50 - 52, 55 - 57

Exercises 5.2

In Exercises 1–6, list the transformations needed to transform the graph of $h(x) = 2^x$ into the graph of the given function. (Section 3.4 may be helpful.)

1. $f(x) = 2^x - 5$
2. $g(x) = -(2^x)$
3. $k(x) = 3(2^x)$
4. $g(x) = 2^{x-1}$
5. $f(x) = 2^{x+2} - 5$
6. $g(x) = -5(2^{x-1}) + 7$

In Exercises 7–13, list the transformations needed to transform the graph of $h(x) = 3^x$ into the graph of the given function. (Section 3.4 may be helpful.)

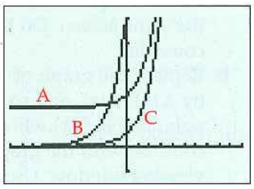
7. $f(x) = 3^x + 4$
8. $g(x) = 3^{-x}$
9. $k(x) = \frac{1}{4}(3^x)$
10. $g(x) = 3^{0.4x}$
11. $f(x) = 3^{2-x}$
12. $f(x) = 8 + 5(3^x)$
13. $g(x) = 4(3^{-0.15x})$

In Exercises 14–19, sketch a complete graph of the function.

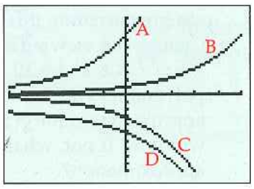
14. $f(x) = 4^{-x}$
15. $f(x) = \left(\frac{5}{2}\right)^{-x}$
16. $f(x) = 2^{3x}$
17. $g(x) = 3^{\frac{x}{2}}$
18. $f(x) = 2^{5-x}$
19. $g(x) = 2^{x-5}$

In Exercises 20–21, match the functions to the graphs. Assume $a > 1$ and $c > 1$.

20. $f(x) = a^x$ **C**
$g(x) = a^x + 3$ **A**
$h(x) = a^{x+5}$ **B**



21. $f(x) = c^x$ **B**
$g(x) = -3c^x$ **C**
$h(x) = c^{x+5}$ **A**
$k(x) = -3c^x - 2$ **D**



In Exercises 22–29, find a viewing window (or windows) that shows a complete graph of the function.

22. $k(x) = e^{-x}$
23. $f(x) = e^{-x^2}$
24. $f(x) = \dfrac{e^x + e^{-x}}{2}$
25. $h(x) = \dfrac{e^x - e^{-x}}{2}$
26. $g(x) = 2^x - x$
27. $k(x) = \dfrac{2}{e^x + e^{-x}}$
28. $f(x) = \dfrac{5}{1 + e^{-x}}$
29. $g(x) = \dfrac{10}{1 + 9e^{-\frac{x}{2}}}$

In Exercises 30–34, determine whether the function is even, odd, or neither. (See *Excursion* 3.4A.)

30. $f(x) = 10^x$ **neither**
31. $g(x) = 2^x - x$ **neither**
32. $f(x) = \dfrac{e^x + e^{-x}}{2}$ **even**
33. $f(x) = \dfrac{e^x - e^{-x}}{2}$ **odd**
34. $f(x) = e^{-x^2}$ **even**
35. Use the Big-Little concept (see Section 4.4) to explain why $e^x + e^{-x}$ is approximately equal to e^x when x is large.

In Exercises 36–39, find the average rate of change of the function. (See Section 3.7).

36. $f(x) = x(2^x)$ as x goes from 1 to 3 **11**
37. $g(x) = 3^{x^2-x}$ as x goes from -1 to 1 **-4**
38. $h(x) = 5^{-x^2}$ as x goes from -1 to 0 **$\frac{4}{5}$**
39. $f(x) = e^x - e^{-x}$ as x goes from -3 to -1 **≈ 8.84**

In Exercises 40–43, find the difference quotient of the function. (See Section 3.7.)

