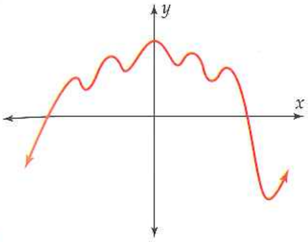
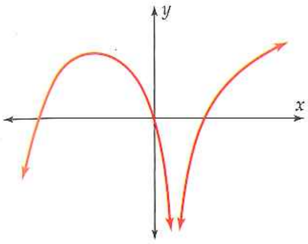
1–12, 15–24

Exercises 4.3

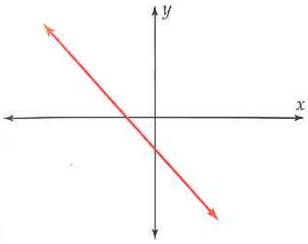
In Exercises 1–6, decide whether the given graph could possibly be the graph of a polynomial function.

1.


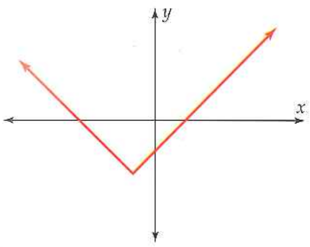
Yes

2.


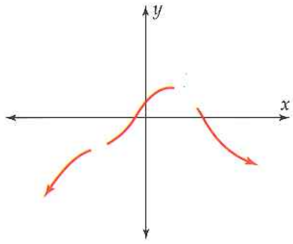
No

3.


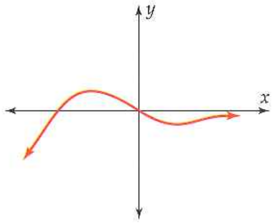
Yes

4.


No

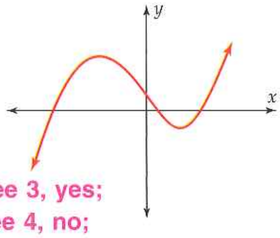
5.


No

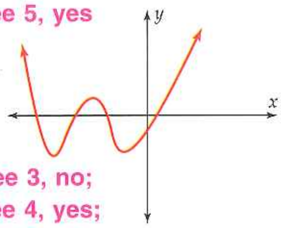
6.


Yes

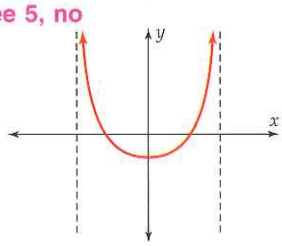
In Exercises 7–12, determine whether the given graph could possibly be the graph of a polynomial function of degree 3, degree 4, or degree 5.

7.


Degree 3, yes; degree 4, no; degree 5, yes

8.


Degree 3, no; degree 4, yes; degree 5, no

9.


No

Exercises 4.3

ANSWERS

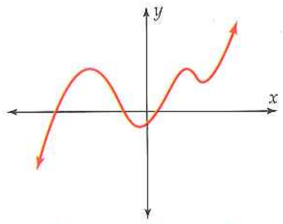
13. The graphs have the same *shape* in the window with $-40 \le x \le 40$ and $-1000 \le y \le 5000$ but don't look identical.

16. -2 and 0 are zeros of even multiplicity; 3 is a zero of odd multiplicity

17. -2 and -1 are zeros of odd multiplicity; 2 is a zero of even multiplicity

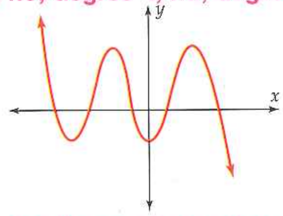
18. -2 and 2 are both zeros of even multiplicity

10.



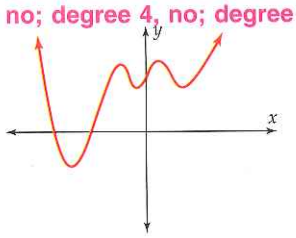
Degree 3, no; degree 4, no; degree 5, yes

11.



Degree 3, no; degree 4, no; degree 5, yes

12.



No

In Exercises 13 and 14, find a viewing window in which the graph of the given polynomial function f appears to have the same general shape as the graph of its leading term.

