How many days did it take for 6000 people to become infected? **9.9 days**

After 2 weeks, how many people were infected? **≈6986**

ical Thinking For each positive integer n, let f_n the polynomial function whose rule is

$$f_n(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots - \frac{x^n}{n}$$

ere the sign of the last term is + if n is odd − if n is even. In the viewing window with $\le x \le 1$ and $-4 \le y \le 1$, graph $= \ln(1 + x)$ and $f_4(x)$ on the same screen. what values of x does f_4 appear to be a good roximation of g?

.5 ≤ x ≤ 0.5

ical Thinking Using the viewing window in rcise 70, find a value of n for which the graph he function f_n (as defined in Exercise 70) appears to coincide with the graph of $g(x) = \ln(1 + x)$. Use the trace feature to move from graph to graph to see how good this approximation actually is.

= 30 gives an approximation with a maximum or of 0.00001 when $-0.7 \le x \le 0.7$.

72. A bicycle store finds that N the number of bikes sold, is related to d, the number of dollars spent on advertising.

$$N = 51 + 100 \cdot \ln\left(\frac{d}{100} + 2\right)$$

a. How many bikes will be sold if nothing is spent on advertising? if $1000 is spent? if $10,000 is spent?

b. If the average profit is $25 per bike, is it worthwhile to spend $1000 on advertising? What about $10,000?

c. What are the answers in part **b** if the average profit per bike is $35?

72. a. ≈120 bikes; ≈299 bikes; ≈513 bikes

b. If d = $1000, they make $7475 in profit. Less the advertising costs that's almost $6500 in profit. If d = $10,000, that's $12,825 profit. Less the advertising costs that's only $2825 left over. Spending $1000 on ads is a better idea.

c. If profit is $35 per bike, then spending $1000 on advertising gives a profit of $10,465, or $9465 after the ads are paid for. Spending $10,000 on ads gives $17,955 in profits. Less ad costs, that's $7955. Better, but still spending $1000 pays off the most handsomely.

Properties and Laws of Logarithms

ves

properties and laws of thms to simplify and ate expressions

The definitions of common and natural logarithms differ only in their bases. Therefore, common and natural logarithms share the same basic properties and laws.

Basic Properties of Logarithms

Logarithms are only defined for positive real numbers. That is,

log v and ln v are defined only when $v > 0$.

The graphs of $y = \log x$ and $y = \ln x$ both contain the point (1, 0) because $10^0 = 1$ and $e^0 = 1$.

$$\log 1 = 0 \quad \text{and} \quad \ln 1 = 0$$

Any number o the zero power, ero, is 1.

$= 1$, where $x \neq 0$

The values of $\log 10^4$ and $\ln e^9$ can be found by writing equivalent exponential statements.

If $\log 10^4 = x$, then $10^x = 10^4$. So $x = 4$.
If $\ln e^9 = x$, then $e^x = e^9$. So $x = 9$.

In general,

$$\log 10^k = k, \quad \text{for every real number } k.$$
$$\ln e^k = k, \quad \text{for every real number } k.$$

Section 5.5 Properties and Laws of Logarithms

Teaching Notes

Have students review Figure 5.4-8 on page 359. Remind them that it can represent either of the following function pairs:

- the exponential function $f(x) = 10^x$ and its inverse, the logarithmic function $f(x) = \log x$
- the exponential function $f(x) = e^x$ and its inverse, the logarithmic function $f(x) = \ln x$

Encourage them to remember this figure to serve as a "snapshot" summary of these basic properties of exponential and logarithmic functions:

- $10^0 = 1$ and $e^0 = 1$
- $\log 1 = 0$ and $\ln 1 = 0$
- both logarithmic functions are defined only for positive values of x

To help students understand the statements at the bottom of this page, stress that *a logarithm is an exponent.*

- The *log* of any number is the exponent that must be applied to the base 10 to produce that number. So, $\log 10^k = k$.
- The *ln* of any number is the exponent that must be applied to the base e to produce that number. So, $\ln e^k = k$.

