Exercises 5.1

ANSWERS

46. $-(a+b)+(a+b)^{\frac{1}{3}}$

67. $x^{\frac{7}{6}}-x^{\frac{11}{6}}$

68. $3x^2+2$

69. $x-y$

70. $2x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{3}{2}}+2x^{\frac{1}{3}}y^{\frac{1}{2}}-y^2$

76. $\dfrac{5+5\sqrt{3}-\sqrt{10}-\sqrt{30}}{15}$

1-4, 22, 24, 30-35, 41-48, 57-63 odds

Exercises 5.1

Note: Unless directed otherwise, assume all letters represent positive real numbers.

In Exercises 1–15, evaluate each expression without using a calculator.

1. $\sqrt{144}$ **12**
2. $\sqrt[3]{64}$ **4**
3. $\sqrt[4]{16}$ **2**
4. $\sqrt[3]{-27}$ **−3**
5. $\sqrt{0.0081}$ **0.09**
6. $\sqrt{0.000169}$ **0.013**
7. $\sqrt[3]{0.008}$ **0.2**
8. $\sqrt[3]{-0.125}$ **−0.5**
9. $\sqrt{0.5^6}$ **0.125**
10. $\sqrt{(-3)^4}$ **9**
11. $27^{\frac{4}{3}}$ **81**
12. $81^{-\frac{1}{4}}$ **$\frac{1}{3}$**
13. $(-64)^{\frac{2}{3}}$ **16**
14. $\left(-\frac{1}{64}\right)^{-\frac{2}{3}}$ **16**
15. $16^{-\frac{3}{2}}$ **$\frac{1}{64}$**

In Exercises 16–40, simplify each expression without using a calculator.

16. $\sqrt{3^{15}}$ **$3^7\sqrt{3}$**
17. $\sqrt[3]{12^{16}}$ **$12^5(\sqrt[3]{12})$**
18. $\sqrt{0.08^{12}}$ **$(0.08)^6$**
19. $\sqrt{(-11)^{28}}$ **11^{14}**
20. $\sqrt[3]{(-0.05)^{24}}$ **$(-0.05)^8$**
21. $\sqrt[3]{0.4^{18}}$ **$(0.4)^6$**
22. $\sqrt{6}\cdot\sqrt{12}$ **$6\sqrt{2}$**
23. $\sqrt{8}\cdot\sqrt{96}$ **$16\sqrt{3}$**
24. $\sqrt[3]{18}\cdot\sqrt[3]{12}$ **6**
25. $\sqrt[3]{-32}\cdot\sqrt[3]{16}$ **−8**
26. $\dfrac{\sqrt{10}}{\sqrt{8}\cdot\sqrt{5}}$ **$\frac{1}{2}$**
27. $\dfrac{\sqrt{6}}{\sqrt{14}\cdot\sqrt{63}}$ **$\frac{\sqrt{3}}{21}$**
28. $\dfrac{\sqrt[3]{324}}{\sqrt[3]{6}\cdot\sqrt[3]{2}}$ **3**
29. $\dfrac{\sqrt[3]{54}}{\sqrt[3]{32}\cdot\sqrt[3]{-4}}$ **$-\frac{3}{4}$**
30. $\sqrt{27}+2\sqrt{3}$ **$5\sqrt{3}$**
31. $4\sqrt{5}-\sqrt{20}$ **$2\sqrt{5}$**
32. $(1+\sqrt{3})(2-\sqrt{3})$ **$-1+\sqrt{3}$**
33. $(3+\sqrt{2})(3-\sqrt{2})$ **7**
34. $(4-\sqrt{3})(5+2\sqrt{3})$ **$14+3\sqrt{3}$**
35. $(2\sqrt{5}-4)(3\sqrt{5}+2)$ **$22-8\sqrt{5}$**
36. $(3\sqrt{2}-4\sqrt{6})^2$ **$114-48\sqrt{3}$**
37. $5\sqrt{20}-\sqrt{45}+2\sqrt{80}$ **$15\sqrt{5}$**
38. $\sqrt[3]{40}+2\sqrt[3]{135}-5\sqrt[3]{320}$ **$-12\sqrt[3]{5}$**
39. $\dfrac{2^{\frac{11}{2}}\cdot 2^{-7}\cdot 2^{-5}}{2^3\cdot 2^{\frac{1}{2}}\cdot 2^{-10}}$ **1**
40. $\dfrac{(3^2)^{-\frac{1}{2}}(9^4)^{-1}}{27^{-3}}$ **1**