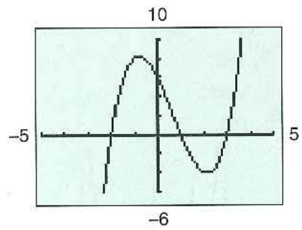
13. $f(x) = x^4 - 6x^3 + 9x^2 - 3$

14. $f(x) = x^3 - 5x^2 + 4x - 2$

$-20 \le x \le 20,\ -1000 \le y \le 1000$

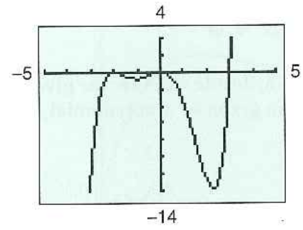
In Exercises 15–18, the graph of a polynomial function is shown. List each zero of the polynomial and state whether its multiplicity is even or odd.

15.

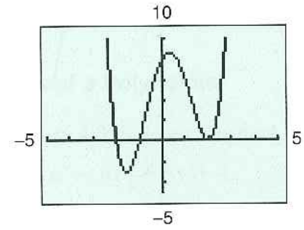


-2, 1, and 3 are zeros of odd multiplicity

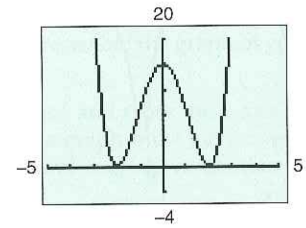
16.



17.

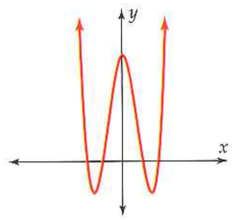


18.

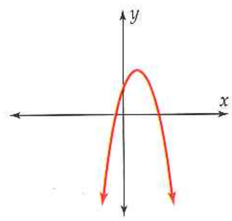


In Exercises 19–24, use your knowledge of polynomial graphs, *not* a calculator, to match the given function with one of graphs a–f.

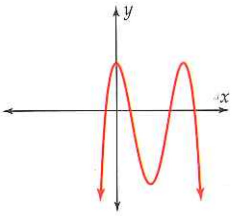
a.



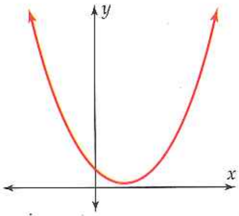
b.



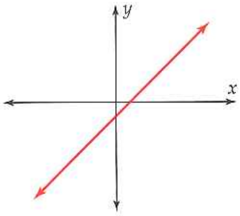
c.



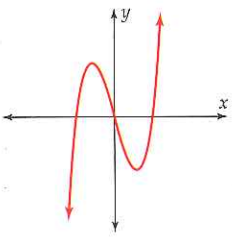
d.



e.



f.



19. $f(x) = 2x - 3$ **e** **20.** $g(x) = x^2 - 4x + 7$ **d**

21. $g(x) = x^3 - 4x$ **f** **22.** $f(x) = x^4 - 5x^2 + 4$ **a**

23. $f(x) = -x^4 + 6x^3 - 9x^2 + 2$ **c**

24. $g(x) = -2x^2 + 3x + 1$ **b**

In Exercises 25–28, graph the function in the standard viewing window and explain why that graph cannot possibly be complete.

25. $f(x) = 0.01x^3 - 0.2x^2 - 0.4x + 7$

26. $g(x) = 0.01x^4 + 0.1x^3 - 0.8x^2 - 0.7x + 9$

27. $h(x) = 0.005x^4 - x^2 + 5$

28. $f(x) = 0.001x^5 - 0.01x^4 - 0.2x^3 + x^2 + x - 5$

In Exercises 29–34, find a single viewing window that shows a complete graph of the function.

29. $f(x) = x^3 + 8x^2 + 5x - 14$
$-9 \le x \le 3$ and $-20 \le y \le 40$

30. $g(x) = x^3 - 3x^2 - 4x - 5$
$-3 \le x \le 5$ and $-20 \le y \le 5$

31. $g(x) = -x^4 - 3x^3 + 24x^2 + 80x + 15$
$-6 \le x \le 6$ and $-60 \le y \le 320$

32. $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$
$0 \le x \le 5$ and $-2 \le y \le 10$

33. $f(x) = 2x^5 - 3.5x^4 - 10x^3 + 5x^2 + 12x + 6$
$-3 \le x \le 4$ and $-35 \le y \le 20$

34. $g(x) = x^5 + 8x^4 + 20x^3 + 9x^2 - 27x - 7$
$-5 \le x \le 2$ and $-18 \le y \le 20$

In Exercises 35–40, find a complete graph of the function and list the viewing window(s) that show(s) this graph.

