In Exercises 41–44, state the magnitude on the Richter scale of an earthquake that satisfies the given condition.

41. 100 times stronger than the zero quake
2

42. $10^{4.7}$ times stronger than the zero quake
4.7

43. 350 times stronger than the zero quake
≈2.54

44. 2500 times stronger than the zero quake
≈3.4

Exercises 45–48 deal with the energy intensity i of a sound, which is related to the loudness of the sound by the function $L(i) = 10 \cdot \log\left(\frac{i}{i_0}\right)$, where i_0 is the minimum intensity detectable by the human ear and $L(i)$ is measured in decibels. Find the decibel measure of the sound.

45. ticking watch (intensity is 100 times i_0)
20 decibels

46. soft music (intensity is 10,000 times i_0)
40 decibels

47. loud conversation (intensity is 4 million times i_0)
≈66 decibels

48. Victoria Falls in Africa (intensity is 10 billion times i_0)
100 decibels

49. How much louder is the sound in Exercise 46 than the sound in Exercise 45?
100 times

50. The perceived loudness L of a sound of intensity I is given by $L = k \cdot \ln I$, where k is a certain constant. By how much must the intensity be increased to double the loudness? (That is, what must be done to I to produce $2L$?)
***I* must be squared to double *L*.**

51. Compute each of the following pairs of numbers.
a. $\log 18$ and $\frac{\ln 18}{\ln 10}$ **b.** $\log 8950$ and $\frac{\ln 8950}{\ln 10}$
c. What do the results in parts **a** and **b** suggest?

52. Find each of the following logarithms.
a. $\log 8.753$ **b.** $\log 87.53$ **c.** $\log 875.3$
d. $\log 8753$ **e.** $\log 87{,}530$
f. How are the numbers 8.753, 87.53, 875.3, 8753, and 87,530 related to one another? How are their logarithms related? State a general conclusion that this evidence suggests.

5.5.A *Excursion:* Logarithmic Functions to Other Bases

Objectives

- Evaluate logarithms to any base with and without a calculator
- Solve exponential and logarithmic equations to any base by using an equivalent equation
- Identify transformations of logarithmic functions to any base
- Use properties and laws of logarithms to simplify and evaluate logarithmic expressions to any base

Common and natural logarithms were defined by considering the inverse functions of the exponential functions $f(x) = 10^x$ and $f(x) = e^x$. In this section, you will see that a similar procedure can be carried out with any positive number b in place of 10 and e.

NOTE In the discussion below, b is a fixed positive number with $b > 1$. The discussion on exponents and logarithms to base b is also valid for $0 < b < 1$, but in that case the graphs have a different shape.

Defining Logarithmic Functions to Other Bases

Because $f(x) = b^x$ is an increasing function, it is a one-to-one function and therefore has an inverse function. (See Section 3.6) Recall that the graphs of inverse functions are reflections of one another across the line $y = x$. An exponential function $f(x) = b^x$ and its inverse function are graphed in Figure 5.5.A-1.

---

51. a. ≈1.2553
b. ≈3.9518
c. $\log x = \frac{\ln x}{\ln 10}$

52. a. ≈0.9422
b. ≈1.9422
c. ≈2.9422
d. ≈3.9422
e. ≈4.9422
f. The sequence of numbers increases by a factor of 10. The logarithms increase by 1; on a logarithmic scale, each 1 unit increase corresponds to an increase by a factor of 10.

Section 5.5.A *Excursion:* Logarithmic Functions to Other Bases

Math Background

The functions $f(x) = b^x$ and $g(x) = \log_b x$ are inverses. Recall the method of finding an inverse (page 207). To find the inverse of the function $f(x) = b^x$, rewrite the function using y, exchange the x and y variables, and then solve the equation for y:

$$f(x) = b^x$$
$$y = b^x$$
$$x = b^y$$
$$y = \log_b x$$
$$f^{-1}(x) = \log_b x$$

The equation $y = \log_b x$ above is where we "solve the equation for y." It might not seem like "solving" in the customary sense. We can think of it as giving a name to the inverse function. That is, we replace $x = b^y$ with its equivalent equation $y = \log_b x$.

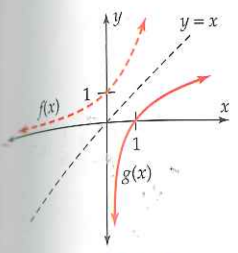


Figure 5.5.A-1

This inverse function g is called the **logarithmic function to the base b.** The value of $g(x)$ at the number x is denoted $\log_b x$ and is called the **logarithm to the base b** of the number x.