In Exercises 41–56, simplify each expression.

41. $\sqrt{16a^8b^{-2}}$ **$4\frac{a^4}{b}$**
42. $\sqrt{24x^6y^{-4}}$ **$\frac{2\sqrt{6}x^3}{y^2}$**
43. $\dfrac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$ **$\frac{d^5}{2\sqrt{c}}$**
44. $\dfrac{\sqrt{a^{-10}b^{-12}}}{\sqrt{a^{14}d^{-4}}}$ **$\frac{d^2}{a^{12}b^6}$**
45. $\sqrt[9]{(4x+2y)^{18}}$ **$(4x+2y)^2$**
46. $\sqrt[3]{a+b}\cdot\sqrt[3]{-(a+b)^2}+\sqrt[3]{a+b}$
47. $\sqrt{x^7}\cdot x^{\frac{5}{2}}\cdot x^{-\frac{3}{2}}$ **$x^{\frac{9}{2}}$**
48. $(x^{\frac{1}{2}}y^3)(x^0y^7)^{-2}$ **$\frac{x^{\frac{1}{2}}}{y^{11}}$**
49. $(c^{\frac{2}{5}}d^{-\frac{2}{3}})(c^6d^3)^{\frac{4}{3}}$ **$c^{\frac{42}{5}}d^{\frac{10}{3}}$**
50. $\left(\dfrac{r^{\frac{2}{3}}}{s^{\frac{1}{5}}}\right)^{\frac{15}{9}}$ **$\frac{r^{\frac{10}{9}}}{s^{\frac{1}{3}}}$**
51. $\dfrac{(7a)^2(5b)^{\frac{3}{2}}}{(5a)^{\frac{3}{2}}(7b)^4}$ **$\frac{a^{\frac{1}{2}}}{49b^{\frac{5}{2}}}$**
52. $\dfrac{(6a)^{\frac{1}{2}}\sqrt{ab}}{a^2b^{\frac{3}{2}}}$ **$\frac{6^{\frac{1}{2}}}{ab}$**
53. $\dfrac{(2a)^{\frac{1}{2}}(3b)^{-2}(4a)^{\frac{3}{5}}}{(4a)^{-\frac{3}{2}}(3b)^2(2a)^{\frac{1}{5}}}$ **$\frac{2^{\frac{9}{2}}a^{\frac{12}{5}}}{3^4b^4}$**
54. $\dfrac{(a^{\frac{3}{4}}b)^2(ab^{\frac{1}{4}})^3}{(ab)^{\frac{1}{2}}(bc)^{-\frac{1}{4}}}$ **$a^4b^{\frac{5}{2}}c^{\frac{1}{4}}$**
55. $(a^{x^2})^{\frac{1}{x}}$ **a^x**
56. $\dfrac{(b^x)^{x-1}}{b^{-x}}$ **b^{x^2}**

In Exercises 57–66, write each expression without radicals, using only positive exponents.

