Exercises 4.2

When asked to find the zeros of a polynomial, find exact zeros whenever possible and approximate the other zeros.

In Exercises 1–12, find all the rational zeros of the polynomial.

1. $x^3 + 3x^2 - x - 3$ **$x = \pm 1$ or -3**
2. $x^3 - x^2 - 3x + 3$ **$x = 1$**
3. $x^3 + 5x^2 - x - 5$ **$x = \pm 1$ or -5**
4. $3x^3 + 8x^2 - x - 20$ **$x = \frac{4}{3}$**
5. $2x^5 + 5x^4 - 11x^3 + 4x^2$ *Hint:* The Rational Zero Test can only be used on polynomials with nonzero constant terms. Factor $f(x)$ as a product of a power of x and a polynomial $g(x)$ with nonzero constant term. Then use the Rational Zero Test on $g(x)$. **$x = -4, 0, 1$ or $\frac{1}{2}$**
6. $2x^6 - 3x^5 - 7x^4 - 6x^3$ **$x = 0$ or 3**
7. $\frac{1}{12}x^3 - \frac{1}{12}x^2 - \frac{2}{3}x + 1$ *Hint:* The Rational Zero Test can only be used on polynomials with integer coefficients. Note that $f(x)$ and $12f(x)$ have the same zeros. (Why?) **$x = -3$ or 2**
8. $\frac{2}{3}x^4 + \frac{1}{2}x^3 - \frac{5}{4}x^2 - x - \frac{1}{6}$ **$x = -\frac{1}{4}$ or $-\frac{1}{2}$**
9. $\frac{1}{3}x^4 - x^3 - x^2 + \frac{13}{3}x - 2$ **$x = 2$**
10. $\frac{1}{3}x^7 - \frac{1}{2}x^6 - \frac{1}{6}x^5 + \frac{1}{6}x^4$ **$x = 0$ or $\frac{1}{2}$**
11. $0.1x^3 - 1.9x + 3$ **$x = -5, 2,$ or 3**
12. $0.05x^3 + 0.45x^2 - 0.4x + 1$ **$x = -10$**

In Exercises 13–18, factor the polynomial as a product of linear factors and a factor $g(x)$ such that $g(x)$ is either a constant or a polynomial that has no rational zeros.

13. $2x^3 - 4x^2 + x - 2$ **$(x - 2)(2x^2 + 1)$**
14. $6x^3 - 5x^2 + 3x - 1$ **$(2x - 1)(3x^2 - x + 1)$**
15. $x^6 + 2x^5 + 3x^4 + 6x^3$ **$x^3(x^2 + 3)(x + 2)$**
16. $x^5 - 2x^4 + 2x^3 - 3x + 2$ **$(x - 1)^2(x + 1)(x^2 - x + 2)$**
17. $x^5 - 4x^4 + 8x^3 - 14x^2 + 15x - 6$ **$(x - 2)(x - 1)^2(x^2 + 3)$**
18. $x^5 + 4x^3 + x^2 + 6x$ **$x(x^4 + 4x^2 + x + 6)$**

In Exercises 19–22, use the Bounds Test to find lower and upper bounds for the real zeros of the polynomial.

19. $x^3 + 2x^2 - 7x + 20$ **Lower −5; upper 2**
20. $x^3 - 15x^2 - 16x + 12$ **Lower −2; upper 16**
21. $-x^5 - 5x^4 + 9x^3 + 18x^2 - 68x + 176$ *Hint:* The Bounds Test applies only to polynomials with a positive leading coefficient. The polynomial $f(x)$ has the same zeros as $-f(x)$. Why? **Lower −7; upper 3**
22. $-0.002x^3 - 5x^2 + 8x - 3$ **Lower −2502; upper 2**

In Exercises 23–36, find all real zeros of the polynomial.