35. $f(x) = 0.1x^5 + 3x^4 - 4x^3 - 11x^2 + 3x + 2$

36. $g(x) = x^4 - 48x^3 - 101x^2 + 49x + 50$

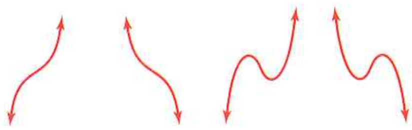
37. $g(x) = 0.03x^3 - 1.5x^2 - 200x + 5$
$-90 \le x \le 120$ and $-15{,}000 \le y \le 5000$

38. $f(x) = 0.25x^6 + 0.25x^5 - 35x^4 - 7x^3 + 823x^2 + 25x - 2750$
$-12 \le x \le 12$ and $-45{,}000 \le y \le 5000$

39. $g(x) = 2x^3 - 0.33x^2 - 0.006x + 5$

40. $f(x) = 0.3x^5 + 2x^4 - 7x^3 + 2x^2$

41. **a.** Explain why the graph of a cubic polynomial function has either two local extrema or none at all. *Hint:* If it had only one, what would the graph look like when $|x|$ is very large?
b. Explain why the general shape of the graph of a cubic polynomial function must be one of the following.



a. **b.** **c.** **d.**

42. The figure shows an incomplete graph of an even polynomial function f of fourth degree. (Even functions were defined in *Excursion* 3.4.A.)
a. Find the zeros of f.
b. Explain why
$$f(x) = k(x - a)(x - b)(x - c)(x - d)$$
where a, b, c, d are the zeros of f.

42. **a.** $x = -4, -2, 2,$ or 4
b. We have four distinct zeros of a fourth degree polynomial. Each zero corresponds to a linear factor of f.
c. $k = 0.25$
d. $(0, 16)$ is a local maximum; $(\pm 3.162, -9)$ are local minima
e. Increasing on $(-3.162, 0)$ and $(3.162, \infty)$ decreasing on $(-\infty, -3.162)$ and $(0, 3.162)$

25. The graph in the standard viewing window does not rise at the far right as does the graph of the highest degree term x^3, so it is not complete.

26. The graph in the standard viewing window does not rise at the far left as does the graph of the highest degree term x^4, so it is not complete.

27. The graph in the standard viewing window does not rise at the far left and far right as does the graph of the highest degree term $0.005x^4$, so it is not complete.

28. The graph in the standard viewing window does not rise at the far right and fall at the far left as does the graph of the highest degree term $0.001x^5$, so it is not complete.

35. Left half: $-33 \le x \le -2$ and $-50{,}000 \le y \le 250{,}000$; right half: $-2 \le x \le 3$ and $-20 \le y \le 30$

36. Overall: $-20 \le x \le 60$ and $-700{,}000 \le y \le 100000$; near y-axis: $-3 \le x \le 2$ and $-100 \le y \le 100$

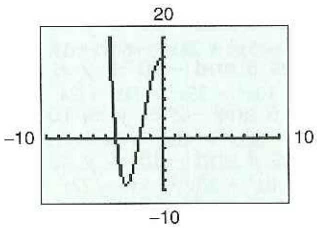
39. Overall: $-3 \le x \le 3$ and $-20 \le y \le 20$; near y-axis: $-0.1 \le x \le 0.2$ and $4.997 \le y \le 5.001$

40. Overall: $-10 \le x \le 10$ and $-500 \le y \le 2500$; near x-axis: $0 \le x \le 3$ and $-10 \le y \le 5$; near y-axis: $-0.5 \le x \le 0.5$ and $-0.1 \le y \le 0.1$

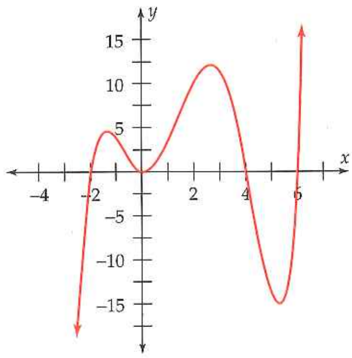
41. **a.** The graph of a cubic polynomial (degree 3) has at most $3 - 1 = 2$ local extrema. When $|x|$ is large, the graph resembles the graph of ax^3, that is, one end shoots upward and the other end downward. If the graph had only one local extremum, both ends of the graph would go in the same direction (both up or down). Thus, the graph of a cubic polynomial has either two local extrema or none.
b. These are the only possible shapes for a graph that has 0 or 2 local extrema, 1 point of inflection, and resembles the graph of ax^3 when $|x|$ is large.

