

1. All real numbers except $-\frac{5}{2}$
2. All real numbers except 3 and $-\frac{1}{2}$
3. All real numbers except $3 \pm \sqrt{5}$
4. All real numbers except 0 and $\pm\sqrt{6}$
5. All real numbers except $\pm\sqrt{2}$ and 1
6. All real numbers except $x \approx 1.00948$ and $x \approx -2.76232$
7. Vertical asymptotes $x = -1$ and $x = 6$
8. Vertical asymptotes $x = 0$ and $x = \frac{-7 \pm \sqrt{41}}{2}$
9. Hole at $x = 0$; vertical asymptote $x = -1$
10. Hole at $x = 0$; no vertical asymptotes
11. Vertical asymptote at $x = -2$; hole at $x = 2$
12. Hole at $x = 3$; vertical asymptote at $x = -2$
13. $y = 3$; any window with $-115 \leq x \leq 110$
14. $y = \frac{3}{2}$; any window with $-20 \leq x \leq 20$
15. $y = -1$; any window with $-31 \leq x \leq 35$
16. $y = 0$; any window with $-20 \leq x \leq 20$
17. $y = \frac{5}{2}$; any window with $-40 \leq x \leq 42$
18. $y = 16$; window: $-500 \leq x \leq 500$ and $0 \leq y \leq 20$
19. Asymptote: $y = x$; window: $-14 \leq x \leq 14$ and $-15 \leq y \leq 15$
20. $y = x^2 - 2x + 2$; any window with $-100 \leq y \leq 100$
21. Asymptote: $y = x^2 - x$; window: $-15 \leq x \leq 6$ and $-40 \leq y \leq 240$
22. $y = x + 4$; standard viewing window

24-34 evens 27, 33, 40, 44, 46

Exercises 4.4

In Exercises 1-6, find the domain of the function.

1. $f(x) = \frac{-3x}{2x+5}$
2. $g(x) = \frac{x^3+x+1}{2x^2-5x-3}$
3. $h(x) = \frac{6x-5}{x^2-6x+4}$
4. $g(x) = \frac{x^3-x^2-x-1}{x^5-36x}$
5. $f(x) = \frac{x^5-2x^3+7}{x^3-x^2-2x+2}$
6. $h(x) = \frac{x^5-5}{x^4+12x^3+60x^2+50x-125}$

In Exercises 7-12, use algebra to determine the location of the vertical asymptotes and holes in the graph of the function.

7. $f(x) = \frac{x^2+4}{x^2-5x-6}$
8. $g(x) = \frac{x-5}{x^3+7x^2+2x}$
9. $f(x) = \frac{x}{x^3+2x^2+x}$
10. $g(x) = \frac{x}{x^3+5x}$
11. $f(x) = \frac{x^2-4x+4}{(x+2)(x-2)^3}$
12. $h(x) = \frac{x-3}{x^2-x-6}$

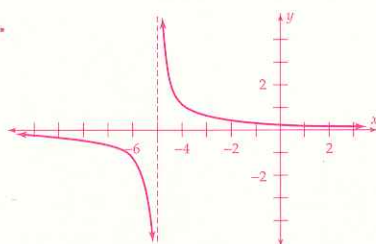
In Exercises 13-22, find the horizontal or other asymptote of the graph of the function when $|x|$ is large, and find a viewing window in which the ends of the graph are within 0.1 of this asymptote.

13. $f(x) = \frac{3x-2}{x+3}$
14. $g(x) = \frac{3x^2+x}{2x^2-2x+4}$
15. $h(x) = \frac{5-x}{x-2}$
16. $f(x) = \frac{4x^2-5}{2x^3-3x^2+x}$
17. $g(x) = \frac{5x^3-8x^2+4}{2x^3+2x}$
18. $h(x) = \frac{8x^5-6x^3+2x-1}{0.5x^5+x^4+3x^2+x}$
19. $f(x) = \frac{x^3-1}{x^2-4}$
20. $g(x) = \frac{x^3-4x^2+6x+5}{x-2}$
21. $h(x) = \frac{x^3+3x^2-4x+1}{x+4}$
22. $f(x) = \frac{x^3+3x^2-4x+1}{x^2-x}$

In Exercises 23-50, analyze the function algebraically: list its vertical asymptotes, holes, and horizontal asymptote. Then sketch a complete graph of the function.

