3 Exercises

Assignment Guide
ign *Guided Practice* exercises
necessary.

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Basic 18–27, 37–46
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mework Quick Check
ckly check key concepts.
rcises: 18, 22, 27, 28, 34, 46

wers
ossible answer: $(x) = x^2 - 7x + 12$
ossible answer: $(x) = x^2 + 8x + 16$
ossible answer: $(x) = x^2 - 3x$
ossible answer: $(x) = x^2 - 4x - 5$
ossible answer: $(x) = x^2 - 8x + 12$
ossible answer: $(x) = x^2 - 6x + 9$
No; possible answer: the relationship between the building height and jump time is quadratic, not linear; therefore, a jump that is half as high will not last half as long.

State Resources

5-3 Exercises

GUIDED PRACTICE

1. **Vocabulary** The solutions of the equation $3x^2 + 2x + 5 = 0$ are its __?__. (*roots* or *zeros*) **roots**

SEE EXAMPLE 1 p. 333

Find the zeros of each function by using a graph and table.

2. $f(x) = x^2 + 4x - 5$ **−5, 1**
3. $g(x) = -x^2 + 6x - 8$ **2, 4**
4. $f(x) = x^2 - 1$ **−1, 1**

SEE EXAMPLE 2 p. 334

Find the zeros of each function by factoring.

5. $f(x) = x^2 - 7x + 6$ **1, 6**
6. $g(x) = 2x^2 - 5x + 2$ **$\frac{1}{2}$, 2**
7. $h(x) = x^2 + 4x$ **−4, 0**
8. $f(x) = x^2 + 9x + 20$ **−5, −4**
9. $g(x) = x^2 - 6x - 16$ **−2, 8**
10. $h(x) = 3x^2 + 13x + 4$ **−4, $-\frac{1}{3}$**

SEE EXAMPLE 3 p. 335

11. **Archery** The height h of an arrow in feet is modeled by $h(t) = -16t^2 + 63t + 4$, where t is the time in seconds since the arrow was shot. How long is the arrow in the air? **4 s**

SEE EXAMPLE 4 p. 336

Find the roots of each equation by factoring.

12. $x^2 - 6x = -9$ **3**
13. $5x^2 + 20 = 20x$ **2**
14. $x^2 = 49$ **−7, 7**

SEE EXAMPLE 5 p. 337

Write a quadratic function in standard form for each given set of zeros.

15. 3 and 4
16. −4 and −4
17. 3 and 0

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–26	2
27	3
28–33	4
34–36	5

Extra Practice
Skills Practice p. S12
Application Practice p. S36

Find the zeros of each function by using a graph and table.

18. $f(x) = -x^2 + 4x - 3$ **1, 3**
19. $g(x) = x^2 + x - 6$ **−3, 2**
20. $f(x) = x^2 - 9$ **−3, 3**

Find the zeros of each function by factoring.

21. $f(x) = x^2 + 11x + 24$ **−8, −3**
22. $g(x) = 2x^2 + x - 10$ **$-\frac{5}{2}$, 2**
23. $h(x) = -x^2 + 9x$ **0, 9**
24. $f(x) = x^2 - 15x + 54$ **6, 9**
25. $g(x) = x^2 + 7x - 8$ **−8, 1**
26. $h(x) = 2x^2 - 12x + 18$ **3**

27. **Biology** A bald eagle snatches a fish from a lake and flies to an altitude of 256 ft. The fish manages to squirm free and falls back down into the lake. Its height h in feet can be modeled by $h(t) = 256 - 16t^2$, where t is the time in seconds. How many seconds will the fish fall before hitting the water? **4 s**

Find the roots of each equation by factoring.

28. $x^2 + 8x = -16$ **−4**
29. $4x^2 = 81$ **$-\frac{9}{2}$, $\frac{9}{2}$**
30. $9x^2 + 12x + 4 = 0$ **$-\frac{2}{3}$**
31. $36x^2 - 9 = 0$ **$-\frac{1}{2}$, $\frac{1}{2}$**
32. $x^2 - 10x + 25 = 0$ **5**
33. $49x^2 = 28x - 4$ **$\frac{2}{7}$**

Write a quadratic function in standard form for each given set of zeros.