Because the functions $f(x) = b^x$ and $g(x) = \log_b x$ are inverse functions,

$$\log_b v = u \quad \text{if and only if} \quad b^u = v.$$

Because all logarithms are exponents, every statement about logarithms is equivalent to a statement about exponents.

Logarithmic statement $\log_b v = u$	Equivalent exponential statement $b^u = v$
$\log_3 81 = 4$	$3^4 = 81$
$\log_4 64 = 3$	$4^3 = 64$
$\log_{125} 5 = \frac{1}{3}$	$125^{\frac{1}{3}} = 5$
$\log_8\left(\frac{1}{4}\right) = -\frac{2}{3}$	$8^{-\frac{2}{3}} = \frac{1}{4}$

Example 1 **Evaluating Logarithms to Other Bases**

Without using a calculator, find each value.

a. $\log_2 16$ **b.** $\log_{\frac{1}{3}} 9$ **c.** $\log_5(-25)$

Solution

a. If $\log_2 16 = x$, then $2^x = 16$. Because $2^4 = 16$, $\log_2 16 = 4$.

b. If $\log_{\frac{1}{3}} 9 = x$, then $\left(\frac{1}{3}\right)^x = 9$. Because $\left(\frac{1}{3}\right)^{-2} = 9$, $\log_{\frac{1}{3}} 9 = -2$

c. If $\log_5(-25) = x$, then $5^x = -25$. Because there is no real number exponent of 5 that produces a negative number, $\log_5(-25)$ is not defined.

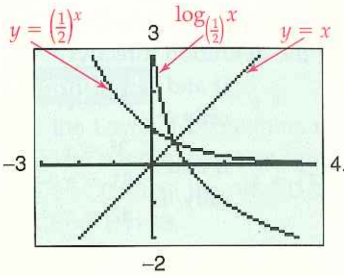
Example 2 **Solving Logarithmic Equations**

Solve each equation for x.

a. $\log_5 x = 3$ **b.** $\log_6 1 = x$ **c.** $\log_{\frac{1}{6}}(-3) = x$ **d.** $\log_6 6 = x$

Math Background

The **NOTE** mentions that graphs of logarithmic functions with bases between 0 and 1 have shapes that are different than those shown in the text. This is because graphs of exponential functions with bases between 0 and 1 have different shapes than graphs of exponential functions with bases greater than 1 (see page 336). The graphs of $y = \left(\frac{1}{2}\right)^x$ and its inverse, $y = \log_{\left(\frac{1}{2}\right)} x$ are shown below.



Teaching Notes

To prepare students for the definition of a **logarithm to the base b,** build on the two bases they are familiar with. Show the following alternate ways to write $\log x$ and $\ln x$: $\ln x = \log_e x$ and $\log x = \log_{10} x$. Then say that any positive number b other than 1 can be used as a base.

For the last row of the table, ask students to show why $8^{-\frac{2}{3}} = \frac{1}{4}$

$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$$

Example Notes

Remind students that a logarithm is an exponent.

In **Example 1a,** they can ask themselves, "What exponent must be applied to the base 2 to produce 16?"

In **Example 2a,** they can ask themselves, "What do I get when I apply the exponent 3 to the base 5?"

ADDITIONAL EXAMPLES

Example 1

Without using a calculator, find each value.

a. $\log_3 27$ 3

b. $\log_{\frac{1}{2}} 32$ −5

c. $\log_4(-64)$ undefined

Teaching Notes

For the **Basic Properties of Logarithms** box, have students review the corresponding box on page 364. They should see that the information is identical in both boxes. Ask them which Examples just completed illustrate property 1. Examples 1c and 2c

For the **Laws of Logarithms** box, have students review the corresponding boxes on pages 365–366.

Example Notes

Example 3 can also be solved by applying the definition directly:

$\log_b v$	$= u$	if and only if	$b^u = v$
$\log_3(x - 1) = 4$		if and only if	$3^4 = x - 1$ $81 = x - 1$ $82 = x$

Estimation can be helpful in checking the reasonableness of answers in **Example 4**. We know that $\log_7 1 = 0$, $\log_7 7 = 1$ and $\log_7 49 = 2$. So, *estimates* of the answers are:

4a slightly greater than 1
4b between 0 and 1
4c slightly less than 2

Encourage students to check actual answers as follows:

For **Example 4a,** $\log_7 10 = 1.1833$. Therefore, $7^{1.1833}$ should equal 10.
$7^{1.1833} \approx 10.0001$ ✓

For **Example 4b,** $\log_7 2.5 = 0.4709$. Therefore, $7^{0.4709}$ should equal 2.5.
$7^{0.4709} \approx 2.5001$ ✓