23. $f(x) = \frac{1}{x+5}$
24. $q(x) = \frac{-7}{x-6}$
25. $k(x) = \frac{-3}{2x+5}$
26. $g(x) = \frac{-4}{2-x}$
27. $f(x) = \frac{3x}{x-1}$
28. $p(x) = \frac{x-2}{x}$
29. $f(x) = \frac{2-x}{x-3}$
30. $g(x) = \frac{3x-2}{x+3}$
31. $f(x) = \frac{1}{x(x+1)^2}$
32. $g(x) = \frac{x}{2x^2-5x-3}$
33. $f(x) = \frac{x-3}{x^2+x-2}$
34. $g(x) = \frac{x+2}{x^2-1}$
35. $h(x) = \frac{(x^2+6x+5)(x+5)}{(x+5)^3(x-1)}$
36. $f(x) = \frac{x^2-1}{x^3-2x^2+x}$
37. $f(x) = \frac{-4x^2+1}{x^2}$
38. $k(x) = \frac{x^2+1}{x^2-1}$
39. $q(x) = \frac{x^2+2x}{x^2-4x-5}$
40. $F(x) = \frac{x^2+x}{x^2-2x+4}$
41. $p(x) = \frac{(x+3)(x-3)}{(x-5)(x+4)(x+3)}$
42. $p(x) = \frac{x^3+3x^2}{x^4-4x^2}$
43. $f(x) = \frac{x^2-x-6}{x-2}$
44. $k(x) = \frac{x^2+x-2}{x}$
45. $Q(x) = \frac{4x^2+4x-3}{2x-5}$
46. $K(x) = \frac{3x^2-12x+15}{3x+6}$
47. $f(x) = \frac{x^3-2}{x-1}$
48. $p(x) = \frac{x^3+8}{x+1}$
49. $q(x) = \frac{x^3-1}{x-2}$
50. $f(x) = \frac{x^4-1}{x^2}$

23.



vertical asymptote $x = -5$
horizontal asymptote $y = 0$

24-50. See pp. 1063-1066.

In Exercises 51–60, find a viewing window or windows that show(s) a complete graph of the function using asymptotes, intercepts, end behavior, and holes. Be alert for hidden behavior.

$$51. f(x) = \frac{x^3 + 4x^2 - 5x}{(x^2 - 4)(x^2 - 9)}$$

$$52. g(x) = \frac{x^2 + x - 6}{x^3 - 19x + 30}$$

$$53. h(x) = \frac{2x^2 - x - 6}{x^3 + x^2 - 6x}$$

$$54. f(x) = \frac{x^3 - x + 1}{x^4 - 2x^3 - 2x^2 + x - 1}$$

$$55. f(x) = \frac{2x^4 - 3x^2 + 1}{3x^4 - x^2 + x - 1}$$

$$56. g(x) = \frac{x^4 + 2x^3}{x^5 - 25x^3}$$

$$57. h(x) = \frac{3x^2 + x - 4}{2x^2 - 5x}$$

$$58. f(x) = \frac{2x^2 - 1}{3x^3 + 2x + 1}$$

$$59. g(x) = \frac{x - 4}{2x^3 - 5x^2 - 4x + 12}$$

$$60. h(x) = \frac{x^2 - 9}{x^3 + 2x^2 - 23x - 60}$$

In Exercises 61–66, find a viewing window or windows that show(s) a complete graph of the function—if possible, with no erroneous vertical line segments. Be alert for hidden behavior.

$$61. f(x) = \frac{2x^2 + 5x + 2}{2x + 7}$$

$$62. g(x) = \frac{2x^3 + 1}{x^2 - 1}$$

$$63. h(x) = \frac{x^3 - 2x^2 + x - 2}{x^2 - 1}$$

$$64. f(x) = \frac{3x^3 - 11x - 1}{x^2 - 4}$$

$$65. g(x) = \frac{2x^4 + 7x^3 + 7x^2 + 2x}{x^3 - x + 50}$$

$$66. h(x) = \frac{2x^3 + 7x^2 - 4}{x^2 + 2x - 3}$$

67. a. Graph $f(x) = \frac{1}{x}$ in the viewing window with $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$.

- b. Without using a calculator, describe how the graph of $g(x) = \frac{2}{x}$ can be obtained from the graph of $f(x)$. *Hint:* $g(x) = 2f(x)$

- c. Without using a calculator, describe how the graphs of each of the following functions can be obtained from the graph of $f(x)$.

$$h(x) = \frac{1}{x} + 4 \quad k(x) = \frac{1}{x-3} \quad t(x) = \frac{1}{x+2}$$

- d. Without using a calculator, describe how the graph of $p(x) = \frac{2}{x-3} + 4$ can be obtained

from the graph of $f(x) = \frac{1}{x}$.

- e. Show that the function $p(x)$ of part d is a rational function by rewriting its rule as the quotient of two first-degree polynomials.