34. 5 and −1
35. 6 and 2
36. 3 and 3

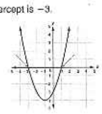
Find the zeros of each function.

37. $f(x) = 6x - x^2$ **0, 6**
38. $g(x) = x^2 - 25$ **−5, 5**
39. $h(x) = x^2 - 12x + 36$ **6**
40. $f(x) = 3x^2 - 12$ **−2, 2**
41. $g(x) = x^2 - 22x + 121$ **11**
42. $h(x) = 30 + x - x^2$ **−5, 6**
43. $f(x) = x^2 - 11x + 30$ **5, 6**
44. $g(x) = x^2 - 8x - 20$ **−2, 10**
45. $h(x) = 2x^2 + 18x + 28$ **−7, −2**

5-3 READING STRATEGIES

A **zero** of a function is the value of x that makes $f(x) = 0$. Two different ways of finding the zeros of a function, graphing and factoring, are compared below.

Finding Zeros by Using a Graph
$f(x) = x^2 - 2x - 3$
The graph opens upward.
The vertex is $(-1, -4)$
The y-intercept is -3.



The zeros are 1 and −3.

Finding Zeros by Factoring
$f(x) = x^2 + 2x - 3$
Set the function equal to zero.
$x^2 + 2x - 3 = 0$
Factor.
$(x - 1)(x + 3) = 0$
Set each factor equal to 0 and solve.
$x - 1 = 0$
$x = 1$
$x + 3 = 0$
$x = -3$
The zeros are 1 and −3

Answer each question.

1. How can you use a graph to find the zeros of a quadratic function?
Possible answer: The points where the function crosses the x-axis are the zeros of the function.
2. Can a quadratic function have more than two zeros? Explain.
No; possible answer: a parabola can cross the x-axis at most at two points.
3. Consider the function $f(x) = (x - 1)(x + 1)$.
a. What are the zeros of the function? **1 and −1**
b. Which method did you use? Why?
Factoring; possible answer: since the function was already factored, I just set each factor equal to 0 and solved for x.
4. A quadratic function opens down and its vertex is $(0, -3)$. How many zeros does this function have? Explain.
No zeros exist for this function. Possible answer: Since the parabola opens down and its vertex is below the x-axis, the graph will not cross the x-axis.
5. Compare and contrast the two methods of finding the zeros of a quadratic function. Describe when you would use one or the other method.
Possible answer: I would check some factors of c to see if I could easily factor the equation. If not, then I would make a graph.

5-3 RETEACH

Solve the equation $ax^2 + bx + c = 0$ to find the roots of the equation.
Find the roots of $x^2 + 2x - 15 = 0$ to find the zeros of $f(x) = x^2 + 2x - 15$.

$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$ — Factor, then multiply to check.
$(x + 5) = 0$ or $(x - 3) = 0$ — Set each factor equal to 0.
$x = -5$ or $x = 3$ — Solve each equation for x.

To check the roots, substitute each root into the original equation:

Equation:	$x^2 + 2x - 15 = 0$	$x^2 + 2x - 15 = 0$
Root:	$x = -5$	$x = 3$
Check:	$(-5)^2 + 2(-5) - 15$	$(3)^2 + 2(3) - 15$
	$25 - 10 - 15 = 0$ ✓	$9 + 6 - 15 = 0$ ✓

The **roots** of $x^2 + 2x - 15 = 0$ are −5 and 3.
The **zeros** of $f(x) = x^2 + 2x - 15$ are −5 and 3.
The roots of the equation are the zeros of the function.

Find the zeros of each function by factoring. Set the function equal to 0, factor, set each factor equal to 0, and then solve each equation.

