CHAPTER 5

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

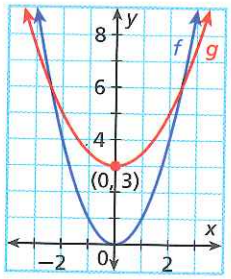
1. The number $5i$ can be classified as both a(n) ___?___ and a ___?___.
2. The value of the input x that makes the output $f(x)$ equal zero is called the ___?___.
3. The ___?___ is the point at which the parabola intersects the axis of symmetry.
4. The type and number of solutions to a quadratic equation can be determined by finding the ___?___.
5. When a parabola opens upward, the y-value of the vertex is the ___?___ of a quadratic function.

5-1 Using Transformations to Graph Quadratic Functions *(pp. 315–322)*

EXAMPLES

■ **Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph $g(x) = \frac{1}{2}x^2 + 3$.**

$g(x) = \frac{1}{2}x^2 + 3$ is f vertically compressed by a factor of $\frac{1}{2}$ and translated 3 units up.



■ **Use the description to write a quadratic function in vertex form. The function $f(x) = x^2$ is translated 1 unit right to create g.**

translation 1 unit right: $h = 1$

$g(x) = a(x - h)^2 + k \rightarrow g(x) = (x - 1)^2$

EXERCISES

Graph each function by using a table.

6. $f(x) = -x^2 - 2x$
7. $f(x) = \frac{1}{2}x^2 + 3x - 4$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

8. $g(x) = 4(x - 2)^2$
9. $g(x) = -2(x + 1)^2$
10. $g(x) = \frac{1}{3}x^2 - 3$
11. $g(x) = -(x + 2)^2 + 6$

Use the description to write each quadratic function in vertex form.

12. $f(x) = x^2$ is reflected across the x-axis and translated 3 units down to create g.
13. $f(x) = x^2$ is vertically stretched by a factor of 2 and translated 4 units right to create g.
14. $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{4}$ and translated 1 unit left to create g.

5-2 Properties of Quadratic Functions in Standard Form *(pp. 323–330)*

EXAMPLE

■ For $f(x) = -x^2 + 2x + 3$, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

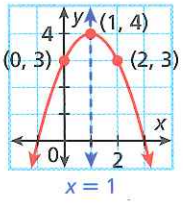
a. Because $a < 0$, the parabola opens downward.

b. axis of symmetry:
$x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$

c. $f(1) = -1^2 + 2(1) + 3 = 4$
The vertex is $(1, 4)$.

d. Because $c = 3$, the y-intercept is 3.

e.



EXERCISES

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

15. $f(x) = x^2 - 4x + 3$ **16.** $g(x) = x^2 + 2x + 3$

17. $h(x) = x^2 - 3x$ **18.** $j(x) = \frac{1}{2}x^2 - 2x + 4$

Find the minimum or maximum value of each function.

19. $f(x) = x^2 + 2x + 6$ **20.** $g(x) = 6x - 2x^2$

21. $f(x) = x^2 - 5x + 1$ **22.** $g(x) = -2x^2 - 8x + 10$

23. $f(x) = -x^2 - 4x + 8$ **24.** $g(x) = 3x^2 + 7$

5-3 Solving Quadratic Equations by Graphing and Factoring *(pp. 333–340)*

EXAMPLES

■ Find the roots of $x^2 + x = 30$ by factoring.

$x^2 + x - 30 = 0$ *Rewrite in standard form.*
$(x - 5)(x + 6) = 0$ *Factor.*
$x - 5 = 0$ or $x + 6 = 0$ *Zero Product Property.*
$x = 5$ or $x = -6$ *Solve each equation.*

■ Write a quadratic function with zeros 8 and −8.

$x = 8$ or $x = -8$ *Write zeros as solutions.*
$x - 8 = 0$ or $x + 8 = 0$ *Set equations equal to 0.*
$(x - 8)(x + 8) = 0$ *Converse Zero Product Property*
$f(x) = x^2 - 64$ *Replace 0 with f(x).*

EXERCISES

Find the roots of each equation by factoring.

25. $x^2 - 7x - 8 = 0$ **26.** $x^2 - 5x + 6 = 0$

27. $x^2 = 144$ **28.** $x^2 - 21x = 0$

29. $4x^2 - 16x + 16 = 0$ **30.** $2x^2 + 8x + 6 = 0$

31. $x^2 + 14x = 32$ **32.** $9x^2 + 6x + 1 = 0$

Write a quadratic function in standard form for each given set of zeros.