- f. If r , s , and t are constants, describe how the graph of $q(x) = \frac{r}{x+s} + t$ can be obtained from the graph of $f(x) = \frac{1}{x}$.

- g. Show that the function $q(x)$ of part f is a rational function by rewriting its rule as the quotient of two first-degree polynomials.

68. The graph of $f(x) = \frac{2x^3 - 2x^2 - x + 1}{3x^3 - 3x^2 + 2x - 1}$ has a vertical asymptote. Find a viewing window that demonstrates this fact.

69. a. Find the difference quotient of $f(x) = \frac{1}{x}$ and express it as a single fraction in lowest terms.

- b. Use the difference quotient in part a to determine the average rate of change of $f(x)$ as x changes from 2 to 2.1, from 2 to 2.01, and from 2 to 2.001. Estimate the instantaneous rate of change of $f(x)$ at $x = 2$.

- c. Use the different quotient in part a to determine the average rate of change of $f(x)$ as x changes from 3 to 3.1, from 3 to 3.01, and from 3 to 3.001. Estimate the instantaneous rate of change of $f(x)$ at $x = 3$.

- d. How are the estimated instantaneous rates of change of $f(x)$ at $x = 2$ and $x = 3$ related to the values of $g(x) = \frac{-1}{x^2}$ at $x = 2$ and $x = 3$?

70. Do Exercise 69 for the functions $f(x) = \frac{1}{x^2}$ and

$$g(x) = \frac{-2}{x^3}.$$

71. a. When $x \geq 0$, what rational function has the same graph as $f(x) = \frac{x-1}{|x|-2}$? *Hint:* Use the definition of absolute value.

- b. When $x < 0$, what rational function has the same graph as $f(x) = \frac{x-1}{|x|-2}$? See the hint for part a.

51. Overall: $-5 \leq x \leq 4.4$ and $-8 \leq y \leq 4$; hidden area near origin: $-2 \leq x \leq 2$ and $-0.5 \leq y \leq 0.5$; hidden area near $x = -5$: $-15 \leq x \leq -3$ and $-0.07 \leq y \leq 0.02$

52. $-8 \leq x \leq 8$ and $-5 \leq y \leq 5$

53. $-9.4 \leq x \leq 9.4$ and $-4 \leq y \leq 4$; there is a hole at $x = 2$.

54. $-5 \leq x \leq 8$ and $-3 \leq y \leq 2$

55. Overall: $-4.7 \leq x \leq 4.7$ and $-2 \leq y \leq 2$; there is a hole at $x = -1$; to see the vertical asymptote, use $0.65 \leq x \leq 0.75$ and $-3 \leq y \leq 3$.

56. $-12 \leq x \leq 12$ and $-4 \leq y \leq 4$ there is a hole at $x = 0$

57. For vertical asymptotes and x -intercepts: $-4.7 \leq x \leq 4.7$ and $-8 \leq y \leq 8$; to see graph get close to the horizontal asymptote: $-40 \leq x \leq 35$ and $-2 \leq y \leq 3$

58. $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$

59. Overall: $-4.7 \leq x \leq 4.7$ and $-2 \leq y \leq 2$; hidden area near $x = 4$: $3 \leq x \leq 15$ and $-0.02 \leq y \leq 0.01$

60. $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$; there is a hole at $x = -3$

61. $-15.5 \leq x \leq 8.5$ and $-16 \leq y \leq 8$

62. $-5 \leq x \leq 5$ and $-7 \leq y \leq 7$

63. $-4.7 \leq x \leq 4.7$ and $-12 \leq y \leq 8$

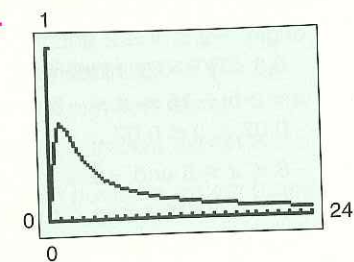
64. $-4 \leq x \leq 4$ and $-12 \leq y \leq 12$

65. Overall: $-13 \leq x \leq 7$ and $-20 \leq y \leq 20$; hidden area near the origin: $-2.5 \leq x \leq 1$ and $-0.02 \leq y \leq 0.02$

66. $-6 \leq x \leq 6$ and $-12 \leq y \leq 12$

- 67–71. See p. 1066.

about 26%



- b. The horizontal asymptote is $c = 0$. The amount of the drug in the bloodstream gets vanishingly small.
d. There is about 55.9% in the blood at time 1.12.