By definition, log 678 is the exponent to which 10 must be raised to pro-
duce 678.

$$10^{\log 678} = 678$$

Similarly, ln 54 is the exponent to which e must be raised to produce 54

$$e^{\ln 54} = 54$$

In general,

$$\mathbf{10^{\log v} = v \text{ and } e^{\ln v} = v, \qquad \text{for every } v > 0.}$$

The facts presented above are summarized in the table below.

Basic Properties of Logarithms

Common logarithms	Natural logarithms
1. log v is defined only when $v > 0$	**1. ln v is defined only when $v > 0$**
2. log 1 = 0 and log 10 = 1	**2. ln 1 = 0 and ln e = 1**
3. log $10^k = k$ for every real number k	**3. ln $e^k = k$ for every real number k**
4. $10^{\log v} = v$ for every $v > 0$	**4. $e^{\ln v} = v$ for every $v > 0$**

Properties 3 and 4 are restatements of the fact that the composition of inverse functions produces the identity function.

That is, if $f(x) = 10^x$ and $g(x) = \log x$, then

$$(f \circ g)(x) = f(\log x) = 10^{\log x} = x \text{ for all } x > 0$$
$$(g \circ f)(x) = g(10^x) = \log 10^x = x \text{ for all } x$$

Analogous statements are true for $f(x) = e^x$ and $g(x) = \ln x$.

The properties of logarithms can be used to simplify expressions and solve equations. For example, applying Property 3 with $k = 2x^2 + 7x + 9$ allows you to rewrite the expression $\ln e^{2x^2+7x+9}$ as $2x^2 + 7x + 9$.

Example 1 Solving Equations by Using Properties of Logarithms

Use the basic properties of logarithms to solve the equation $\ln(x + 1) = 2$.

Solution

Because $f(x) = e^x$ is a function, if $\ln(x + 1) = 2$, then $e^{\ln(x+1)} = e^2$.

$$e^{\ln(x+1)} = e^2$$
$$x + 1 = e^2 \qquad \textit{Apply Property 4 with } v = x + 1$$
$$x = e^2 - 1$$
$$x \approx 6.3891$$

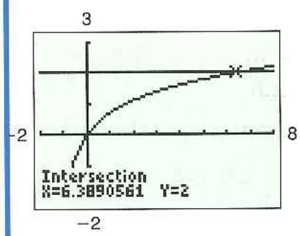


Figure 5.5-1

The intersection of the graphs of $Y_1 = \ln(x + 1)$ and $Y_2 = 2$, shown in Figure 5.5-1, confirms the solution.

Teaching Notes

To help students understand that $10^{\log 678} = 678$, again stress that *a logarithm is an exponent.* log 678 is the exponent that must be applied to the base 10 to produce 678, so of course, if that exponent is applied to the base 10, then 678 must be produced! That is, $10^{\log 678} = 678$. Ask students to use similar reasoning to explain why $e^{\ln 54} = 54$.

Point out that all the bold statements in the text on pages 363 and 364 are summarized in the table, **Basic Properties of Logarithms.**

Have students refer back to the **Composition of a Function and its Inverse** (page 211). Remind them that $(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$ and $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$.

Using g instead of f^{-1} establishes properties 3 and 4 in the table. Use the concept that inverse functions "undo each other" to reinforce this idea. Neglecting restrictions on x, properties 3 and 4 can be summarized as follows:

- $\log 10^x = 10^{\log x} = x$, and
- $\ln e^x = e^{\ln x} = x$

To explain that k can be any real number in property 3, write $10^{\square}$ and $e^{\square}$ on the board. Ask students if they can think of any number that would make either expression undefined if written in the box. no

To explain that v must be positive in property 4, remind them that both logarithmic functions are defined only for positive values (recall the logarithm graph in their "snapshot").

Illustrate the first **Product Law of Logarithms** as follows:

Let $v = 10^2, w = 10^3$. Then:

$$\log(v \cdot w) = \log(10^2 \cdot 10^3) = \log 10^5 = 5$$

$$\log v + \log w = \log 10^2 + \log 10^3 = 2 + 3 = 5$$

Ask students to illustrate the second law.