57. $\sqrt[3]{a^2+b^2}$ **$(a^2+b^2)^{\frac{1}{3}}$**
58. $\sqrt[4]{a^3-b^3}$ **(a^3-b^3)**
59. $\sqrt[4]{\sqrt[4]{a^3}}$ **$a^{\frac{3}{16}}$**
60. $\sqrt{\sqrt[3]{a^3b^4}}$ **$a^{\frac{1}{2}}b^{\frac{2}{3}}$**
61. $\sqrt[5]{t}\cdot\sqrt{16t^5}$ **$4t^{\frac{27}{10}}$**
62. $\sqrt{x}\cdot\sqrt[3]{x^2}\cdot\sqrt[4]{x^3}$ **$x^{\frac{23}{12}}$**
63. $(\sqrt[3]{xy^2})^{-\frac{3}{5}}$ **$\frac{1}{x^{\frac{1}{5}}y^{\frac{2}{5}}}$**
64. $\left(\sqrt[4]{r^{14}s^{-\frac{21}{5}}}\right)^{-\frac{3}{7}}$ **$\frac{s^{\frac{9}{20}}}{r^{\frac{3}{2}}}$**
65. $\dfrac{c}{(c^{\frac{5}{6}})^{42}(c^{51})^{-\frac{2}{3}}}$ **1**
66. $(c^{\frac{5}{6}}-c^{-\frac{5}{6}})^2$ **$c^{\frac{5}{3}}-2+\frac{1}{c^{\frac{5}{3}}}$**

In Exercises 67–72, simplify each expression.

67. $x^{\frac{1}{2}}(x^{\frac{2}{3}}-x^{\frac{4}{3}})$
68. $x^{\frac{1}{2}}(3x^{\frac{3}{2}}+2x^{-\frac{1}{2}})$
69. $(x^{\frac{1}{2}}+y^{\frac{1}{2}})(x^{\frac{1}{2}}-y^{\frac{1}{2}})$
70. $(x^{\frac{1}{3}}+y^{\frac{1}{2}})(2x^{\frac{1}{3}}-y^{\frac{3}{2}})$
71. $(x+y)^{\frac{1}{2}}[(x+y)^{\frac{1}{2}}-(x+y)]$ **$x+y-(x+y)$**
72. $(x^{\frac{1}{3}}+y^{\frac{1}{3}})(x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}})$ **$x+y$**

In Exercises 73–78, rationalize the denominator and simplify your answer.

73. $\dfrac{3}{\sqrt{8}}$ **$\frac{3\sqrt{2}}{4}$**
74. $\dfrac{2}{\sqrt{6}}$ **$\frac{\sqrt{6}}{3}$**
75. $\dfrac{3}{2+\sqrt{12}}$ **$\frac{3\sqrt{3}-}{4}$**
76. $\dfrac{1+\sqrt{3}}{5+\sqrt{10}}$
77. $\dfrac{2}{\sqrt{x}+2}$ **$\frac{2\sqrt{x}-4}{x-4}$**
78. $\dfrac{\sqrt{x}}{\sqrt{x}-\sqrt{c}}$ **$\frac{x+\sqrt{xc}}{x-c}$**

In Exercises 79–84, factor the given expression. For example, $x - x^{\frac{1}{2}} - 2 = (x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} + 1)$.

79. $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6$

80. $x^{\frac{2}{5}} + 11x^{\frac{1}{5}} + 30$

81. $x + 4x^{\frac{1}{2}} + 3$

82. $x^{\frac{1}{3}} + 7x^{\frac{1}{6}} + 10$

83. $x^{\frac{4}{5}} - 81$

84. $x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + 9$

In Exercises 85–88, rationalize the numerator and simplify your answer. Assume $h \neq 0$.

85. $\dfrac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$

86. $\dfrac{2\sqrt{x+h+3} - 2\sqrt{x+3}}{h}$

87. $\dfrac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h}$

88. $\dfrac{\sqrt{(x+h)^2-(x+h)} - \sqrt{x^2-x}}{h}$

89. Some restrictions are necessary when defining fractional powers of a negative number.
a. Explain why the equations $x^2 = -4$, $x^4 = -4$, $x^6 = -4$, etc., have no real solutions. Conclude that $c^{\frac{1}{2}}, c^{\frac{1}{4}}, c^{\frac{1}{6}}$ cannot be defined when $c = -4$.
b. Since $\frac{1}{3}$ is the same as $\frac{2}{6}$, it should be true that $c^{\frac{1}{3}} = c^{\frac{2}{6}}$, that is, that $\sqrt[3]{c} = \sqrt[6]{c^2}$. Show that this is false when $c = -8$.