Section 4.3 **Graphs of Polynomial Functions** 271

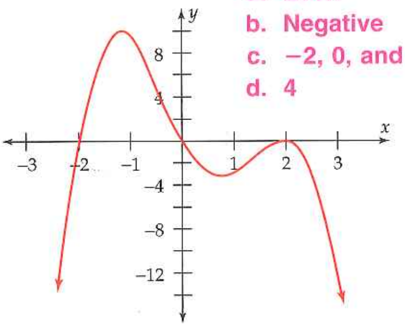
c. Experiment with your calculator to find the value of k that produces the graph in the figure.
d. Find all local extrema of f.
e. List the approximate intervals on which f is increasing and those on which it is decreasing.



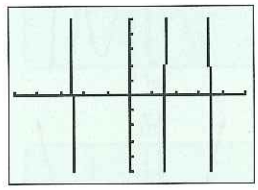
43. A complete graph of a polynomial function g is shown below.
a. Is the degree of $g(x)$ even or odd? **Odd**
b. Is the leading coefficient of $g(x)$ positive or negative? **Positive**
c. What are the real zeros of $g(x)$? **−2, 0, 4, and 6**
d. What is the smallest possible degree of $g(x)$? **5**



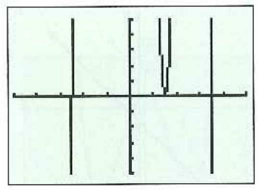
44. Do Exercise 43 for the polynomial function g whose complete graph is shown here.
a. Even
b. Negative
c. −2, 0, and 2
d. 4



45. The figure below is a partial view of the graph of a cubic polynomial whose leading coefficient is negative. Which of the patterns shown in Exercise 41 does this graph have? **d**



46. The figure below is a partial view of the graph of a fourth-degree polynomial. Sketch the general shape of the graph and state whether the leading coefficient is positive or negative. **Negative**



In Exercises 47–56, sketch a complete graph of the function. Label each x-intercept and the coordinates of each local extremum; find intercepts and coordinates exactly when possible, otherwise approximate them.

47. $f(x) = x^3 - 3x^2 + 4$ **48.** $g(x) = 4x - \frac{4x^3}{3}$

49. $h(x) = 0.25x^4 - 2x^3 + 4x^2$

50. $f(x) = 0.25x^4 - \frac{2x^3}{3}$

51. $g(x) = 3x^3 - 18.5x^2 - 4.5x - 45$

52. $h(x) = 2x^3 + x^2 - 4x - 2$

53. $f(x) = x^5 - 3x^3 + x + 1$

54. $g(x) = 0.25x^4 - x^2 + 0.5$

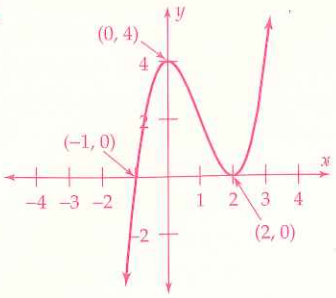
55. $h(x) = 8x^4 + 22.8x^3 - 50.6x^2 - 94.8x + 138.6$

56. $f(x) = 32x^6 - 48x^4 + 18x^2 - 1$

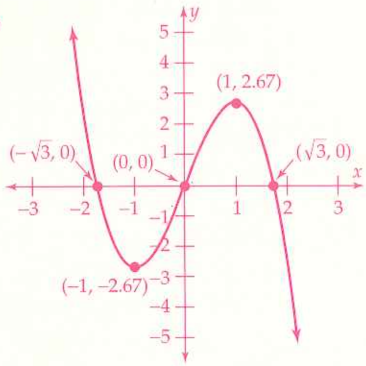
57. *Critical Thinking*
a. Graph $g(x) = 0.01x^3 - 0.06x^2 + 0.12x + 3.92$ in the viewing window with $-3 \le x \le 3$ and $0 \le y \le 6$ and verify that the graph appears to coincide with the horizontal line $y = 4$ between

52–56. ***See p. 1063.***

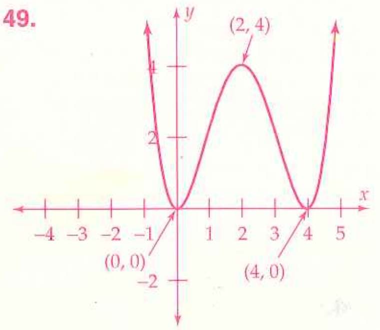
47.



