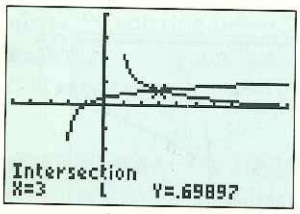
ADDITIONAL EXAMPLES

Example 11

Solve the equation
$\log(x + 2) = 1 - \log(x - 1)$.
Confirm your solution with a graph.

$$\log(x+2) = 1 - \log(x-1)$$
$$\log(x+2) + \log(x-1) = 1$$
$$\log[(x+2)(x-1)] = 1$$
$$\log(x^2 + x - 2) = 1$$
$$10^{\log(x^2+x-2)} = 10^1$$
$$x^2 + x - 2 = 10$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$x = -4$; $x = 3$

Because $\log(x + 2)$ and $\log(x - 1)$ are not defined for $x = -4$, it cannot be a solution. Therefore, the only solution is $x = 3$.



Exercises 5.6

ANSWERS

9. $x = \frac{\ln 5}{\ln 3} \approx 1.465$

11. $x = \frac{\ln 3}{\ln 1.5} \approx 2.7095$

13. $x = \frac{\ln 3 - 5\ln 5}{\ln 5 + 2\ln 3} \approx -1.825$

14. $x = \frac{\ln 4 - 2\ln 3}{3\ln 4 - \ln 3} \approx -0.2650$

15. $x = \frac{\ln 2 - \ln 3}{3\ln 2 + \ln 3} \approx -0.1276$

16. $z = \frac{-3\ln 3}{\ln 3 - \ln 2} \approx -8.1285$

17. $x = \frac{(\ln 5)}{2} \approx 0.805$

19. $x = \frac{(-\ln 3.5)}{1.4} \approx -0.895$

20. $x = -3\ln\left(\frac{5.6}{3.4}\right) \approx -1.4970$

21. $x = \frac{2\ln\left(\frac{5}{2.1}\right)}{\ln 3} \approx 1.579$

22. $x = \frac{3\ln\left(\frac{14}{7.8}\right)}{\ln 5} \approx 1.0903$

Solution

$$\log(x-16) = 2 - \log(x-1)$$
$$\log(x-16) + \log(x-1) = 2$$
$$\log[(x-16)(x-1)] = 2$$
$$\log(x^2 - 17x + 16) = 2$$
$$10^{\log(x^2-17x+16)} = 10^2$$
$$x^2 - 17x + 16 = 100$$
$$x^2 - 17x - 84 = 0$$
$$(x+4)(x-21) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 21 = 0$$
$$x = -4 \qquad x = 21$$

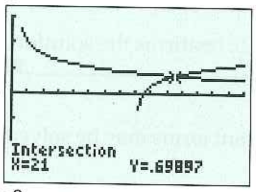


Figure 5.6-12

Because $\log(x - 16)$ and $\log(x - 1)$ are not defined for $x = -4$, it cannot be a solution. Therefore, the only solution is $x = 21$. The intersection of the graphs of $Y_1 = \log(x - 16)$ and $Y_2 = 2 - \log(x - 1)$, shown in Figure 5.6-12, confirms the solution.

1-5, 7-18, 35-42, 53-62

Exercises 5.6

In Exercises 1–8, solve the equation without using logarithms.

1. $3^x = 81$ $x = 4$
2. $3^x + 3 = 30$ $x = 3$
3. $3^{x+1} = 9^{5x}$ $x = \frac{1}{9}$
4. $4^{5x} = 16^{2x-1}$ $x = -2$
5. $3^{5x}9^{x^2} = 27$ $x = \frac{1}{2}$ or -3
6. $2^{x^2+5x} = \frac{1}{16}$ $x = -1$ or -4
7. $9^{x^2} = 3^{-5x-2}$ $x = -2$ or $-\frac{1}{2}$
8. $4^{x^2-1} = 8^x$ $x = -\frac{1}{2}$ or 2

In Exercises 9–29, solve the equation. Give exact answers (in terms of natural logarithms). Then use a calculator to find an approximate answer.