1. $f(x) = 4x^2 - 24x$
$4x^2 - 24x = 0$
$4x(x - \underline{6}) = 0$
$\underline{4x} = 0$ or $x - \underline{6} = 0$
$x = 0$ or $x = 6$

2. $f(x) = x^2 + 4x + 3$
$x^2 + 4x + 3 = 0$
$(x + \underline{3})(x + \underline{1}) = 0$
$x + \underline{3} = 0$ or $x + \underline{1} = 0$
$x = -3$ or $x = -1$

3. $f(x) = x^2 - 5x + 4$
$x^2 - 5x + 4 = 0$
$(x - 4)(x - 1) = 0$
$x - 4 = 0$ or $x - 1 = 0$
$x = 4$ or $x = 1$

4. $f(x) = 3x^2 + 12x$
$3x^2 + 12x = 0$
$3x(x + 4) = 0$
$3x = 0$ or $x + 4 = 0$
$x = 0$ or $x = -4$

LINK Entertainment

The Guinness world record for the greatest number of people juggling at one time was set in 1998 by 1508 people, each of whom juggled at least 3 objects for 10 seconds.

46. **Movies** A stuntwoman jumps from a building 73 ft high and lands on an air bag that is 9 ft tall. Her height above ground h in feet can be modeled by $h(t) = 73 - 16t^2$, where t is the time in seconds.

   a. **Multi-Step** How many seconds will the stuntwoman fall before touching the air bag? (*Hint:* Find the time t when the stuntwoman's height above ground is 9 ft.) **2 s**

   b. **What if...?** Suppose the stuntwoman jumps from a building that is half as tall. Will she be in the air for half as long? Explain.

47. **Entertainment** A juggler throws a ball into the air from a height of 5 ft with an initial vertical velocity of 16 ft/s.

   a. Write a function that can be used to model the height h of the ball in feet t seconds after the ball is thrown. $h(t) = -16t^2 + 16t + 5$

   b. How long does the juggler have to catch the ball before it hits the ground? **1.25 s**

Find the roots of each equation.

48. $x^2 - 2x + 1 = 0$ **1**
49. $x^2 + 6x = -5$ **−5, −1**
50. $25x^2 + 40x = -16$ $-\frac{4}{5}$
51. $9x^2 + 6x = -1$ $-\frac{1}{3}$
52. $5x^2 = 45$ **−3, 3**
53. $x^2 - 6 = x$ **−2, 3**

For each function, (a) find its vertex, (b) find its y-intercept, (c) find its zeros, and (d) graph it.

54. $f(x) = x^2 + 2x - 8$
55. $g(x) = x^2 - 16$
56. $h(x) = x^2 - x - 12$
57. $f(x) = -2x^2 + 4x$
58. $g(x) = x^2 - 5x - 6$
59. $h(x) = 3x^2 + x - 4$

60. **Geometry** The hypotenuse of a right triangle is 2 cm longer than one leg and 4 cm longer than the other leg.

   a. Let x represent the length of the hypotenuse. Use the Pythagorean Theorem to write an equation that can be solved for x.

   b. Find the solutions of the equation from part **a.** $x = 10$ or $x = 2$

   c. Are both solutions reasonable in the context of the problem situation? Explain.

Geometry **Find the dimensions of each rectangle.**

61. Rectangle: $A = 80$ ft², width x, length $x + 16$ **20 ft by 4 ft**

62. Rectangle: $A = 210$ cm², width x, length $x + 1$ **15 cm by 14 cm**

63. Rectangle: $A = 50$ m², width $x - 3$, length $x + 2$ **10 m by 5 m**

64. **Critical Thinking** Will a function whose rule can be factored as a binomial squared ever have two different zeros? Explain.

65. **Write About It** Explain how the Zero Product Property can be used to help determine the zeros of quadratic functions.

MULTI-STEP TEST PREP

66. This problem will prepare you for the Multi-Step Test Prep on page 364.

   A baseball player hits a ball toward the outfield. The height h of the ball in feet is modeled by $h(t) = -16t^2 + 22t + 3$, where t is the time in seconds. In addition, the function $d(t) = 85t$ models the horizontal distance d traveled by the ball.

   a. If no one catches the ball, how long will it stay in the air? **1.5 s**

   b. What is the horizontal distance that the ball travels before it hits the ground? **127.5 ft**

Teaching Tip **Geometry** For **Exercise 60,** remind students that the Pythagorean Theorem states that $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.

