 6x + 4y = 3 \end{cases}$ $\underline{\left(\frac{1}{3}, \frac{1}{4}\right)}$

5. $\begin{cases} 2x - 2y = 14 \\ x + 4y = -13 \end{cases}$ $\underline{(3, -4)}$

6. $\begin{cases} y - x = 17 \\ 2y + 3x = -11 \end{cases}$ $\underline{(-9, 8)}$

7. $\begin{cases} x + 6y = 1 \\ 2x - 3y = 32 \end{cases}$ $\underline{(13, -2)}$

8. $\begin{cases} -\frac{1}{2}x + y = 4 \\ \frac{1}{3}x - y = -3 \end{cases}$ $\underline{(-6, 1)}$

9. $\begin{cases} 3x + y = -15 \\ 2x - 3y = 23 \end{cases}$ $\underline{(-2, -9)}$

10. $\begin{cases} 5x - 2y = -48 \\ 2x + 3y = -23 \end{cases}$ $\underline{(-10, -1)}$

11. $\begin{cases} 4x - 3y = -9 \\ 5x - y = 8 \end{cases}$ $\underline{(3, 7)}$

12. $\begin{cases} 3x - 3y = -1 \\ 12x - 2y = 16 \end{cases}$ $\underline{\left(\frac{5}{3}, 2\right)}$

13. At a bakery, Riley bought 3 bagels and 2 muffins for $7.25. Karen bought 5 bagels and 4 muffins for $13.25. What is the cost of each item? $\underline{\text{bagel: \$1.25; muffin: \$1.75}}$

14. A chemist has a beaker of a 3% acid solution and a beaker of a 7% acid solution. He needs to make 75 mL of a 4% acid solution.

a. Complete the table.

	3% solution	+	7% solution	=	4% solution
Amount of Solution (mL)	x	+	y	=	75
Amount of Acid (mL)	$0.03x$	+	$0.07y$	=	0.04(75)

b. Use the information in the table to write a system of linear equations. $\begin{cases} x + y = 75 \\ 0.03x + 0.07y = 3 \end{cases}$

c. Solve the system of equations to find how much he will use from each beaker.

56.25 mL of the 3% solution; 18.75 mL of the 7% solution

LESSON 6-3

Reteach

Solving Systems by Elimination

Elimination can be used to solve a system of equations by adding terms vertically. This will cause one of the variables to be eliminated. It may be necessary to multiply one or both equations by some number to use this method.

I. Elimination may require no change to either equation.

$\begin{cases} 3x + y = 6 \\ 5x - y = 10 \end{cases}$

Adding vertically will eliminate y.

$3x + y = 6$

$\underline{5x - y = 10}$

$8x + 0 = 16$

II. Elimination may require multiplying one equation by an appropriate number.

$\begin{cases} 2x + 5y = 9 \\ x - 3y = 10 \end{cases}$

Multiply bottom equation by −2.

$2x + 5y = 9$

$\underline{-2(x - 3y) = -2(10)}$

$2x + 5y = 9$

$\underline{-2x + 6y = -20}$

$0 + 11y = -11$

III. Elimination may require multiplying both equations by different numbers.

$\begin{cases} 5x + 3y = 2 \\ 4x + 2y = 10 \end{cases}$

Multiply the top by −2 and the bottom by 3.

$-2(5x + 3y = 2)$

$\underline{3(4x + 2y = 10)}$

$-10x + -6y = -4$

$\underline{12x + 6y = 30}$

$2x + 0 = 26$

Solve each system by elimination.

1. $\begin{cases} 2x - y = 20 \\ 3x + 2y = -19 \end{cases}$ $\underline{(3, -14)}$

2. $\begin{cases} 3x + 2y = 10 \\ 3x - 2y = 14 \end{cases}$ $\underline{(4, -1)}$

3. $\begin{cases} x + y = 12 \\ 2x + y = 6 \end{cases}$ $\underline{(-6, 18)}$

4. $\begin{cases} 3x - y = 2 \\ -8x + 2y = 4 \end{cases}$ $\underline{(-4, -14)}$

LESSON 6-3

Reteach

Solving Systems by Elimination (continued)

A system of equations can be solved by graphing, substitution, or elimination.

- Use graphing if both equations are solved for *y*, or if you want an estimate of the solution.
- Use substitution if either equation is solved for a variable, or has a variable with a coefficient of 1 or −1.
- Use elimination if both equations have the same variable with the same or opposite coefficients.

It may be necessary to manipulate your equations to get them in any of the three forms above.

Solve $\begin{cases} \mathbf{y = 3 - x} \\ \mathbf{2x - y = 6} \end{cases}$.

One equation is solved for a variable. Use substitution.

$\boxed{3 - x}$

$2x - y = 6$ *Substitute x + 2 for y.*

$2x - (3 - x) = 6$

$3x - 3 = 6$

$\underline{+3 \quad +3}$

$3x = 9$

$x = 3$

Substitute $x = 3$ into one of the original equations to find the value of *y*.

$y = 3 - x$

$y = 3 - 3$

$y = 0$ The solution is $(3, 0)$.

Solve $\begin{cases} \mathbf{-2x - y = -5} \\ \mathbf{3x + y = -1} \end{cases}$.

The equations have the same variable with opposite coefficients. Use elimination.

$-2x - y = -5$

$\underline{3x + y = -1}$

$x + 0 = -6$

$x = -6$

Substitute $x = -6$ into one of the original equations to find the value of *y*.

$3x + y = -1$

$3(-6) + y = -1$

$-18 + y = -1$

$\underline{+18 \qquad +18}$

$y = 17$

The solution is $(-6, 17)$.

Solve each system by any method.