For **Example 4c,** $\log_7 48 = 1.9894$. Therefore, $7^{1.9894}$ should equal 48.
$7^{1.9894} \approx 47.9996$ ✓

Solution

a. If $\log_5 x = 3$, then $5^3 = x$. Therefore, $x = 125$.

b. If $\log_6 1 = x$, then $6^x = 1$. Therefore, $x = 0$.

c. If $\log_{\frac{1}{6}}(-3) = x$, then $\left(\frac{1}{6}\right)^x = -3$. Because no real power of $\frac{1}{6}$ is a negative number, $\log_{\frac{1}{6}}(-3) = x$ has no real solution.

d. If $\log_6 6 = x$, then $6^x = 6$. Therefore, $x = 1$.

Basic Properties of Logarithms to Other Bases

Logarithms are only defined for positive real numbers. That is,

$$\log_b v \text{ is defined only when } v > 0.$$

The graph of $y = \log_b x$ contains the point (1, 0) because $b^0 = 1$ for all $b > 0$. That is,

$$\log_b 1 = 0$$

The value of $\log_5 5^4$ can be found by writing an equivalent exponential statement.

$$\text{If } \log_5 5^4 = x, \text{ then } 5^x = 5^4. \text{ So } x = 4.$$

In general,

$$\log_b b^k = k \quad \text{for every real number } k.$$

By definition, $\log_3 104$ is the exponent to which 3 must be raised to produce 104. Therefore,

$$3^{\log_3 104} = 104.$$

In general,

$$b^{\log_b v} = v \text{ for every } v > 0.$$

The facts presented above are summarized in the table below.

Basic Properties of Logarithms

For $b > 0$ and $b \neq 1$,

1. **$\log_b v$ is defined only when $v > 0$**
2. **$\log_b 1 = 0$ and $\log_b b = 1$**
3. **$\log_b b^k = k$ for every real number k**
4. **$b^{\log_b v} = v$ for every $v > 0$**

Properties 3 and 4 are restatements of the fact that the composition of inverse functions produces the identity function.

If $f(x) = b^x$ and $g(x) = \log_b x$, then

$$(f \circ g)(x) = f(\log_b x) = b^{\log_b x} = x \text{ for all } x > 0$$
$$(g \circ f)(x) = g(b^x) = \log_b b^x = x \text{ for all } x$$

Equations that involve both logarithmic and constant terms may be solved by using basic properties of logarithms.

$$b^{\log_b v} = v \text{ for } b > 0 \text{ and } b \neq 1$$

Example 3 **Solving Logarithmic Equations**

Solve the equation $\log_3(x - 1) = 4$.

Solution

$$\log_3(x-1) = 4$$
$$3^{\log_3(x-1)} = 3^4 \qquad \textit{exponentiate both sides}$$
$$x - 1 = 3^4 \qquad b^{\log_b v} = v$$
$$x = 82$$

Laws of Logarithms to Other Bases

Because all logarithms are a form of exponents, the laws of exponents translate to the corresponding laws of logarithms to any base.

Laws of Logarithms

For all b, v, w, and k, with b, v, and w positive and $b \neq 1$:

Product Law: $\log_b(vw) = \log_b v + \log_b w$

Quotient Law: $\log_b\left(\frac{v}{w}\right) = \log_b v - \log_b w$

Power Law: $\log_b(v^k) = k \log_b v$

Example 4 **Using the Laws of Logarithms**

Use the Laws of Logarithms to evaluate each expression, given that $\log_7 2 = 0.3562$, $\log_7 3 = 0.5646$, and $\log_7 5 = 0.8271$.

a. $\log_7 10$ **b.** $\log_7 2.5$ **c.** $\log_7 48$

Solution

a. Use the Product Law.

$$\log_7 10 = \log_7(2 \cdot 5) = \log_7 2 + \log_7 5 = 0.3562 + 0.8271 = 1.1833$$

ADDITIONAL EXAMPLES

Example 2

Solve each equation for x.

a. $\log_6 x = 2$ $x = 36$

b. $\log_4 1 = x$ $x = 0$

c. $\log_{\frac{1}{5}}(-10) = x$ no real solution

d. $\log_7 7 = x$ $x = 1$

Example 3

Solve the equation $\log_5(x + 2) = 2$.

$$\log_5(x+2) = 2$$
$$5^{\log_5(x+2)} = 5^2$$
$$x + 2 = 25$$
$$x = 23$$

Example 4

Use the Laws of Logarithms to evaluate each expression, given that $\log_5 3 = 0.6826$, $\log_5 4 = 0.8614$, and $\log_5 6 = 1.1133$.