$$\begin{aligned} \text{a. } C(x) &= \frac{3}{100}(2x^2) + \frac{1.25}{100} \left(4x \cdot \frac{1000}{x^2} \right) \\ &= 0.06x^2 + \frac{50}{x} \end{aligned}$$

b. $x \approx 16.85$ in.

$$\text{a. } C(x) = \frac{2800 + 3.5x^2}{x}$$

- b. speeds of about 13.91 and 57.52 miles per hour
c. about 28.28 miles per hour

$$\text{a. } c(x) = \frac{20 + x}{50 + x}$$

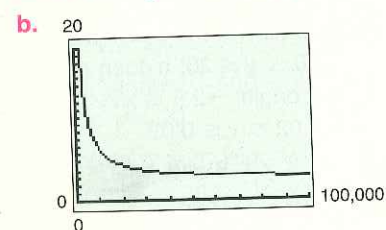
- b. between 25 gallons and 100 gallons

c. $x = 50$ gallons

$$\text{a. } P(x) = \frac{500 + x^2}{x}$$

- b. between 10 and 50 meters

$$\text{a. } a(x) = \frac{c(x)}{x} = \frac{40,000 + 2.60x}{x}$$



- c. $y = 2.60$; the average cost can never be below \$2.60.

b. a. $h_1 = h - 2$

b. $h_1 = \frac{150}{\pi r^2} - 2$

c. $V = \pi(r - 1)^2 \left(\frac{150}{\pi r^2} - 2 \right)$

- d. r must be more than 1 since the walls are 1 foot thick.

- c. Use parts a and b to explain why the graph of $f(x) = \frac{x-1}{|x|-2}$ has two vertical asymptotes. What are they? Confirm your answer by graphing the function.

72. The percentage c of a drug in a person's bloodstream t hours after its injection is approximated by $c(t) = \frac{5t}{4t^2 + 5}$.

- a. Approximately what percentage of the drug is in the person's bloodstream after four and a half hours?
b. Graph the function c in an appropriate window for this situation.
c. What is the horizontal asymptote of the graph? What does it tell you about the amount of the drug in the bloodstream?
d. At what time is the percentage the highest? What is the percentage at that time?

73. A box with a square base and a volume of 1000 cubic inches is to be constructed. The material for the top and bottom of the box costs \$3 per 100 square inches and the material for the sides costs \$1.25 per 100 square inches.

- a. If x is the length of a side of the base, express the cost of constructing the box as a function of x .
b. If the side of the base must be at least 6 inches long, for what value of x will the cost of the box be \$20?

74. A truck traveling at a constant speed on a reasonably straight, level road burns fuel at the rate of $g(x)$ gallons per mile, where x is the speed of the truck in miles per hour and $g(x)$ is given by $g(x) = \frac{800 + x^2}{200x}$.

- a. If fuel costs \$1.40 per gallon, find the rule of the cost function $c(x)$ that expresses the cost of fuel for a 500-mile trip as a function of the speed. *Hint:* $500 \cdot g(x)$ gallons of fuel are needed to go 500 miles. (Why?)
b. What driving speed will make the cost of fuel for the trip \$250?
c. What driving speed will minimize the cost of fuel for the trip?

75. Pure alcohol is being added to 50 gallons of a coolant mixture that is 40% alcohol.

- a. Find the rule of the concentration function $c(x)$ that expresses the percentage of alcohol in the resulting mixture as a function of the number x of gallons of pure alcohol that are added. *Hint:* The final mixture contains $50 + x$ gallons.

(Why?) So $c(x)$ is the amount of alcohol in the final mixture divided by the total amount $50 + x$. How much alcohol is in the original 50-gallon mixture? How much is in the final mixture?

- b. How many gallons of pure alcohol should be added to produce a mixture that is at least 60% alcohol and no more than 80% alcohol?
c. Determine algebraically the exact amount of pure alcohol that must be added to produce a mixture that is 70% alcohol.

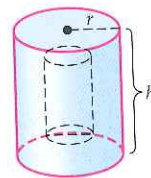
76. A rectangular garden with an area of 250 square meters is to be located next to a building and fenced on three sides, with the building acting as a fence on the fourth side.

- a. If the side of the garden parallel to the building has length x meters, express the amount of fencing needed as a function of x .
b. For what values of x will less than 60 meters of fencing be needed?