Let $v = e^2$, $w = e^3$. Then:

$$\ln(v \cdot w) = \ln(e^2 \cdot e^3) = \ln e^5 = 5$$

$$\ln v + \ln w = \ln e^2 + \ln e^3 = 2 + 3 = 5$$

Laws of Logarithms

The Product Law of Exponents states that $b^m b^n = b^{m+n}$. Because logarithms are exponents, the following law holds.

Product Law of Logarithms

> **For all $v, w > 0$,**
>
> $$\log(vw) = \log v + \log w$$
> $$\ln(vw) = \ln v + \ln w.$$

Proof According to Property 4 of logarithms, $10^{\log v} = v$ and $10^{\log w} = w$. Then, by the Product Law of Exponents:

$$vw = 10^{\log v} \cdot 10^{\log w} = 10^{\log v + \log w}$$

Again by Property 4 of logarithms:

$$10^{\log vw} = vw$$

Therefore, $10^{\log vw} = 10^{\log v + \log w}$; and because exponential functions are one-to-one, $\log vw = \log v + \log w$. A similar argument can be made for natural logarithms.

Example 2 Using the Product Law of Logarithms

Use the Product Law of Logarithms to evaluate each logarithm.

a. Given that $\log 3 = 0.4771$ and $\log 11 = 1.0414$, find $\log 33$.

b. Given that $\ln 7 = 1.9459$ and $\ln 9 = 2.1972$, find $\ln 63$.

Solution

a. $\log 33 = \log(3 \cdot 11) = \log 3 + \log 11 = 0.4771 + 1.0414 = 1.5185$

b. $\ln 63 = \ln(7 \cdot 9) = \ln 7 + \ln 9 = 1.9459 + 2.1972 = 4.1431$

CAUTION

A common error in applying the Product Law of Logarithms is to write the false statement

$\ln 7 + \ln 9 = \ln(7 + 9) = \ln 16$

instead of the correct statement

$\ln 7 + \ln 9 = \ln(7 \cdot 9) = \ln 63.$

Graphing Exploration

Using the viewing window with $-10 \le x \le 10$ and $-8 \le y \le 8$, graph both functions below on the same screen.

$$f(x) = \ln x + \ln 9 \qquad g(x) = \ln(x + 9)$$

Explain how the graph illustrates the caution in the margin.

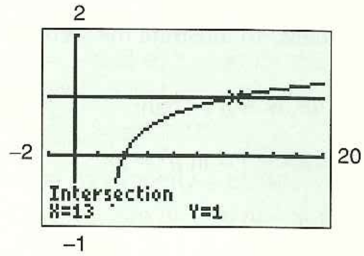
The Quotient Law of Exponents states that $\frac{b^m}{b^n} = b^{m-n}$. When the exponents are logarithms, the Quotient Law is still valid.

ADDITIONAL EXAMPLES

Example 1

Use the basic properties of logarithms to solve the equation $\log(x - 3) = 1$. $x = 13$

The solution can be confirmed by graphing.



Example 2

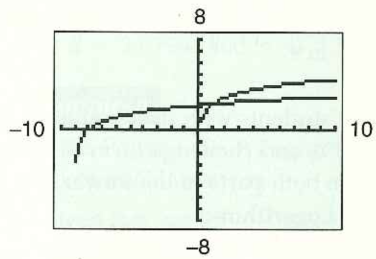
Use the Product Law of Logarithms to evaluate each logarithm.

a. Given that $\log 3 = 0.4771$ and $\log 4 = 0.6021$, find $\log 12$. 1.0792

b. Given that $\ln 3 = 1.0986$ and $\ln 6 = 1.7918$, find $\ln 18$. 2.8904

Teaching Notes

Solution to the **Graphing Exploration:**



The graphs are different, so $\ln x + \ln 9 \neq \ln(x + 9)$ in general. In particular, $\ln 7 + \ln 9 \neq \ln(7 + 9)$.