a. $\log_5 18$

$$\log_5 18 = \log_5(3 \cdot 6) = \log_5 3 + \log_5 6 = 0.6826 + 1.1133 = 1.7959$$

b. $\log_5 0.75$

$$\log_5 0.75 = \log_5\left(\frac{3}{4}\right) = \log_5 3 - \log_5 4 = 0.6826 - 0.8614 = -0.1788$$

c. $\log_5 54$

$$\log_5 54 = \log_5(6 \cdot 3^2) = \log_5 6 + \log_5 3^2 = \log_5 6 + 2\log_5 3 = 1.1133 + 2(0.6826) = 2.4785$$

Teaching Notes

Since calculators only have common and natural logarithms, students need to use the **Change-of-Base Formula** to evaluate logarithms to other bases with the calculator.

Math Background

The following alternate proof of the **Change-of-Base Formula** uses the same concept as the proof in the text, but slightly different notation:

Let $\log_b v = w$.

Then:
$$b^w = v$$
$$\log b^w = \log v$$
$$w \log b = \log v$$
$$w = \frac{\log v}{\log b}$$

Therefore, $\log_b v = \frac{\log v}{\log b}$.

ADDITIONAL EXAMPLES

Example 5

Simplify and write each expression as a single logarithm.

a. $\log_8(x+5) + \log_8 z - \log_8(x^2 + 10x + 25)$ $\log_8 \frac{z}{x+5}$

b. $2 + \log_3\left(\frac{x}{9}\right)$ $\log_3 x$

Example 6

Evaluate $\log_5 7$. 1.2091

COMMON ERROR ALERT

Students may mistakenly use the reciprocal in the **Change-of-Base Formula.** To help students remember the Change-of-Base Formula, note the locations of the variables. In both $\log_b v$ and $\frac{\log v}{\log b}$, b is the "lower" variable and v is the "upper" variable.

They can also check the reasonableness of their result. In **Example 6,** $\log_8 9$ must be greater than 1 because $8^1 = 8$. If a student mistakenly uses $\frac{\log 8}{\log 9}$, the result will be less than 1.

---

b. Use the Quotient Law.

$$\log_7 2.5 = \log_7\left(\frac{5}{2}\right) = \log_7 5 - \log_7 2 = 0.8271 - 0.3562 = 0.4709$$

c. Use the Product and Power Laws.

$$\begin{aligned}\log_7 48 &= \log_7(3 \cdot 2^4)\\ &= \log_7 3 + \log_7 2^4\\ &= \log_7 3 + 4\log_7 2\\ &= 0.5646 + 4(0.3562)\\ &= 1.9894\end{aligned}$$

Example 5 Using the Laws of Logarithms

Simplify and write each expression as a single logarithm.

a. $\log_3(x+2) + \log_3 y - \log_3(x^2 - 4)$

b. $3 - \log_5(125x)$

Solution

a.
$$\begin{aligned}\log_3(x+2) + \log_3 y - \log_3(x^2-4) &= \log_3[(x+2)y] - \log_3(x^2-4)\\ &= \log_3\left[\frac{(x+2)y}{x^2-4}\right]\\ &= \log_3\left[\frac{(x+2)y}{(x+2)(x-2)}\right]\\ &= \log_3 \frac{y}{x-2}\end{aligned}$$

b.
$$\begin{aligned}3 - \log_5(125x) &= 3 - (\log_5 125 + \log_5 x)\\ &= 3 - 3 - \log_5 x\\ &= -\log_5 x\end{aligned}$$

NOTE $-\log_5 x$ can also be expressed as $\log_5 x^{-1}$ or $\log_5\left(\frac{1}{x}\right)$.

Change-of-Base Formula

Scientific and graphing calculators have a LOG key and a LN key for calculating logarithms. No calculators have a key for logarithms to other bases. One way to evaluate logarithms to other bases is to use the formula below.

Change-of-Base Formula

For any positive number v,

$$\log_b v = \frac{\log v}{\log b} \quad \text{and} \quad \log_b v = \frac{\ln v}{\ln b}$$

Proof By Property 4 of the Basic Properties of Logarithms $b^{\log_b v} = v$.

$$b^{\log_b v} = v$$
$$\ln(b^{\log_b v}) = \ln v \quad \textit{take logarithms of both sides}$$
$$\log_b v(\ln b) = \ln v \quad \textit{apply the Power Law}$$
$$\log_b v = \frac{\ln v}{\ln b}$$

A similar argument can be made by taking common logarithms of both sides.

Example 6 Evaluating Logarithms to Other Bases

Evaluate $\log_8 9$.