40. $f(x) = 10^x$
41. $g(x) = 5^{x^2}$
42. $f(x) = 2^x + 2^{-x}$
43. $f(x) = e^x - e^{-x}$

In Exercises 44–49, list all asymptotes of the graph of the function and the approximate coordinates of each local extremum. (See Section 4.3.)

44. $f(x) = x(2^x)$
45. $g(x) = x(2^{-x})$
46. $h(x) = e^{\frac{x^2}{2}}$
47. $k(x) = 2^{x^2-6x+2}$
48. $f(x) = e^{-x^2}$
49. $g(x) = -xe^{\frac{x^2}{20}}$ **No asymptotes; no extrema**

41. $\dfrac{5^{(x+h)^2} - 5^{x^2}}{h}$
42. $\dfrac{(2^{x+h} + 2^{-(x+h)}) - (2^x + 2^{-x})}{h}$
43. $\dfrac{(e^{x+h} - e^{-x-h}) - (e^x - e^{-x})}{h}$
44. The x-axis is a horizontal asymptote; local minimum at $(-1.443, -0.531)$.
45. The x-axis is a horizontal asymptote; local maximum at $(1.44, 0.53)$.
46. No asymptotes; local minimum at $(0, 1)$.
47. No asymptotes; local minimum at $(3, 0.0078)$.
48. The x-axis is a horizontal asymptote; local maximum at $(0, 1)$.

Exercises 5.2

ANSWERS

1. Shift the graph of h vertically 5 units downward.
2. Reflect the graph of h across the x-axis.
3. Stretch the graph of h vertically by a factor of 3.
4. Shift the graph of h horizontally 1 unit to the right.
5. Shift the graph of h horizontally 2 units to the left, then vertically 5 units downward.
6. Shift the graph of h horizontally 1 unit to the right, then reflect it across the x-axis, then stretch it vertically by a factor of 5, then shift it vertically 7 units upward.
7. Shift the graph of h vertically 4 units upward.
8. Reflect the graph of h across the y-axis.
9. Compress the graph of h vertically by a factor of $\frac{1}{4}$.
10. Stretch the graph of h horizontally by a factor of 2.5.
11. Reflect the graph of h across the y-axis, then shift horizontally 2 units to the right.
12. Stretch the graph of h vertically by a factor of 5, then translate it vertically 8 units upward.
13. Reflect the graph of h across the y-axis, stretch horizontally by a factor of $\frac{1}{0.15} = 6\frac{2}{3}$, then stretch vertically by a factor of 4.

14–19. *See pp.* 1066–1067.

22. $-3 \le x \le 3$ and $0 \le y \le 12$
23. $-3 \le x \le 3$ and $0 \le y \le 1$
24. $-4 \le x \le 4$ and $0 \le y \le 10$
25. $-4 \le x \le 4$ and $-10 \le y \le 10$
26. $-10 \le x \le 10$ and $0 \le y \le 20$
27. $-4 \le x \le 4$ and $0 \le y \le 1$
28. $-10 \le x \le 10$ and $0 \le y \le 6$
29. $-5 \le x \le 20$ and $0 \le y \le 10$
35. When x is large, $e^{-x} \approx 0$, so $e^x + e^{-x} \approx e^x + 0 = e^x$.
40. $\dfrac{10^{x+h} - 10^x}{h}$

50. If you deposit \$750 at 2.2% interest, compounded annually and paid from the day of deposit to the day of withdrawal, your balance at time t is given by $B(t) = 750(1.022)^t$. How much will you have after 2 years? after 3 years and 9 months?
≈\$783.36; ≈\$813.77

51. The population of a colony of fruit flies t days from now is given by the function $p(t) = 100 \cdot 3^{\frac{t}{10}}$.
a. What will the population be in 15 days? in 25 days? **≈520; ≈1559**
b. When will the population reach 2500?
≈29.3 days

52. A certain type of bacteria grows according to the function $f(x) = 5000e^{0.4055x}$, where the time x is measured in hours. **≈128,180**
a. What will the population be in 8 hours?
b. When will the population reach 1 million?
≈13.07 hr