Math Background

Notice the statement in the proof of the **Product Law of Logarithms:**

"Therefore, $10^{\log vw} = 10^{\log v + \log w}$; and because exponential functions are one-to-one, $\log vw = \log v + \log w$." Why is it necessary that exponential functions be one-to-one to justify this statement? In this case, the reason is found in the definition of a one-to-one function: If f is a one-to-one function, then $f(u) = f(v)$ implies $u = v$ (i.e. different inputs cannot have the same function value). So, because the exponential function $f(x) = 10^x$ is one-to-one, we have: $f(\log vw) = f(\log v + \log w)$ implies $\log vw = \log v + \log w$. That is, $10^{\log vw} = 10^{\log v + \log w}$ implies $\log vw = \log v + \log w$.

Quotient Law of Logarithms

For all $v, w > 0$,

$$\log\left(\frac{v}{w}\right) = \log v - \log w$$

$$\ln\left(\frac{v}{w}\right) = \ln v - \ln w.$$

The proof of the Quotient Law of Logarithms is similar to the proof of the Product Law of Logarithms.

Example 3 Using the Quotient Law of Logarithms

Use the Quotient Law of Logarithms to evaluate each logarithm.

a. Given that $\log 28 = 1.4472$ and $\log 7 = 0.8451$, find $\log 4$.

b. Given that $\ln 18 = 2.8904$ and $\ln 6 = 1.7918$, find $\ln 3$.

Solution

a. $\log 4 = \log\left(\frac{28}{7}\right) = \log 28 - \log 7 = 1.4472 - 0.8451 = 0.6021$

b. $\ln 3 = \ln\left(\frac{18}{6}\right) = \ln 18 - \ln 6 = 2.8904 - 1.7918 = 1.0986$

CAUTION

Do not confuse $\ln\left(\frac{7}{9}\right) = -0.2513$ with the quotient $\frac{\ln 7}{\ln 9} = 0.8856$. They are *different* numbers.

Graphing Exploration

Using the viewing window with $0 \le x \le 8$ and $-4 \le y \le 2$, graph both functions below on the same screen.

$$f(x) = \ln\left(\frac{x}{9}\right) \qquad g(x) = \frac{\ln x}{\ln 9}$$

Explain how the graph illustrates the caution in the margin.

The Power Law of Exponents, which states that $(b^m)^k = b^{mk}$, can also be translated into a logarithmic statement.

Power Law of Logarithms

For all k and $v > 0$,

$$\log v^k = k \log v,$$

$$\ln v^k = k \ln v.$$

Teaching Notes

Illustrate the first **Quotient Law of Logarithms** with the following example:

Let $v = 10^5, w = 10^3$. Then:

$\log\left(\frac{v}{w}\right) = \log\left(\frac{10^5}{10^3}\right) = \log 10^2 = 2$

$\log v - \log w = \log 10^5 - \log 10^3 = 5 - 3 = 2$

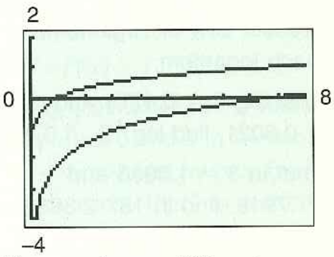
Ask students to illustrate the second law.

Let $v = e^5, w = e^3$. Then:

$\ln\left(\frac{v}{w}\right) = \ln\left(\frac{e^5}{e^3}\right) = \ln e^2 = 2$

$\ln v - \ln w = \ln e^5 - \ln e^3 = 5 - 3 = 2$

Solution to the **Graphing Exploration:**



The graphs are different, so $\ln\left(\frac{x}{9}\right) \neq \frac{\ln x}{\ln 9}$ in general. In particular, $\ln\left(\frac{7}{9}\right) \neq \frac{\ln 7}{\ln 9}$.