5. $\begin{cases} y = x + 3 \\ -2x + y = -4 \end{cases}$ $(7, 10)$

6. $\begin{cases} 4x + y = 10 \\ -2x - y = 4 \end{cases}$ $(7, -18)$

7. $\begin{cases} 2x + y = 8 \\ 3x + 5y = 5 \end{cases}$ $(5, -2)$

LESSON 6-3

Challenge

Solving Systems by Elimination

The elimination method can also be used for a system of three equations in three unknowns.

Three camp leaders purchased equipment for a camping trip.
Max bought 10 sleeping bags, 2 tents, and 1 can of bug repellant for $885.
Carlos bought 5 sleeping bags, 4 tents, and 1 can of bug repellant for $865.
Amy bought 9 sleeping bags, 6 tents, and 6 cans of bug repellant for $1410.
If they made their purchases at the same store, how much did each item cost?

1. Write the 3 equations: $\begin{cases} 10x + 2y + z = 885 \\ 5x + 4y + z = 865 \\ 9x + 6y + 6z = 1410 \end{cases}$

2. Subtract the second equation from the first. $5x - 2y = 20$

3. Multiply the second equation by −6 and add the second and third equations. $-21x - 18y = -3780$

4. The equations in steps 2 and 3 form a linear system in two variables. Solve this system for *x*. $x = 60$

5. Substitute the value of *x* into the first two equations. Write the resulting system. $\begin{cases} 2y + z = 285 \\ 4y + z = 565 \end{cases}$

6. Solve the system in problem 5 for *y* and *z*. $y = 140, z = 5$

7. Write the cost of each item. sleeping bags: $60; tents: $140; bug repellant: $5

LESSON 6-3

Problem Solving

Solving Systems by Elimination

Write the correct answer.

1. Mr. Nguyen bought a package of 3 chicken legs and a package of 7 chicken wings. Ms. Dawes bought a package of 3 chicken legs and a package of 6 chicken wings. Mr. Nguyen bought 45 ounces of chicken. Ms. Dawes bought 42 ounces of chicken. How much did each chicken leg and each chicken wing weigh?

   chicken leg 8 oz.,

   chicken wing 3 oz.

2. Jayce bought 2 bath towels and returned 3 hand towels. His sister Jayna bought 3 bath towels and 3 hand towels. Jayce's bill was $5. Jayna's bill was $45. What are the prices of a bath towel and a hand towel?

   bath towel $10,

   hand towel $5

3. The Lees spent $31 on movie tickets for 2 adults and 3 children. The Macias spent $26 on movie tickets for 2 adults and 2 children. What are the prices for adult and child movie tickets?

   adult ticket $8,

   child ticket $5

4. Last month Stephanie spent $57 on 4 allergy shots and 1 office visit. This month she spent $9 after 1 office visit and a refund for 2 allergy shots from her insurance company. How much does an office visit cost? an allergy shot?

   office visit $25,

   allergy shot $8

Use the chart below to answer questions 5–6. Select the best answer. The chart shows the price per pound for dried fruit.

Dried Fruit Price List			
Pineapple	Apple	Mango	Papaya
$7.50/lb	$7.00/lb	$8.00/lb	$7.25/lb

5. A customer bought 5 pounds of mango and papaya for $37.75. How many pounds of each fruit did the customer buy?

   [✓ **A**] 2 lbs mango and 3 lbs papaya

   B 3 lbs mango and 2 lbs papaya

   C 1 lb mango and 4 lbs papaya

   D 4 lbs mango and 1 lb papaya

6. A store employee made two gift baskets of dried fruit, each costing $100. The first basket had 12 pounds of fruit x and 2 pounds of fruit y. The second basket had 4 pounds of fruit x and 9 pounds of fruit y. Which two fruits did the employee use in the baskets?

   F pineapple and apple

   [✓ **G**] apple and mango

   H mango and papaya

   J papaya and pineapple

LESSON 6-3

Reading Strategies

Connecting Concepts

When solving systems of linear equations using elimination, you will sometimes need to multiply one or both equations by a factor in order to get the same coefficients for a variable. This process is very similar to getting a common denominator for fractions. Look at the example below.

$\begin{cases} 6x - 5y = 16 \\ 4x - 3y = 12 \end{cases}$	To eliminate the *x*-terms, you need to get the same or opposite coefficients for *x* in both equations.
Add: $\frac{5}{6} + \frac{1}{4} = ?$	Think about finding a common denominator for 4 and 6.
4: 4, 8, 12, 16, 20 **6**: 6, 12, 18, 24, 30	Find the least common multiple (LCM) of 4 and 6 by listing their multiples in order. The LCM is 12.
$\begin{cases} 2(6x - 5y = 16) \\ 3(4x - 3y = 12) \end{cases}$	Determine what you have to multiple 4 and 6 by to get 12. Multiply each equation by the appropriate number.
$\begin{cases} 12x - 10y = 32 \\ 12x - 9y = 36 \end{cases}$	Now either add or subtract the equations. In this case, you will subtract the equations.

1. Describe how you would get common *y*-coefficients (instead of *x*-) in the example above.

   Multiply the first equation by 3 and the second equation by 5 to get common coefficients of −15.

2. Show how to get a set of common *y*-coefficients for the system $\begin{cases} 9x - 10y = 7 \\ 5x + 8y = 31 \end{cases}$.

   $\begin{cases} 4(9x - 10y = 7) \\ 5(5x + 8y = 31) \end{cases} \Rightarrow \begin{cases} 36x - 40y = 28 \\ 25x + 40y = 155 \end{cases}$

Solve each system of equations by elimination.

3. $\begin{cases} 9x - 2y = 15 \\ 4x + 3y = -5 \end{cases}$ $(1, -3)$

4. $\begin{cases} 2x - 3y = 50 \\ 7x + 8y = -10 \end{cases}$ $(10, -10)$