9. $3^x = 5$
10. $5^x = 4$ $x = \frac{\ln 4}{\ln 5} \approx 0.8614$
11. $2^x = 3^{x-1}$
12. $4^{x+2} = 2^{x-1}$ $x = \frac{-5\ln 2}{\ln 2} = -5$
13. $3^{1-2x} = 5^{x+5}$
14. $4^{3x-1} = 3^{x-2}$
15. $2^{1-3x} = 3^{x+1}$
16. $3^{z+3} = 2^z$
17. $e^{2x} = 5$
18. $e^{-3x} = 2$ $x = \frac{\ln 2}{-3} \approx -0.2310$
19. $6e^{-1.4x} = 21$
20. $3.4e^{-\frac{x}{3}} = 5.6$
21. $2.1e^{\frac{x}{2}\ln 3} = 5$
22. $7.8e^{\frac{x}{3}\ln 5} = 14$
23. $9^x - 4 \cdot 3^x + 3 = 0$ *Hint:* Note that $9^x = (3^x)^2$; let $u = 3^x$. $x = 0$ or 1
24. $4^x - 6 \cdot 2^x = -8$ $x = 2$ or 1
25. $e^{2x} - 5e^x + 6 = 0$ *Hint:* Let $u = e^x$.
26. $2e^{2x} - 9e^x + 4 = 0$
27. $6e^{2x} - 16e^x = 6$
28. $8e^{2x} + 8e^x = 6$
29. $4^x + 6 \cdot 4^{-x} = 5$

In Exercises 30–32, solve the equation for x.

30. $\frac{e^x + e^{-x}}{e^x - e^{-x}} = t$
31. $\frac{e^x - e^{-x}}{2} = t$
32. $\frac{e^x - e^{-x}}{e^x + e^{-x}} = t$

33. Prove that if $\ln u = \ln v$, then $u = v$. *Hint:* Use the basic property of inverses $e^{\ln v} = v$.

34. **a.** Solve $7^x = 3$ using natural logarithms. Give an exact answer, not an approximation.
 b. Solve $7^x = 3$ using common logarithms. Give an exact answer, not an approximation.
 c. Use the change-of-base formula in *Excursion* 5.5.A to show that your answers in parts a and b are the same.

In Exercises 35–44, solve the equation. (See Example 9.)

35. $\ln(3x - 5) = \ln 11 + \ln 2$ $x = 9$
36. $\log(4x - 1) = \log(x + 1) + \log 2$ $x = \frac{3}{2}$

25. $x = \ln 2 \approx 0.693$ or $x = \ln 3 \approx 1.099$

26. $x = \ln\left(\frac{1}{2}\right) \approx -0.693$ or $x = \ln 4 \approx 1.386$

27. $x = \ln 3 \approx 1.099$

28. $x = \ln\left(\frac{1}{2}\right) \approx -0.693$

29. $x = \frac{\ln 2}{\ln 4} = \frac{1}{2}$ or $x = \frac{\ln 3}{\ln 4} \approx 0.792$

30. $x = \ln\sqrt{\frac{1+t}{t-1}}$

31. $x = \ln(t + \sqrt{t^2 + 1})$

32. $x = \ln\sqrt{\frac{t+1}{1-t}}$

33. If $\ln u = \ln v$, then $e^{\ln u} = e^{\ln v}$, so $u = v$.

34. **a.** $x = \frac{\ln 3}{\ln 7}$
 b. $x = \frac{\log 3}{\log 7}$
 c. $x = \frac{\ln 3}{\ln 7}$

7. $\log(3x - 1) + \log 2 = \log 4 + \log(x + 2)$
$x = 5$

8. $\ln(x + 6) - \ln 10 = \ln(x - 1) - \ln 2$ $x = \frac{11}{4}$

9. $2 \ln x = \ln 36$
$x = 6$

40. $2 \log x = 3 \log 4$
$x = 8$

1. $\ln x + \ln(x + 1) = \ln 3 + \ln 4$
$x = 3$

2. $\ln(6x - 1) + \ln x = \frac{1}{2} \ln 4$ $x = \frac{2}{3}$

3. $\ln x = \ln 3 - \ln(x + 5)$ $x = \frac{-5 + \sqrt{37}}{2}$

4. $\ln(2x + 3) + \ln x = \ln e$ $x = \frac{-3 + \sqrt{9 + 8e}}{4}$

n Exercises 45–52, solve the equation.