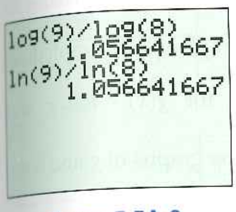


Figure 5.5A-2

Solution

Use the change-of-base formula and a calculator.

$$\log_8 9 = \frac{\log 9}{\log 8} = 1.0566 \quad \text{or} \quad \log_8 9 = \frac{\ln 9}{\ln 8} = 1.0566$$

Graphing Logarithmic Functions to Other Bases

The graph of a logarithmic function to any base b shares characteristics with the graphs of natural logarithms and common logarithms. The following table compares the graphs of exponential and logarithmic functions for base b, where b is any real number, $b > 0$, and $b \neq 1$.

	Exponential function $f(x) = b^x$	Logarithmic function $g(x) = \log_b x$
Domain	all real numbers	all positive real numbers
Range	all positive real numbers	all real numbers
	$f(x)$ increases as x increses	$g(x)$ increases as x increases
	$f(x)$ approaches the x-axis as x decreases	$g(x)$ approaches the y-axis as x approaches 0
Reference points	$\left(-1, \frac{1}{b}\right)$, (0, 1), (1, b)	$\left(\frac{1}{b}, -1\right)$, (1, 0), ($b$, 1)

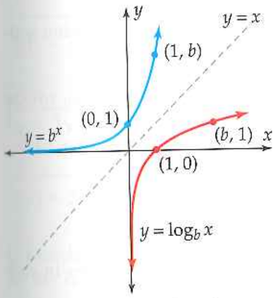


Figure 5.5A-3

Example 7 Transforming Logarithmic Functions

Describe the transformation from $g(x) = \log_2 x$ to $h(x) = \log_2(x + 1) - 3$. Give the domain and range of h.

Teaching Notes

Students will better understand **Graphing Logarithmic Functions to Other Bases** after some paper-and-pencil graphing. Although all logarithmic functions with base $b > 1$ have the basic shape shown in Figure 5.5A-3, they vary in "steepness." To help students understand this, have them graph $y = \log_2 x$ and $y = \log_3 x$ as follows:

First, have them copy and complete the table below:

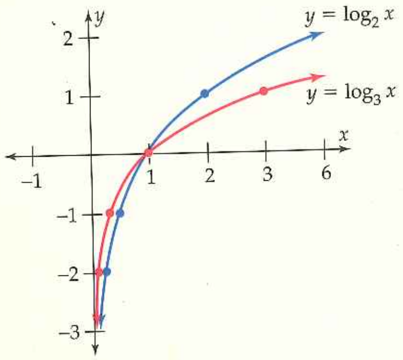
y	−2	−1	0	1
2^y	$\frac{1}{4}$	$\frac{1}{2}$	1	2
3^y	$\frac{1}{9}$	$\frac{1}{3}$	1	3

Next, have them use the values from above to copy and complete the following tables:

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2
$y = \log_2 x$	−2	−1	0	1

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3
$y = \log_3 x$	−2	−1	0	1

Finally, have them graph $y = \log_2 x$ and $y = \log_3 x$ on the same coordinate system:



Notice that both graphs have the same basic shape, but the graph corresponding to the larger base stays closer to the x-axis everywhere except at (1, 0).

ADDITIONAL EXAMPLES

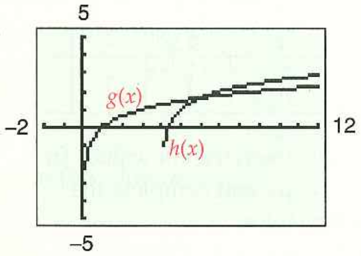
Example 7

Describe the transformation from $g(x) = \log_3 x$ to $h(x) = \log_3(x - 4) + 1$. Give the domain and range of h.

The graph of $h(x)$ is the graph of $g(x)$ after a horizontal translation of 4 units to the right and a vertical translation of 1 unit up.

Domain of h: all real numbers greater than 4

Range of h: all real numbers



Exercises 5.5.A

ANSWERS

1. $\log 0.01 = -2$
2. $\log 1000 = 3$
3. $\log \sqrt[3]{10} = \frac{1}{3}$
4. $\log 3 \approx 0.4771$
5. $\log r = 7k$
6. $\log c = a + b$
7. $\log_7 5{,}764{,}801 = 8$
8. $\log_2\left(\frac{1}{8}\right) = -3$
9. $\log_3\left(\frac{1}{9}\right) = -2$
10. $\log_b 3379 = 14$

Solution

Because $h(x) = g(x + 1) - 3$, its graph is the graph of $g(x) = \log_2 x$ after a horizontal translation of 1 unit to the left and a vertical translation of 3 units down.