Provide students with these values of k and v, and challenge them to illustrate both parts of the **Power Law of Logarithms:**

Let $k = 3$ and $v = 10^4$. Then:

$\log v^k = \log(10^4)^3 = \log 10^{12} = 12$

$k \log v = 3 \log 10^4 = 3 \cdot 4 = 12$

$\ln v^k = \ln(e^4)^3 = \ln e^{12} = 12$

$k \ln v = 3 \ln e^4 = 3 \cdot 4 = 12$

Proof

According to Property 4 of logarithms, $10^{\log v} = v$. Then, by the Power Law of Exponents:

$$v^k = (10^{\log v})^k = 10^{k \log v}$$

Again by Property 4 of logarithms:

$$10^{\log v^k} = v^k$$

So, $10^{\log v^k} = 10^{k \log v}$ and therefore, $\log v^k = k \log v$. A similar argument can be made for natural logarithms.

Example 4 Using the Power Law of Logarithms

Use the Power Law of Logarithms to evaluate each logarithm.

a. Given that $\log 6 = 0.7782$, find $\log \sqrt{6}$.

b. Given that $\ln 50 = 3.9120$, find $\ln \sqrt[3]{50}$.

Solution

a. $\log\sqrt{6} = \log 6^{\frac{1}{2}} = \frac{1}{2}\log 6 = \frac{1}{2}(0.7782) = 0.3891$

b. $\ln\sqrt[3]{50} = \ln 50^{\frac{1}{3}} = \frac{1}{3}\ln 50 = \frac{1}{3}(3.9120) = 1.3040$

The laws of logarithms can be used to simplify various expressions.

Example 5 Simplifying Expressions

Write $\ln 3x + 4\ln x - \ln 3xy$ as a single logarithm.

Solution

$$\begin{aligned}\ln 3x + 4\ln x - \ln 3xy &= \ln 3x + \ln x^4 - \ln 3xy && \textit{Power Law}\\ &= \ln(3x \cdot x^4) - \ln 3xy && \textit{Product Law}\\ &= \ln\left(\frac{3x^5}{3xy}\right) && \textit{Quotient Law}\\ &= \ln\left(\frac{x^4}{y}\right)\end{aligned}$$

Example 6 Simplifying Expressions

Simplify $\ln\left(\frac{\sqrt{x}}{x}\right) + \ln\left(\sqrt[4]{ex^2}\right)$.

Teaching Notes

The proof of the **Quotient Law of Logarithms** is similar to the proof of the **Product Law of Logarithms** that was given with respect to common logarithms on page 365. You may want to have your students prove the Quotient Law of Logarithms using natural logarithms:

According to Property 4 of logarithms, $e^{\ln v} = v$ and $e^{\ln w} = w$. By the Quotient Law of Exponents:

$$\frac{v}{w} = \frac{e^{\ln v}}{e^{\ln w}} = e^{\ln v - \ln w}$$

Again, by Property 4 of logarithms:

$$e^{\ln\left(\frac{v}{w}\right)} = \frac{v}{w}$$

Therefore, $e^{\ln\left(\frac{v}{w}\right)} = e^{\ln v - \ln w}$; and because exponential functions are one-to-one, $\ln\left(\frac{v}{w}\right) = \ln v - \ln w$.

ADDITIONAL EXAMPLES

Example 3

Use the Quotient Law of Logarithms to evaluate each logarithm.

a. Given that $\log 40 = 1.6021$ and $\log 8 = 0.9031$, find $\log 5$. 0.6990

b. Given that $\ln 32 = 3.4657$ and $\ln 8 = 2.0794$, find $\ln 4$. 1.3863

Example 4

Use the Power Law of Logarithms to evaluate each logarithm.

a. Given that $\log 25 = 1.3979$, find $\log \sqrt[4]{25}$.

$\log\sqrt[4]{25} = \frac{1}{4}\log 25 = 0.3495$

b. Given that $\ln 22 = 3.0910$, find $\ln\sqrt{22}$.