33. 2 and −3 **34.** 1 and −1

35. 4 and 5 **36.** −2 and −3

37. −5 and −5 **38.** 9 and 0

5-4 Completing the Square *(pp. 342–349)*

EXAMPLE

■ Solve $x^2 - 8x = 12$ by completing the square.

$x^2 - 8x + \square = 12 + \square$ *Set up equation.*
$x^2 - 8x + 16 = 12 + 16$ *Add $\left(\frac{b}{2}\right)^2$.*
$(x - 4)^2 = 28$ *Factor.*
$x - 4 = \pm\sqrt{28}$ *Take square roots.*
$x = 4 \pm 2\sqrt{7}$ *Solve for x.*

EXERCISES

Solve each equation by completing the square.

39. $x^2 - 16x + 48 = 0$ **40.** $x^2 + 20x + 84 = 0$

41. $x^2 - 6x = 16$ **42.** $x^2 - 14x = 13$

Write each function in vertex form, and identify its vertex.

43. $f(x) = x^2 - 4x + 9$ **44.** $g(x) = x^2 + 2x - 7$

5-5 Complex Numbers and Roots *(pp. 350–355)*

EXAMPLE

■ Solve $x^2 - 22x + 133 = 0$.

$x^2 - 22x + \square = -133 + \square$ *Rewrite.*

$x^2 - 22x + 121 = -133 + 121$ *Add $\left(\frac{b}{2}\right)^2$.*

$(x - 11)^2 = -12$ *Factor.*

$x - 11 = \pm\sqrt{-12}$ *Take square roots.*

$x = 11 \pm 2i\sqrt{3}$ *Solve.*

EXERCISES

Solve each equation.

45. $x^2 = -81$

46. $6x^2 + 150 = 0$

47. $x^2 + 6x + 10 = 0$

48. $x^2 + 12x + 45 = 0$

49. $x^2 - 14x + 75 = 0$

50. $x^2 - 22x + 133 = 0$

Find each complex conjugate.

51. $5i - 4$

52. $3 + i\sqrt{5}$

5-6 The Quadratic Formula *(pp. 356–363)*

EXAMPLES

■ Find the zeros of $f(x) = 3x^2 - 5x + 3$ by using the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Quadratic Formula*

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(3)}}{2(3)}$ *Substitute.*

$= \dfrac{5 \pm \sqrt{-11}}{6} = \dfrac{5}{6} \pm i\dfrac{\sqrt{11}}{6}$ *Simplify.*

■ Find the type and number of solutions for $x^2 + 9x + 20 = 0$.

$b^2 - 4ac = 9^2 - 4(1)(20)$

$= 81 - 80 = 1$

There are two distinct real roots because the discriminant is positive.

EXERCISES

Find the zeros of each function by using the Quadratic Formula.

53. $f(x) = x^2 - 3x - 8$

54. $h(x) = (x - 5)^2 + 12$

55. $f(x) = 2x^2 - 10x + 18$

56. $g(x) = x^2 + 3x + 3$

57. $h(x) = x^2 - 5x + 10$

Find the type and number of solutions for each equation.

58. $2x^2 - 16x + 32 = 0$

59. $x^2 - 6x = -5$

60. $x^2 + 3x + 8 = 0$

61. $x^2 - 246x = -144$

62. $x^2 + 5x = -12$

63. $3x^2 - 5x + 3 = 0$

5-7 Solving Quadratic Inequalities *(pp. 366–373)*

EXAMPLE

■ Solve $x^2 - 4x - 9 \geq 3$ by using algebra.

Write and solve the related equation.

$x^2 - 4x - 12 = 0$ *Write in standard form.*

$(x + 2)(x - 6) = 0$ *Factor.*

$x = -2$ or $x = 6$ *Solve.*

The critical values are -2 and 6. These values divide the number line into three intervals: $x \leq -2$, $-2 \leq x \leq 6$, and $x \geq 6$.

Testing an x-value in each interval gives the solution of $x \leq -2$ or $x \geq 6$.

EXERCISES

Graph each inequality.

64. $y > x^2 + 3x + 4$

65. $y \leq 2x^2 - x - 5$

Solve each inequality by using tables or graphs.

66. $x^2 + 2x - 4 \geq -1$

67. $-x^2 - 5x > 4$

Solve each inequality by using algebra.

68. $-x^2 + 6x < 5$

69. $3x^2 - 25 \leq 2$

70. $x^2 - 3 < 0$

71. $3x^2 + 4x - 3 \leq 1$