Domain of h: The domain of $g(x) = \log_2 x$ is all positive real numbers. The horizontal translation of 1 unit to the left changes the domain to all real numbers greater than −1.

Range of h: The range of $g(x) = \log_2 x$ is all real numbers, so the vertical translation has no effect on the range.

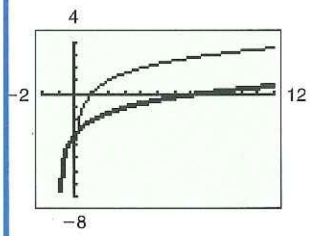


Figure 5.5A-4

The points $\left(\frac{1}{2}, -1\right)$, $(1, 0)$, and $(2, 1)$ on the graph of g are translated to the points $\left(-\frac{1}{2}, -4\right)$, $(0, -3)$, and $(1, -2)$ on the graph of h. To graph these functions with a calculator, graph $Y_1 = \frac{\ln x}{\ln 2}$ for $g(x) = \log_2 x$ and $Y_2 = \frac{\ln(x + 1)}{\ln 2} - 3$ for $h(x) = \log_2(x + 1) - 3$. The graphs of g and h are shown in Figure 5.5A-4.

Exercises 5.5.A

Note: Unless stated otherwise, all letters represent positive numbers and $b \neq 1$.

In Exercises 1–10, translate the given exponential statement into an equivalent logarithmic statement.

1. $10^{-2} = 0.01$
2. $10^3 = 1000$
3. $\sqrt[3]{10} = 10^{\frac{1}{3}}$
4. $10^{0.4771} \approx 3$
5. $10^{7k} = r$
6. $10^{(a+b)} = c$
7. $7^8 = 5{,}764{,}801$
8. $2^{-3} = \frac{1}{8}$
9. $3^{-2} = \frac{1}{9}$
10. $b^{14} = 3379$

In Exercises 11–20, translate the given logarithmic statement into an equivalent exponential statement.

11. $\log 10{,}000 = 4$
    $10^4 = 10{,}000$
12. $\log 0.001 = -3$
    $10^{-3} = 0.001$
13. $\log 750 = 2.8751$
    $10^{2.8751} \approx 750$
14. $\log 0.8 = -0.0969$
    $10^{-0.0969} \approx 0.8$
15. $\log_5 125 = 3$
    $5^3 = 125$
16. $\log_8\left(\frac{1}{4}\right) = -\frac{2}{3}$
    $8^{-\frac{2}{3}} = \frac{1}{4}$
17. $\log_2\left(\frac{1}{4}\right) = -2$
    $2^{-2} = \frac{1}{4}$
18. $\log_2 \sqrt{2} = \frac{1}{2}$
    $2^{\frac{1}{2}} = \sqrt{2}$
19. $\log(x^2 + 2y) = z + w$
    $10^{z+w} = x^2 + 2y$
20. $\log(a + c) = d$
    $10^d = a + c$

In Exercises 21–28, evaluate the given expression without using a calculator.

21. $\log 10^{\sqrt{43}}$
    $\sqrt{43}$
22. $\log_{17}(17^{17})$
    17
23. $\log 10^{\sqrt{x^2+y^2}}$
    $\sqrt{x^2 + y^2}$
24. $\log_{3.5}(3.5^{(x^2-1)})$
    $x^2 - 1$
25. $\log_{16} 4$ $\frac{1}{2}$
26. $\log_2 64$
    6
27. $\log_{\sqrt{3}}(27)$
    6
28. $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$
    −4

In Exercises 29–36, find the missing entries in each table.

29.

x	0	1	2	4
$f(x) = \log_4 x$	Not defined	0	0.5	1

30.

x	$\frac{1}{25}$	5	25	$\sqrt{5}$
$g(x) = \log_5 x$	−2	1	2	$\frac{1}{2}$

31.

x	?	$\frac{1}{6}$	1	216
$h(x) = \log_6 x$	−2	−1	0	3

32.

x	$\frac{10}{3}$	4	6	12
$k(x) = \log_3(x-3)$	−1	0	1	2

33.

x	0	$\frac{1}{7}$	$\sqrt{7}$	49
$f(x) = 2\log_7 x$	Not defined	−2	1	4

34.

x	?	?	100	1000
$g(x) = 3\log x$	6	3	6	9

35.