5. $\ln(x + 9) - \ln x = 1$ $x = \frac{9}{(e - 1)}$

6. $\ln(2x + 1) - 1 = \ln(x - 2)$ $x = \frac{2e + 1}{e - 2}$

7. $\log x + \log(x - 3) = 1$ $x = 5$

8. $\log(x - 1) + \log(x + 2) = 1$ $x = 3$

9. $\log \sqrt{x^2 - 1} = 2$
$x = \pm\sqrt{10001}$

50. $\log \sqrt[3]{x^2 + 21x} = \frac{2}{3}$
$x = -25$ or 4

1. $\ln(x^2 + 1) - \ln(x - 1) = 1 + \ln(x + 1)$
$x = \sqrt{\frac{e + 1}{e - 1}}$

2. $\frac{\ln(x + 1)}{\ln(x - 1)} = 2$
$x = 3$

xercises 53–62 deal with the half-life function $M(x) = c(0.5)^{\frac{x}{h}}$, which was discussed in Section 5.3 and sed in Example 5 of this section.

3. How old is a piece of ivory that has lost 36% of its carbon-14?
≈3689 yr old

4. How old is a mummy that has lost 49% of its carbon-14?
≈5566 yr old

5. Find when part of the Pueblo Benito ruins was built if the doorway timbers have 89.14% of their original carbon-14. (See the image on the first page of this chapter.)
≈950.35 yr old

6. How old is a wooden statue that has only one-third of its original carbon-14?
≈9082 yr old

7. A quantity of uranium decays to two-thirds of its original mass in 0.26 billion years. Find the half-life of uranium.
≈444,000,000 yr

8. A certain radioactive substance loses one-third of its original mass in 5 days. Find its half-life.
≈8.55 days

decimal

59. Krypton-85 loses 6.44% of its mass each year. What is its half-life?
≈10.413 yr

60. Strontium-90 loses 2.5% of its mass each year. What is its half-life?
≈27.38 yr

61. The half-life of a certain substance is 3.6 days. How long will it take for 20 grams to decay to 3 grams?
≈9.853 days

62. The half-life of cobalt-60 is 4.945 years. How long will it take for 25 grams to decay to 15 grams?
≈3.64 yr

Exercises 63–68 deal with the compound interest formula $A = P(1 + r)^t$, which was discussed in Section 5.3 and used in Example 6 of this section.

63. At what annual rate of interest should $1000 be invested so that it will double in 10 years, if interest is compounded quarterly?
≈6.99%

64. Find how long it takes $500 to triple if it is invested at 6% in each compounding period.
a. annually ≈18.85 yr **b.** quarterly ≈18.45 yr **c.** daily ≈18.31 yr

65. **a.** How long will it take to triple your money if you invest $500 at a rate of 5% per year compounded annually? ≈22.5 yr
b. How long will it take at 5% compounded quarterly?
≈22.1 yr

66. At what rate of interest compounded annually should you invest $500 if you want to have $1500 in 12 years?
≈9.587%

67. How much money should be invested at 5% interest compounded quarterly so that 9 years later the investment will be worth $5000? This answer is called the present value of $5000 at 5% interest.
≈$3197.05

68. Find a formula that gives the time needed for an investment of P dollars to double, if the interest rate is r% compounded annually. *Hint:* Solve the compound interest formula for t, when $A = 2P$.

Exercises 69–76 deal with functions of the form $f(x) = Pe^{kx}$, where k is the continuous exponential growth rate. See Example 7.

69. The present concentration of carbon dioxide in the atmosphere is 364 parts per million (ppm) and is increasing exponentially at a continuous yearly rate of 0.4% (that is, $k = 0.004$). How many years will it take for the concentration to reach 500 ppm?
≈79.36 yr

70. The amount P of ozone in the atmosphere is currently decaying exponentially each year at a
≈277.26 yr

68. $t = \frac{\ln 2}{\ln\left(1 + \frac{r}{100}\right)}$

78. a. ≈9.55 years

b. No; $e^{-0.5544t}$ is always greater than 0, so $1 + 199e^{-0.5544t}$ is always greater than 1 for any value of t. Therefore, $\frac{2000}{1 + 199e^{-0.5544t}}$ is always less than 2000.