90. a. Suppose r is a solution of the equation $x^n = c$ and s is a solution of $x^n = d$. Verify that rs is a solution of $x^n = cd$.
b. Explain why part **a** shows that $\sqrt[n]{cd} = \sqrt[n]{c} \cdot \sqrt[n]{d}$.

91. Write laws 3, 4, and 5 of exponents in radical notation in the case when $r = \frac{1}{m}$ and $s = \frac{1}{n}$.

92. a. Graph $f(x) = x^5$ and explain why this function has an inverse function.
b. Show algebraically that the inverse function is $g(x) = x^{\frac{1}{5}}$.

93. If n is an odd positive integer, show that $f(x) = x^n$ has an inverse function and find the rule of the inverse function. *Hint:* Exercise 92 is the case when $n = 5$.

94. A long pendulum swings more slowly than a short pendulum. The time it takes for a pendulum to complete one full swing, or cycle, is called its period. The relationship between the period T (in seconds) of the pendulum and its length x (in meters) is given by the function $T(x) = 2p\sqrt{\frac{x}{9.8}}$. Find the period for pendulums whose lengths are 0.5 m and 1.0 m.
0.5 m: ≈1.4 sec; 1.0 m: ≈2.0 sec

95. In meteorology, the wind chill C can be calculated by using the formula
$C = 0.0817(3.71\sqrt{V} + 5.81 - 0.25V)(t - 91.4) + 91.4$,
where V is the wind speed in miles per hour and t is the air temperature in degrees Fahrenheit. Find the wind chill when the wind speed is 12 miles per hour and the temperature is 35°F.
≈19°F

96. The elevation E in meters above sea level and the boiling point of water, T, in degrees Celsius at that elevation are related by the equation $E \approx 1000(100 - T) + 580(100 - T)^2$. Find the approximate boiling point of water at an elevation of 1600 meters.
≈99°C

97. Accident investigators can usually estimate a motorist's speed s in miles per hour by examining the length d in feet of the skid marks on the road. The estimate of the speed also depends on the road surface and weather conditions. If f represents the coefficient of friction between rubber and the road surface, then $s = \sqrt{30fd}$ gives an estimate of the motorist's speed. The coefficient of friction f between rubber and concrete under wet conditions is 0.4. Estimate, to the nearest mile per hour, a motorist's speed under these conditions if the skid marks are 200 feet long.
49 mph

98. Using a viewing window with $0 \le x \le 4$ and $0 \le y \le 2$, graph the following functions on the same screen.

$$f(x) = x^{\frac{1}{2}} \qquad g(x) = x^{\frac{1}{4}} \qquad h(x) = x^{\frac{1}{6}}$$

In each of the following cases, arrange $x^{\frac{1}{2}}, x^{\frac{1}{4}}$, and $x^{\frac{1}{6}}$ in order of increasing size and justify your answer by using the graphs.
a. $0 < x < 1$ **b.** $x > 1$

99. Using a viewing window with $-3 \le x \le 3$ and $-1.5 \le y \le 1.5$, graph the following functions on the same screen.

$$f(x) = x^{\frac{1}{3}} \qquad g(x) = x^{\frac{1}{5}} \qquad h(x) = x^{\frac{1}{7}}$$

In each of the following cases, arrange $x^{\frac{1}{3}}, x^{\frac{1}{5}}$, and $x^{\frac{1}{7}}$ in order of increasing size and justify your answer by using the graphs.
a. $x < -1$ **b.** $-1 < x < 0$
c. $0 < x < 1$ **d.** $x > 1$