MULTI-STEP TEST PREP **Exercise 66** involves finding the zeros of a quadratic function representing a baseball's height. This exercise prepares students for the Multi-Step Test Prep on page 364.

Answers

54–59. For graphs, see p. A28.

54. $(-1, -9)$; -8; $-4, 2$

55. $(0, -16)$; -16; $-4, 4$

56. $\left(\frac{1}{2}, -12\frac{1}{4}\right)$; -12; $-3, 4$

57. $(1, 2)$; 0; $0, 2$

58. $\left(2\frac{1}{2}, -12\frac{1}{4}\right)$; -6; $-1, 6$

59. $\left(-\frac{1}{6}, -4\frac{1}{12}\right)$; -4; $-1\frac{1}{3}, 1$

60a. Possible answer: $(x - 2)^2 + (x - 4)^2 = x^2$

60c. Possible answer: The solutions represent possible lengths in centimeters of the hypotenuse. If $x = 10$, the triangle would have side lengths of 10 cm, 8 cm, and 6 cm. If $x = 2$, the triangle would have side lengths of 2 cm, 0 cm, and −2 cm. Because length cannot be negative, only the solution $x = 10$ is reasonable.

64–65. See p. A28.

5-3 PROBLEM SOLVING

Erin and her friends launch a rocket from ground level vertically into the air with an initial velocity of 80 feet per second. The height of the rocket, $h(t)$, after t seconds is given by $h(t) = -16t^2 + 80t$.

1. They want to find out how high they can expect the rocket to go and how long it will be in the air.

   a. Use the standard form $f(x) = ax^2 + bx + c$ to find values for a, b, and c. $a = -16, b = 80, c = 0$

   b. Use the coordinates for the vertex of the path of the rocket to find t, the number of seconds the rocket will be in the air before it starts its downward path. $t = \frac{-80}{2(-16)} = 2.5$ seconds

   c. Substitute the value for t in the given function to find the maximum height of the rocket. How high can they expect their rocket to go? 100 feet

   d. Megan points out that the rocket will have a height of zero again when it returns to the ground. How long will the rocket stay in the air? 5 seconds

2. Megan gets ready to launch the same rocket from a platform 21 feet above the ground with the same initial velocity. How long will the rocket stay in the air this time?

   a. Write a function that represents the rocket's path for this launch. $h(t) = -16t^2 + 80t + 21$

   b. Factor the corresponding equation to find the values for t when h is zero. $(4t + 1)(-4t + 21) = 0$

   c. Erin says that the roots of the equation are $t = 5.25$ and $t = -20.25$ and that the rocket will stay in the air 5.5 seconds. Megan says she is wrong. Who is correct? How do you know?

   Possible answer: Erin has the roots correct. Set each factor equal to 0 and solve for t. But the rocket will stay in the air 5.25 seconds. (The negative root represents the time before launch since the rocket is starting at 21 feet, not at ground level.)

Choose the letter for the best answer.

3. Which function models the path of a rocket that lands 3 seconds after launch?

   (A) $h(t) = -16t^2 + 32t + 48$
   B $h(t) = -16t^2 + 32t + 10.5$
   C $h(t) = -16t^2 + 40t + 48$
   D $h(t) = -16t^2 + 40t + 10.5$

4. Megan reads about a rocket whose path can be modeled by the function $h(t) = -16t^2 + 100t + 15$. Which could be the initial velocity and launch height?

   A 15 ft/s; 100 ft off the ground
   B 16 ft/s; 100 ft off the ground
   (C) 100 ft/s; 15 ft off the ground
   D 171 ft/s; 15 ft off the ground

5-3 CHALLENGE

Some equations that are not of the second degree can be rewritten in quadratic form. Once in quadratic form, the equation may be solved by factoring. When you solve an equation in quadratic form, you may obtain a value that does not satisfy the original equation. For this reason, it is important to check all solutions in the original equation.