79. a. $k \approx 0.229$, $c \approx 83.3$

b. ≈12.43 weeks

80. a. in the 13th year; company A; company B

b. in the 31st year; company A; company B

continuous rate of $\frac{1}{4}\%$ (that is, $k = -0.0025$). How long will it take for half the ozone to disappear $\left(\text{that is, when will the amount be } \frac{P}{2}\right)$? Your answer is the half-life of ozone.

71. The population of Brazil increased exponentially from 151 million in 1990 to 173 million in 2000.

a. At what continuous rate was the population growing during this period? **≈1.3601%**

b. Assuming that Brazil's population continues to increase at this rate, when will it reach 250 million? **in 2027**

72. Outstanding consumer debt increased exponentially from \$781.5 billion in 1990 to \$1765.5 billion in 2002. (Source: Federal Reserve Bulletin)

a. At what continuous rate is consumer debt growing? **≈6.79%**

b. Assuming this rate continues, when will consumer debt reach \$2500 billion? **in 2007**

73. The probability P percent of having an accident while driving a car is related to the alcohol level of the driver's blood by the formula $P = e^{kt}$, where k is a constant. Accident statistics show that the probability of an accident is 25% when the blood alcohol level is $t = 0.15$.

a. Find k. Use $P = 25$, not 0.25. **$k \approx 21.459$**

b. At what blood alcohol level is the probability of having an accident 50%? **$t \approx 0.182$**

74. Under normal conditions, the atmospheric pressure (in millibars) at height h feet above sea level is given by $P(h) = 1015e^{-kh}$, where k is a positive constant.

a. If the pressure at 18,000 feet is half the pressure at sea level, find k. **$k \approx 0.0000385$**

b. Using the information from part **a**, find the atmospheric pressure at 1000 feet, 5000 feet, and 15,000 feet. **≈976.7; ≈837.3; ≈569.7**

75. One hour after an experiment begins, the number of bacteria in a culture is 100. An hour later there are 500.

a. Find the number of bacteria at the beginning of the experiment and the number 3 hours later. **20; 2500**

b. How long does it take the number of bacteria at any given time to double? **$\frac{\ln 2}{\ln 5} \approx 0.43$ hr**

76. If the population at time t is given by $S(t) = ce^{kt}$, find a formula that gives the time it takes for the population to double. **$t = \frac{\ln 2}{k}$**

77. The spread of a flu virus in a community of 45,000 people is given by the function

$$f(t) = \frac{45{,}000}{1 + 224e^{-0.889t}},$$

where $f(t)$ is the number of people infected in week t.

a. How many people had the flu at the outbreak of the epidemic? after 3 weeks? **200; ≈2718**

b. When will half the town be infected? **≈6.09 weeks**

78. The beaver population near a certain lake in year t is approximately $p(t) = \frac{2000}{1 + 199e^{-0.5544t}}$.

a. When will the beaver population reach 1000?

b. Will the population ever reach 2000? Why?

79. *Critical Thinking* According to one theory of learning, the number of words per minute N that a person can type after t weeks of practice is given by $N = c(1 - e^{-kt})$, where c is an upper limit that N cannot exceed and k is a constant that must be determined experimentally for each person.

a. If a person can type 50 wpm (words per minute) after 4 weeks of practice and 70 wpm after 8 weeks, find the values of k and c for this person. According to the theory, this person will never type faster than c wpm.

b. Another person can type 50 wpm after 4 weeks of practice and 90 wpm after 8 weeks. How many weeks must this person practice to be able to type 125 wpm?

80. *Critical Thinking* Wendy has been offered two jobs, each with the same starting salary of \$24,000 and identical benefits. Assuming satisfactory performance, she will receive a \$1200 raise each year at the company A, whereas the company B will give her a 4% raise each year.

a. In what year (after the first year) would her salary be the same at either company? Until then, which company pays better? After that, which company pays better?

b. Answer the questions in part **a** assuming that the annual raise at company A is \$1800.