$\ln\sqrt{22} = \frac{1}{2}\ln 22 = 1.5455$

Example 5

Write $\log 8x + 3\log x - \log 2x^2$ as a single logarithm. $\log 4x^2$

xample Notes

n alternate sequence of steps that n be used in **Example 6** is shown low:

$$\left(\frac{\sqrt{x}}{x}\right) + \ln\left(\sqrt[4]{ex^2}\right)$$
$$= \ln\sqrt{x} - \ln x + \ln\left(\sqrt[4]{ex^2}\right)$$
$$= \ln x^{\frac{1}{2}} - \ln x + \ln(ex^2)^{\frac{1}{4}}$$
$$= \frac{1}{2}\ln x - \ln x + \frac{1}{4}\ln(ex^2)$$
$$= \frac{1}{2}\ln x - \ln x + \frac{1}{4}(\ln e + \ln x^2)$$
$$= \frac{1}{2}\ln x - \ln x + \frac{1}{4}(\ln e + 2\ln x)$$
$$= \frac{1}{2}\ln x - \ln x + \frac{1}{4}\ln e + \frac{1}{2}\ln x$$
$$= \frac{1}{4}\ln e$$
$$= \frac{1}{4}$$

DDITIONAL EXAMPLES

xample 6

implify $2\log\left(\frac{\sqrt[4]{m}}{m}\right) + \log\left(\sqrt{100m^3}\right)$.

$$\log\left(\frac{\sqrt[4]{m}}{m}\right) + \log\left(\sqrt{100m^3}\right)$$
$$= 2\log\left(\frac{m^{\frac{1}{4}}}{m}\right) + \log\left(100m^3\right)^{\frac{1}{2}}$$
$$= 2\log\left(m^{-\frac{3}{4}}\right) + \log\left(100m^3\right)^{\frac{1}{2}}$$
$$= -\frac{3}{4}(2)\log m + \frac{1}{2}\log 100m^3$$
$$= -\frac{3}{2}\log m + \frac{1}{2}(\log 100 + \log m^3)$$
$$= -\frac{3}{2}\log m + \frac{1}{2}(2 + 3\log m)$$
$$= -\frac{3}{2}\log m + 1 + \frac{3}{2}\log m$$
$$= 1$$

xample 7

n earthquake in Turkey in 1999 egistered 7.6 on the Richter scale. lumerous aftershocks followed during he next few days, one of which had a magnitude of 5.0. How much more ntense was the ground motion of the arthquake than that of this ftershock?

$10^{2.6} \approx 398$ times more intense

Solution

$$\ln\left(\frac{\sqrt{x}}{x}\right) + \ln\left(\sqrt[4]{ex^2}\right) = \ln\left(\frac{x^{\frac{1}{2}}}{x}\right) + \ln(ex^2)^{\frac{1}{4}}$$
$$= \ln\left(x^{-\frac{1}{2}}\right) + \ln(ex^2)^{\frac{1}{4}}$$
$$= -\frac{1}{2}\ln x + \frac{1}{4}\ln(ex^2) \quad \textit{Power Law}$$
$$= -\frac{1}{2}\ln x + \frac{1}{4}(\ln e + \ln x^2) \quad \textit{Product Law}$$
$$= -\frac{1}{2}\ln x + \frac{1}{4}(\ln e + 2\ln x) \quad \textit{Power Law}$$
$$= -\frac{1}{2}\ln x + \frac{1}{4}\ln e + \frac{1}{2}\ln x$$
$$= \frac{1}{4}\ln e$$
$$= \frac{1}{4} \quad \ln e = 1$$

Applications

A logarithmic scale is a scale that is determined by a logarithmic function. Because logarithmic growth is slow, measurements on a logarithmic scale can sometimes be deceptive. The Richter scale is an illustration of this.