Example

Solve: $x^4 - 29x^2 + 100 = 0$

Solution

		Check
	$x^4 - 29x^2 + 100 = 0$	$x^4 - 29x^2 + 100 = 0$
Rewrite the equation in x^2.	$(x^2)^2 - 29(x^2) + 100 = 0$	$(\pm5)^4 - 29(\pm5)^2 + 100$
Factor.	$(x^2 - 25)(x^2 - 4) = 0$	$= 625 - 725 + 100$
Zero Product Property	$x^2 - 25 = 0$ or $x^2 - 4 = 0$	$= 0$ ✓
Solve for x.	$x = \pm5$ or $x = \pm2$	$x^4 - 29x^2 + 100 = 0$
		$(\pm2)^4 - 29(\pm2)^2 + 100$
		$= 0$ ✓

Check all four values in the original equation, as shown at right.

So, there are four solutions: −5, −2, 2, 5

Determine if each equation can be expressed in quadratic form. If so, write the equation in quadratic form. If not, write *no*. Do not solve.

1. $x^4 + 4x^2 - 5 = 0$ $(x^2)^2 + 4(x^2) - 5 = 0$
2. $x^4 - x^3 + 12 = 0$ No
3. $x^8 - 17x^4 + 16 = 0$ $(x^4)^2 - 17(x^4) + 16 = 0$
4. $x^3 - 2x^2 - 5 = 0$ No
5. $x - 2\sqrt{x} - 8 = 0$ $(\sqrt{x})^2 - 2(\sqrt{x}) - 8 = 0$
6. $(x - 3)^2 - 4(x - 3) - 21 = 0$ It is in quadratic form. Let $y = x - 3$, then $y^2 - 4y - 21 = 0$.

Factor each equation to determine if it contains a quadratic factor. If so, write the factored form of the equation. If not, write *no*. Do not solve.

7. $x^3 - 9x = 0$ $x(x^2 - 9) = 0$
8. $x^5 - 4x^4 - 12x^3 = 0$ $x^3(x^2 - 4x - 12) = 0$
9. $2x^3 - x^2 - 9 = 0$ No

Write each equation in quadratic form and solve. Check your answers.

10. $x^4 - 10x^2 + 9 = 0$ $(x^2)^2 - 10(x^2) + 9 = 0$; $\pm1, \pm3$
11. $9x^4 - 18x^2 + 8 = 0$ $9(x^2)^2 - 18(x^2) + 8 = 0$; $\pm\sqrt{\frac{2}{3}}, \pm\sqrt{\frac{4}{3}}$
12. $x - 3\sqrt{x} - 4 = 0$ $(\sqrt{x})^2 - 3(\sqrt{x}) - 4 = 0$; 16

5-3 PRACTICE A

5-3 PRACTICE C

5-3 PRACTICE B

Find the zeros of each function by using a graph and a table.

1. $f(x) = x^2 + 5x + 6$

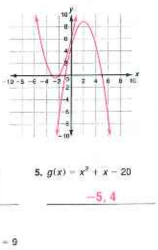
x	−4	−3	−2	−1	0
$f(x)$	2	0	0	2	6

−2 and −3

2. $g(x) = -x^2 + 4x + 5$

x	−2	0	2	4	6
$f(x)$	−7	5	9	5	−7

−1 and 5



Find the zeros of each function by factoring.

3. $h(x) = -x^2 - 6x - 9$ −3
4. $f(x) = 2x^2 + 9x + 4$ −0.5, −4
5. $g(x) = x^2 + x - 20$ −5, 4

Find the roots of each equation by factoring.

6. $12x = 9x^2 + 4$ $\frac{2}{3}$
7. $16x^2 = 9$ −0.75, 0.75

Write a quadratic function in standard form for each given set of zeros.

8. −2 and 7 $f(x) = x^2 - 5x - 14$
9. 1 and −8 $f(x) = x^2 + 7x - 8$

Solve.

10. The quadratic function that approximates the height of a javelin throw is $h(t) = -0.08t^2 + 4.48$, where t is the time in seconds after it is thrown and h is the javelin's height in feet. How long will it take for the javelin to hit the ground? About 7.5 s