79. $(x^{\frac{1}{3}} + 3)(x^{\frac{1}{3}} - 2)$

80. $(x^{\frac{1}{5}} + 6)(x^{\frac{1}{5}} + 5)$

81. $(x^{\frac{1}{2}} + 3)(x^{\frac{1}{2}} + 1)$

82. $(x^{\frac{1}{6}} + 5)(x^{\frac{1}{6}} + 2)$

83. $(x^{\frac{2}{5}} + 9)(x^{\frac{1}{5}} + 3)(x^{\frac{1}{5}} - 3)$

84. $(x^{\frac{1}{3}} - 3)^2$

85. $\dfrac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$

86. $\dfrac{2}{\sqrt{x+h+3} + \sqrt{x+3}}$

87. $\dfrac{2x+h}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$

88. $\dfrac{2x+h-1}{\sqrt{(x+h)^2-(x+h)} + \sqrt{x^2-x}}$

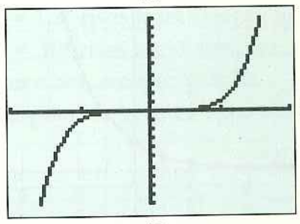
89. a. The square (or any even power) of a real number is never negative. Graphically these equations lie strictly above or on the x-axis.
b. $\sqrt[3]{-8} = -2$, whereas $\sqrt[6]{(-8)^2} = 2$

90. a. We know that $r^n = c$, $s^n = d$. Now $(rs)^n = r^n s^n = cd$
b. Since $\sqrt[n]{a} = a^{\frac{1}{n}}$, if $\sqrt[n]{c} = x$, $\sqrt[n]{d} = y$, then $\sqrt[n]{cd} = xy$

91. $\sqrt[n]{\sqrt[m]{c}} = \sqrt[mn]{c}$; $\sqrt[m]{cd} = \sqrt[m]{c}\sqrt[m]{d}$; $\sqrt[m]{\dfrac{c}{d}} = \dfrac{\sqrt[m]{c}}{\sqrt[m]{d}}$

92. a.

10

−2  2

−10

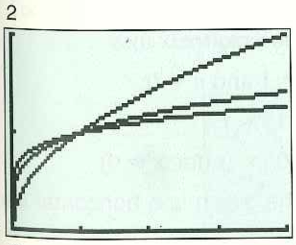
The function has an inverse since it passes the horizontal line test and so is one-to-one.

b. $y = x^5$; $y^{\frac{1}{5}} = (x^5)^{\frac{1}{5}}$; $y^{\frac{1}{5}} = x$
Now switch variables and use function notation to have the inverse $g(x) = x^{\frac{1}{5}}$.

93. When n is an odd positive integer, if $a < b$, $a^n < b^n$. Therefore, $f(x) = x^n$ is an increasing function and thus is one-to-one. Therefore, $f(x) = x^n$ has an inverse if n is an odd positive integer. The inverse is $g(x) = \sqrt[n]{x}$.

98.

2



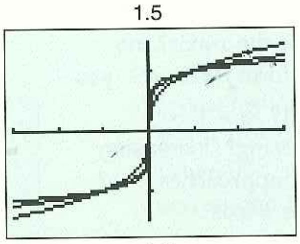
0 4

0

a. $x^{\frac{1}{2}} < x^{\frac{1}{4}} < x^{\frac{1}{6}}$
b. $x^{\frac{1}{6}} < x^{\frac{1}{4}} < x^{\frac{1}{2}}$

99.

1.5

−3  3

−1.5

a. $x^{\frac{1}{3}} < x^{\frac{1}{5}} < x^{\frac{1}{7}}$
b. $x^{\frac{1}{7}} < x^{\frac{1}{5}} < x^{\frac{1}{3}}$
c. $x^{\frac{1}{3}} < x^{\frac{1}{5}} < x^{\frac{1}{7}}$
d. $x^{\frac{1}{7}} < x^{\frac{1}{5}} < x^{\frac{1}{3}}$