53. According to data from the National Center for Health Statistics, the life expectancy at birth for a person born in year x is approximated by the function below.

$$D(x) = \frac{79.257}{1 + 9.7135 \times 10^{24} \cdot e^{-0.0304x}}$$
$$(1900 \le x \le 2050)$$

a. What is the life expectancy of someone born in 1980? in 2000? **≈74.06 yr; ≈76.34 yr**
b. In what year was life expectancy at birth 60 years? **1930**

54. The number of subscribers, in millions, to basic cable TV can be approximated by the function

$$g(x) = \frac{76.7}{1 + 16 \cdot 0.8444^x}$$

where $x = 0$ corresponds to 1970. (Source: The Cable TV Financial Datebook and The Pay TV Newsletter)
a. Estimate the number of subscribers in 1995 and in 2005. **≈62.2 million; ≈73.5 million**
b. When does the number of subscribers reach 70 million? **2000**
c. According to this model, will the number of subscribers ever reach 90 million?
No; 76.7 million is the cap.

55. The estimated number of units that will be sold by a certain company t months from now is given by $N(t) = 100{,}000e^{-0.09t}$.
a. What are the current sales ($t = 0$)? What will sales be in 2 months? in 6 months?
a. 100,000; ≈83,527; ≈58,275
b. No. The graph continues to decrease toward zero.
b. From examining the graph, do you think tha sales will ever start to increase again? Expla

56. a. The function $g(t) = 1 - e^{-0.0479t}$ gives the percentage of the population (expressed as a decimal) that has seen a new TV show t wee after it goes on the air. What percentage of people have seen the show after 24 weeks?
b. Approximately when will 90% of the people have seen it?
a. ≈68.3%. b. ≈48 wk

57. a. The beaver population near a certain lake in year t is approximated by the function $p(t) = \frac{2000}{1 + 199e^{-0.5544t}}$. What is the populatio now (when $t = 0$) and what will it be in 5 years
b. Approximately when will there be 1000 beavers?
a. 10; ≈149 b. ≈9.55 yr

58. *Critical Thinking* Look back at Section 4.3, where the basic properties of graphs of polynomial functions were discussed. Then review the basic properties of the graph of $f(x) = a^x$ discussed in this section. Using these various properties, give an argument to show that for any fixed positive number a, where $a \ne 1$, it is not possible to find a polynomial function $g(x) = c_n x^n + \cdots + c_1 x + c_0$ such that $a^x = g(x)$ for all numbers x. In other words, *no exponential function is a polynomial function.*

59. *Critical Thinking* For each positive integer n, let f_n be the polynomial function below.

$$f_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$$

a. Using the viewing window with $-4 \le x \le 4$ and $-5 \le y \le 55$, graph $g(x) = e^x$ and $f_4(x)$ on the same screen. Do the graphs appear to coincide?
b. Replace the graph of $f_4(x)$ by that of $f_5(x)$, then by $f_6(x)$, $f_7(x)$, and so on until you find a polynomial $f_n(x)$ whose graph appears to coincide with the graph of $g(x) = e^x$ in this viewing window. Use the trace feature to move from graph to graph at the same value of x to see how accurate this approximation is.
c. Change the viewing window so that $-6 \le x \le 6$ and $-10 \le y \le 400$. Is the polynomial you found in part **b** a good approximation for $g(x)$ in this viewing window? If not, what polynomial is a good approximation?

58. Any non-constant polynomial must either "blow up" or "blow down" as $|x|$ increases. Therefore, since any exponential of the form $f(x) = a^x$ where a is not 1 has the x-axis as a horizontal asymptote in either the positive or negative direction, no polynomial can be the same as an exponential of this form.

59. a. Not entirely
b. The graph of $f_8(x)$ appears to coincide with the graph of $g(x)$ on most calculator screens; when $-2.4 \le x \le 2.4$, the maximum error is at most 0.01.
c. Not at the right side of the viewing window; $f_{12}(x)$