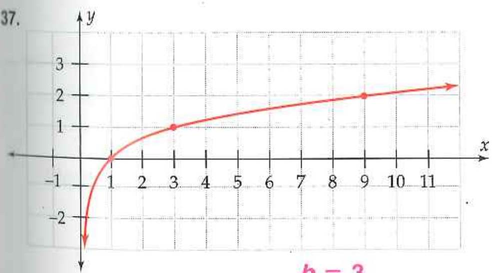
x	−2.75	−1	1	29
$h(x) = 3\log_2(x+3)$	−6	3	6	15

36.

x	$\frac{1}{e}$	1	e	e^2
$k(x) = 2\ln x$	−2	0	2	4

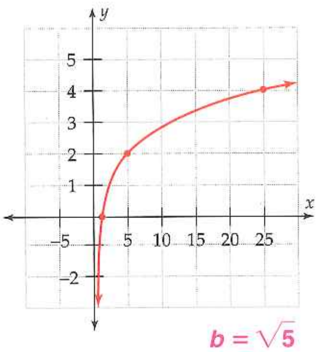
In Exercises 37–40, a graph or a table of values is given for the function $f(x) = \log_b x$. Find b.

37.



$b = 3$

38.



$b = \sqrt{5}$

39.

x	0.05	1	400	$2\sqrt{5}$
$f(x)$	−1	0	2	$\frac{1}{2}$

$b = 20$

40.

x	$\frac{1}{25}$	1	5	125
$f(x)$	−4	0	2	6

$b = \sqrt{5}$

In Exercises 41–46, solve each equation for x.

41. $\log_3 243 = x$ 5

42. $\log_{81} 27 = x$ $\frac{3}{4}$

43. $\log_{27} x = \frac{1}{3}$ 3

44. $\log_5 x = -4$ $\frac{1}{625}$

45. $\log_x 64 = 3$ 4

46. $\log_x\left(\frac{1}{9}\right) = -\frac{2}{3}$ 27

In Exercises 47–60, write the given expression as the logarithm of a single quantity. (See Example 5.)

47. $2\log x + 3\log y - 6\log z$ $\log\frac{x^2y^3}{z^6}$

48. $5\log_8 x - 3\log_8 y + 2\log_8 z$

49. $\log x - \log(x+3) + \log(x^2-9)$ $\log(x^2-3x)$

50. $\log_3(y+2) + \log_3(y-3) - \log_3 y$

51. $\frac{1}{2}\log_2(25c^2)$ $\log_2(5c)$

52. $\frac{1}{3}\log_2(27b^6)$ $\log_2(3b^2)$

53. $-2\log_4(7c)$

54. $\frac{1}{3}\log_5(x+1)$

55. $2\ln(x+1) - \ln(x+2)$ $\ln\left(\frac{(x+1)^2}{x+2}\right)$

56. $\ln(z-3) + 2\ln(z+3)$ $\ln((z-3)(z+3)^2)$

48. $\log_8\left(\frac{x^5z^2}{y^3}\right)$

50. $\log_3\left(\frac{(y+2)(y-3)}{y}\right)$

53. $\log_4\left(\frac{1}{49c^2}\right)$

54. $\log_5\sqrt[3]{x+1}$

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57. $\log_2(2x) - 1$ **$\log_2(x)$** **58.** $2 - \log_5(25z)$ **$\log_5(z^{-1})$**

59. $2\ln(e^2 - e) - 2$ **$\ln(e^2 - 2e + 1)$** **60.** $4 - 4\log_5(20)$ **$\log_5\left(\frac{1}{256}\right)$**

In Exercises 61–68, use a calculator and the change-of-base formula to evaluate the logarithm.

61. $\log_2 10$ **≈3.3219** **62.** $\log_2 22$ **≈4.4594** **63.** $\log_7 5$ **≈0.8271**

64. $\log_5 7$ **≈1.2091** **65.** $\log_{500} 1000$ **≈1.1115** **66.** $\log_{500} 250$ **≈0.8885**

67. $\log_{12} 56$ **≈1.6199** **68.** $\log_{12} 725$ **≈2.6505**

In Exercises 69–72, describe the transformation from *f* to *g*, and give the domain and range of *g*.

69. $f(x) = \log_5 x$ and $g(x) = \log_5(3x - 4)$

70. $f(x) = \log_7 x$ and $g(x) = -2 \cdot \log_7(x + 5)$

71. $f(x) = \log_2 x$ and $g(x) = \frac{1}{3} \cdot \log_2(x - 1) + 7$

72. $f(x) = \log_4 x$ and $g(x) = 3\log_4(-2x)$

In Exercises 73–78, answer true or false. Explain your answer.