The magnitude $R(i)$ of an earthquake on the Richter scale is given by $R(i) = \log\left(\frac{i}{i_0}\right)$, where i is the amplitude of the ground motion of the earthquake and i_0 is the amplitude of the ground motion of the *zero earthquake*. A moderate earthquake might have 1000 times the ground motion of the zero earthquake, or $i = 1000i_0$. Its magnitude would be

$$\log\left(\frac{1000i_0}{i_0}\right) = \log 1000 = 3$$

An earthquake with 10 times this ground motion, or $i = 10{,}000i_0$, would have a magnitude of

$$\log\left(\frac{10{,}000i_0}{i_0}\right) = \log 10{,}000 = 4$$

So a tenfold increase in ground motion produces only a 1-point change on the Richter scale. In general,

> increasing the ground motion by a factor of 10^k increases the Richter magnitude by k units.

NOTE The **zero earthquake** has ground motion amplitude of less than 1 micron on a standard seismograph 100 kilometers from the epicenter.

Example 7 Richter Scale

The 1989 World Series earthquake in San Francisco measured 7.0 on the Richter scale, and the great earthquake of 1906 measured 8.3. How much more intense was the ground motion of the 1906 earthquake than that of the 1989 earthquake?

Solution

The difference in Richter magnitude is $8.3 - 7.0 = 1.3$. Therefore, the 1906 earthquake was $10^{1.3} \approx 20$ times more intense than the 1989 earthquake in terms of ground motion. ■

ercises 5.5

xercises 1–4, solve each equation by using the c properties of logarithms.**

$\log(x-3) = 2$ **103**
2. $\log(2x) = 3$ **500**
$\ln(x+4) = -1$ **≈−3.63**
4. $5 + \ln(x-1) = 8$ **≈21.09**

xercises 5–10, use laws of logarithms and the val- given below to evaluate each logarithmic ression.

$\log 7 = 0.8451$ $\log 5 = 0.6990$
$\log 3 = 0.4771$ $\log 2 = 0.3010$

$\log 8$ **≈0.9030**
6. $\log 12$ **≈1.0791**
$\log\left(\frac{5}{7}\right)$ **≈−0.1461**
8. $\log\left(\frac{3}{14}\right)$ **≈−0.6690**
$\log 0.6$ **≈−0.2219**
10. $\log 1.5$ **≈0.1761**

xercises 11–20, write the given expression as a sin- logarithm.

$\ln x^2 + 3\ln y$ **$\ln(x^2y^3)$**
12. $\ln 2x + 2\ln x - \ln 3y$ **$\ln\left(\frac{2x^3}{3y}\right)$**
$\log(x^2-9) - \log(x+3)$ **$\log(x-3)$**
$\log 3x - 2[\log x - \log(2+y)]$ **$\log\left(\frac{3(2+y)^2}{x}\right)$**
$2\ln x - 3(\ln x^2 + \ln x)$ **$\ln(x^{-7})$**
$\ln\left(\frac{e}{\sqrt{x}}\right) - \ln\left(\sqrt{ex}\right)$ **$\ln\left(\frac{\sqrt{e}}{x}\right)$**
17. $3\ln(e^2 - e) - 3$ **$3\ln(e-1)$**
$2 - 2\log 20$ **$\log\frac{1}{4}$**
19. $\log 10x + \log 20y - 1$ **$\log(20xy)$**
$\ln(e^2x) + \ln(ey) - 3$ **$\ln xy$**

xercises 21–26, let $u = \ln x$ and $v = \ln y$. Write the n expression in terms of u and v. For example, $\ln x^3y = \ln x^3 + \ln y = 3\ln x + \ln y = 3u + v$.

$\ln(x^2y^5)$ **$2u + 5v$**
22. $\ln(x^3y^2)$ **$3u + 2v$**
$\ln(\sqrt{x}\cdot y^2)$ **$\frac{1}{2}u + 2v$**
24. $\ln\left(\frac{\sqrt{x}}{y}\right)$ **$\frac{1}{2}u - v$**
25. $\ln\left(\sqrt[3]{x^2\sqrt{y}}\right)$ **$\frac{2}{3}u + \frac{1}{6}v$**
26. $\ln\left(\frac{\sqrt{x^2y}}{\sqrt[3]{y}}\right)$ **$u + \frac{1}{6}v$**

27. a. Graph $y = x$ and $y = e^{\ln x}$ in separate viewing windows. For what values of x are the graphs identical?
b. Use the properties of logarithms to explain your answer in part **a.**

28. a. Graph $y = x$ and $y = \ln e^x$ in separate viewing windows. For what values of x are the graphs identical?
b. Use the properties of logarithms to explain your answer in part **a.**

In Exercises 29–34, use graphical or algebraic means to determine whether the statement is true or false.