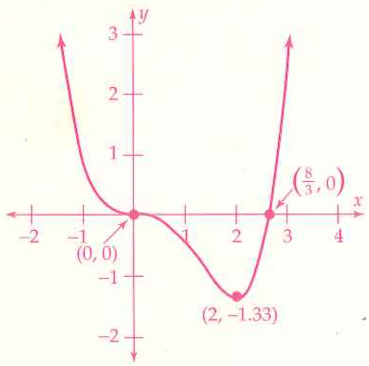
48.



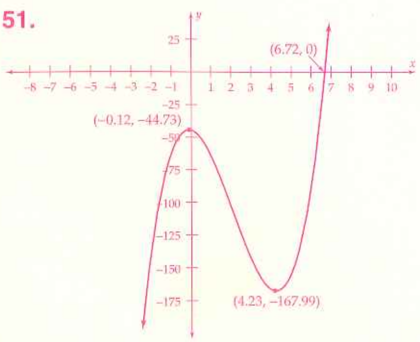
49.



50.



51.



$x = 1$ and $x = 3$. In other words, it appears that every x with $1 \le x \le 3$ is a solution of the equation

$$0.01x^3 - 0.06x^2 + 0.12x + 3.92 = 4.$$

Explain why this is impossible. Conclude that the actual graph is not horizontal between $x = 1$ and $x = 3$.

b. Use the trace feature to verify that the graph is actually rising from left to right between $x = 1$ and $x = 3$. Find a viewing window that shows this.

c. Show that it is not possible for the graph of a polynomial $f(x)$ to contain a horizontal segment. *Hint:* A horizontal line segment is part of the horizontal line $y = k$ for some constant k. Adapt the argument in part **a**, which is the case $k = 4$.

58. *Critical Thinking*

a. Let $f(x)$ be a polynomial of odd degree. Explain why $f(x)$ must have at least one real zero. *Hint:* Why must the graph of f cross the x-axis, and what does this mean?

b. Let $g(x)$ be a polynomial of even degree, with a negative leading coefficient and a positive constant term. Explain why $g(x)$ must have at least one positive and at least one negative zero.

59. *Critical Thinking* The graph of

$$f(x) = (x + 18)(x^2 - 20)(x - 2)^2(x - 10)$$

has x-intercepts at each of its zeros, that is, at $x = -18, \pm\sqrt{20} \approx \pm4.472, 2$, and 10. It is also true that $f(x)$ has a relative minimum at $x = 2$.

a. Draw the x-axis and mark the zeros of $f(x)$. Then use the fact that $f(x)$ has degree 6 (Why?) to sketch the general shape of the graph, as was done for cubics in Exercise 41.

b. Now graph $f(x)$ in the standard viewing window. Does the graph resemble your sketch? Does it even show all the x-intercepts between -10 and 10?

c. Graph $f(x)$ in the viewing window with $-19 \le x \le 11$ and $-10 \le y \le 10$. Does this window include all the x-intercepts, as it should?

d. List viewing windows that give a complete graph of $f(x)$.

57. a. The solutions are zeros of $g(x) - 4 = 0.01x^3 - 0.06x^2 + 0.12x - 0.08$. This polynomial has degree 3 and hence has at most 3 zeros.

b. $1 \le x \le 3$ and $3.99 \le y \le 4.01$

c. Suppose $f(x)$ has degree n. If the graph of $f(x)$ had a horizontal segment lying on the line $y = k$ for some constant k, then the equation $f(x) = k$ would have infinitely many solutions. But the polynomial $f(x) - k$ has degree n and thus has at most n roots. Hence the equation $f(x) = k$ has at most n solutions, which means the graph cannot have a horizontal segment.

58–59. ***See p.* 1063.**

4.3.A *Excursion:* Polynomial Models

Objectives

- Fit a polynomial model to data

Linear regression was used in Section 1.5 to construct a linear function that modeled a set of data points. When the scatter plot of the data points looks more like a higher degree polynomial graph than a straight line, similar least squares regression procedures are available on most calculators for constructing quadratic, cubic, and quartic polynomial functions to model the data.

Example 1 A Polynomial Model

The following data, which is based on statistics from the Department of Health and Human Services, gives the *cumulative* number of reported cases of AIDS in the United States from 1982 through 2000. Find a quadratic, a cubic, and a quartic regression equation and determine which equation best models the data.

Section 4.3.A *Excursion:* Polynomial Models

Math Background

Non-linear regression is sometimes called curvilinear regression. Polynomial regression is one type of curvilinear regression. Other types of curvilinear regression include exponential regression, power regression, and logarithmic regression.