73. $\log_b\left(\frac{r}{5}\right) = \log_b r - \log_b 5$ **True**

74. $\frac{\log_b a}{\log_b c} = \log_b\left(\frac{a}{c}\right)$ **False**

75. $\frac{\log_b r}{t} = \log_b(r^{\frac{1}{t}})$ **True**

76. $\log_b(cd) = \log_b c + \log_b d$ **True**

77. $\log_5(5x) = 5(\log_5 x)$ **False**

78. $\log_b(ab)^t = t(\log_b a) + t$ **True**

79. Which is larger: 397^{398} or 398^{397}? *Hint:* $\log 397 \approx 2.5988$ and $\log 398 \approx 2.5999$ and $f(x) = 10^x$ is an increasing function.
397^{398}

80. If $\log_b 9.21 = 7.4$ and $\log_b 359.62 = 19.61$, then what is $\frac{\log_b 359.62}{\log_b 9.21}$?
2.65

In Exercises 81–84, assume that *a* and *b* are positi[ve] with $a \neq 1$ and $b \neq 1$.

81. Express $\log_b u$ in terms of logarithms to the ba[se] **$\log_b u = \frac{\log}{\log}$**

82. Show that $\log_b a = \frac{1}{\log_a b}$.

83. How are $\log u$ and $\log_{100} u$ related?
$\log_{10} u = 2\log_{100} u$

84. Show that $a^{\log b} = b^{\log a}$.

85. If $\log_b x = \frac{1}{2}\log_b v + 3$, show that $x = (b^3)\sqrt{v}$.

86. Graph the functions $f(x) = \log x + \log 7$ and $g(x) = \log(x + 7)$ on the same screen. For what values of *x* is it true that $f(x) = g(x)$? What do you conclude about the statement $\log 6 + \log 7 =$ $\log(6 + 7)$?

87. Graph the functions $f(x) = \log\left(\frac{x}{4}\right)$ and $g(x) = \frac{\log x}{\log 4}$. Are they the same? What does this say about a statement such as $\log\left(\frac{48}{4}\right) = \frac{\log 48}{\log 4}$

In Exercises 88–90, sketch a complete graph of the function, labeling any holes, asymptotes, or loc[al] extrema.

88. $f(x) = \log_5 x + 2$

89. $h(x) = x\log x^2$

90. $g(x) = \log_{20} x^2$

69. Horizontal shift of $\frac{4}{3}$ units to the right, then compress horizontally by a factor of $\frac{1}{3}$.
Domain: all real numbers $> \frac{4}{3}$
Range: all real numbers

70. Reflect across the *x*-axis, then a vertical stretch by a factor of 2, then a horizontal shift of 5 units to the left.
Domain: all real numbers > -5
Range: all real numbers

71. Compress the graph vertically by a factor of $\frac{1}{3}$, then a horizontal shift of 1 unit to the right, then a vertical shift of 7 units upward.
Domain: all real numbers > 1
Range: all real numbers

72. Stretch vertically by a factor of 3, then compress horizontally by a factor of $\frac{1}{2}$, then reflect across the *y*-axis.
Domain: all real numbers < 0
Range: all real numbers

82. $\log_b a = \frac{\ln a}{\ln b} = \frac{1}{\frac{\ln b}{\ln a}} = \frac{1}{\log_a b}$

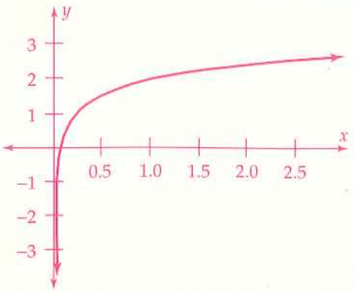
84. Let $y = a^{\log b}$.
Then $\log y = \log\left(a^{\log b}\right)$
$= (\log b)(\log a)$
$= (\log a)(\log b)$
$= \log\left(b^{\log a}\right)$.
Therefore, $y = b^{\log a}$ and $a^{\log b} = b^{\log a}$.

85. $\log_b x = \frac{1}{2}\log_b v + 3$
$= \log_b\sqrt{v} + \log_b b^3$
$= \log_b\left(b^3 \cdot \sqrt{v}\right)$
hence $x = b^3\sqrt{v}$

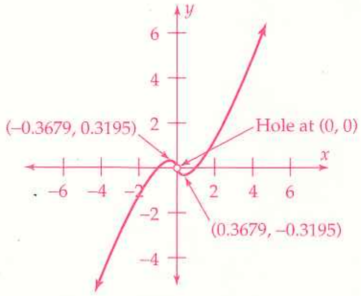
86. $f(x) = g(x)$ only when $x = \frac{7}{6}$, so the statement is false.

87. $f(x) = g(x)$ only when $x \approx 0.123$, so the statement is false.

88. The *y*-axis is a vertical asymptote.



89.



90. The *y*-axis is a vertical asymptote.