29. $\ln|x| = |\ln x|$
30. $\ln\left(\frac{1}{x}\right) = \frac{1}{\ln x}$
31. $\log x^5 = 5\log x$
32. $e^{x\ln x} = x^x \quad (x > 0)$
33. $\ln x^3 = (\ln x)^3$
34. $\log\sqrt{x} = \sqrt{\log x}$

In Exercises 35 and 36, find values of a and b for which the statement is false.

35. $\frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$

36. $\log(a+b) = \log a + \log b$

37. If $\ln b^7 = 7$, what is b? **$b = e$**

38. Suppose $f(x) = A\ln x + B$, where A and B are constants. If $f(1) = 10$ and $f(e) = 1$, what are A and B? **$A = -9, B = 10$**

39. If $f(x) = A\ln x + B$ and $f(e) = 5$ and $f(e^2) = 8$, find A and B. **$A = 3, B = 2$**

40. Show that $g(x) = \ln\left(\frac{x}{1-x}\right)$ is the inverse function of $f(x) = \frac{1}{1+e^{-x}}$. (See Section 3.6.)

$$(f\circ g)(x) = f\left(\ln\left(\frac{x}{1-x}\right)\right) = \frac{1}{1+e^{-\left(\ln\left(\frac{x}{1-x}\right)\right)}} = \frac{1}{1+e^{\ln\left(\frac{1-x}{x}\right)}} = \frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{x}{x+(1-x)} = x$$

and

$$(g\circ f)(x) = g\left(\frac{1}{1+e^{-x}}\right) = \ln\left(\frac{\frac{1}{1+e^{-x}}}{1-\frac{1}{1+e^{-x}}}\right) = \ln\left(\frac{1}{1+e^{-x}-1}\right) = \ln\left(\frac{1}{e^{-x}}\right) = \ln e^x = x$$

Real-World Application

The Richter scale, developed by Charles Richter (1900–1985), assigns a magnitude to each earthquake, based on the amplitude of measured seismic waves. Because the scale is logarithmic, an increase of 10 in seismic amplitude results in an increase of only 1 in the assigned magnitude. A logarithmic scale is well suited for this purpose because there is such a great range in magnitudes of earthquakes. For example, if one earthquake has 100 times the seismic amplitude as another earthquake, the Richter scale number assigned to it is only 2 greater.

Exercises 5.5

ANSWERS

27. a. For all $x > 0$
b. According to the fourth property of natural logarithms on page 364, $e^{\ln x} = x$ for every $x > 0$.

28. a. for all values of x
b. This is true because $f(x) = \ln x$ and $g(x) = e^x$ are inverse functions. $(f\circ g)(x) = \ln e^x = x$

29. False; the right side is not defined when $x < 0$, but the left side is.

30. False; Let $x = 1$ for example. The left side of the equation has the value 0 while the right side is undefined.

31. True by the Power Law

32. True; $e^{x\ln x} = e^{\ln x^x} = x^x$

33. False; the graph of the left side differs from the graph of the right side.

34. False; For example, let $x = e^{16}$. The left side of the equation has the value 8 while the right side has the value 4.

35. Answers may vary: $\frac{\log 3}{\log 2} \approx 1.585$ and $\log\left(\frac{3}{2}\right) = 0.1761$ thus $\frac{\log 3}{\log 2} \neq \log\left(\frac{3}{2}\right)$

36. Answers may vary; $\log(3+2) \approx 0.6990$ and $\log 3 + \log 2 \approx 0.7782$; thus $\log(3+2) \neq \log 3 + \log 2$