77. A certain company has fixed costs of \$40,000 and variable costs of \$2.60 per unit.

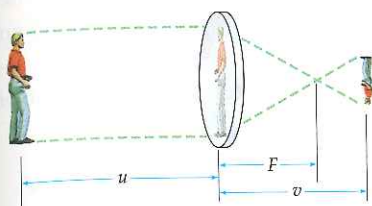
- a. Let x be the number of units produced. Find the rule of the average cost function. (The average cost is the cost of the units divided by the number of units.)
b. Graph the average cost function in a window with $0 \leq x \leq 100,000$ and $0 \leq y \leq 20$.
c. Find the horizontal asymptote of the average cost function. Explain what the asymptote means in this situation, that is, how low can the average cost possibly be?

78. Radioactive waste is stored in a cylindrical tank, whose exterior has radius r and height h as shown in the figure. The sides, top, and bottom of the tank are one foot thick and the tank has a volume of 150 cubic feet including top, bottom, and walls.



- a. Express the interior height h_1 (that is, the height of the storage area) as a function of h .
b. Express the interior height as a function of r .
c. Express the volume of the interior as a function of r .
d. Explain why r must be greater than 1.

79. The relationship between the fixed focal length F of a camera, the distance u from the object being photographed to the lens, and the distance v from the lens to the film is given by $\frac{1}{F} = \frac{1}{u} + \frac{1}{v}$.



- a. If the focal length is 50 mm, express v as a function of u .
 b. What is the horizontal asymptote of the graph of the function in part a?
 c. Graph the function in part a when $50 \text{ mm} < u < 35,000 \text{ mm}$.
80. a. ≈ 9.801 meters per second per second

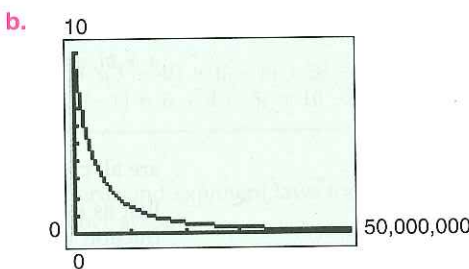
- d. When you focus the camera on an object, the distance between the lens and the film is changed. If the distance from the lens to the camera changes by less than 0.1 millimeter, the object will remain in focus. Explain why you have more latitude in focusing on distant objects than on very close ones.

80. The formula for the gravitational acceleration in units of meters per second squared of an object relative to the earth is

$$g(r) = \frac{3.987 \times 10^{14}}{(6.378 \times 10^6 + r)^2}$$

where r is the distance in meters above the earth's surface.

- a. What is the gravitational acceleration at the earth's surface?
 b. Graph the function $g(r)$ for $r \geq 0$.
 c. Can you ever escape the pull of gravity? Does the graph have any r -intercepts?



- c. no; no

4.5

Complex Numbers

Objectives

- Write complex numbers in standard form
- Perform arithmetic operations on complex numbers
- Find the conjugate of a complex number
- Simplify square roots of negative numbers
- Find all solutions of polynomial equations

If restricted to nonnegative numbers, you cannot solve the equation $x + 5 = 0$. Enlarging the number system to include negative integers makes it possible to find the solution to this equation. By enlarging the number system to include rational numbers, it is possible to solve equations that have no integer solution, such as $3x = 7$. Similarly, the equation $x^2 = 2$ has no rational solution, but $x = \sqrt{2}$ and $x = -\sqrt{2}$ are real number solutions. The idea of enlarging a number system to include solutions to equations that cannot be solved in a particular number system is a natural one.

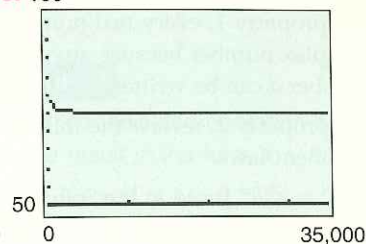
Complex Numbers

Equations such as $x^2 = -1$ and $x^2 = -4$ have no solutions in the real number system because $\sqrt{-1}$ and $\sqrt{-4}$ are not real numbers. In order to solve such equations, that is, to find the square roots of negative numbers, the number system must be enlarged again. There is a number system, called the **complex number system**, with the desired properties.

79. a. $v = \frac{50u}{u - 50}$

b. $v = 50$

c. 100



- d. If the object is close, a small change in u leads to a large change in v . However, when u is large, a small change in u leads to nearly no change in v , so that u may change substantially while the object stays in focus.

section

4.5

Complex Numbers

Teaching Notes

Point out that when you build a new number system from a smaller one, the new one should contain the smaller one. In particular, all natural numbers are integers, all integers are rational numbers, all rational numbers are real numbers, and, in the present setting, all real numbers are complex numbers.

You may want to review (from Section 2.2):

- The solutions of a quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number, and the equation has no real solutions.

In this section, students will learn that when $b^2 - 4ac < 0$, there are solutions of $ax^2 + bx + c = 0$ that are complex numbers.