23. $2x^3 - 5x^2 + x + 2$ **$x = 1, 2,$ or $-\frac{1}{2}$**
24. $t^4 - t^3 + 2t^2 - 4t - 8$ **$x = 2$ or -1**
25. $6x^3 - 11x^2 + 6x - 1$
26. $z^3 + z^2 + 2z + 2$ **$x = -1$**
27. $x^4 + x^3 - 19x^2 + 32x - 12$ **$x = 2$ or $\frac{-5 \pm \sqrt{37}}{2}$**
28. $3x^5 + 2x^4 - 7x^3 + 2x^2$ **$x = -2, 0, \frac{1}{3},$ or 1**
29. $2x^5 - x^4 - 10x^3 + 5x^2 + 12x - 6$ **$x = \frac{1}{2}$ or $\pm\sqrt{2}$ or $\pm\sqrt{3}$**
30. $x^5 - x^3 + x$ **$x = 0$**
31. $x^6 - 4x^5 - 5x^4 - 9x^2 + 36x + 45$ **$x = -1, 5,$ or $\pm\sqrt{3}$**
32. $x^5 + 3x^4 - 4x^3 - 11x^2 - 3x + 2$ **$x = -1, 2,$ or $\frac{-3 \pm \sqrt{13}}{2}$**
33. $3x^4 + 2x^3 - 4x^2 + 4x - 1$ **$x = \frac{1}{3}$ or $x \approx -1.8393$**
34. $x^5 + 8x^4 + 20x^3 + 9x^2 - 27x - 27$ **$x = -3$ or $x \approx 1.1479$**
35. $x^4 - 48x^3 - 101x^2 + 49x + 50$ **$x \approx -2.2470$ or -0.5550 or 0.8019 or $x = 50$**
36. $3x^7 + 8x^6 - 13x^5 - 36x^4 - 10x^3 + 21x^2 + 41x + 10$

37. a. Show that $\sqrt{2}$ is an irrational number. *Hint:* $\sqrt{2}$ is a zero of $x^2 - 2$. Does this polynomial have any rational zeros?
    b. Show that $\sqrt{3}$ is irrational.

38. Graph $f(x) = 0.001x^3 - 0.199x^2 - 0.23x + 6$ in the standard viewing window.
    a. How many zeros does $f(x)$ appear to have? Without changing the viewing window, explain why $f(x)$ must have an additional zero. *Hint:* Each zero corresponds to a factor of $f(x)$. What does the rest of the factorization consist of?
    b. Find all the zeros of $f(x)$.

39. According to the FBI, the number of people murdered each year per 100,000 population can be approximated by the polynomial function $f(x) = 0.0011x^4 - 0.0233x^3 + 0.1144x^2 + 0.0126x + 8.1104$ $(0 \le x \le 10)$, where $x = 0$ corresponds to 1987.

40.

41.

42.

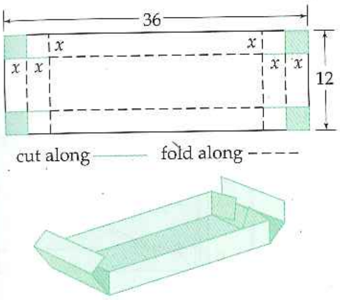
Exercises 4.2
ANSWERS

5. $x = 1, \frac{1}{2},$ or $\frac{1}{3}$
6. $x = -\frac{5}{3}$ or 2, $x \approx 1.19869$, -0.286462, or -2.91223
7. a. The only possible rational zeros of $f(x) = x^2 - 2$ are ± 1 or ± 2. But $\sqrt{2}$ is a zero of $f(x)$ and $\sqrt{2} \neq \pm 1$ or ± 2. Hence, $\sqrt{2}$ is irrational.
   b. $\sqrt{3}$ is a zero of $x^2 - 3$ whose only possible rational zeros are ± 1 or ± 3. But $\sqrt{3} \neq \pm 1$ or ± 3.
8. a. We see two real zeros and so there are two distinct linear factors of f. If we divide these out we will be left with another linear factor which must correspond to another real zero.
   b. $x = 5, -6,$ or 200
9. a. 8.6378 people per 100,000
   b. 1995
   c. 1991

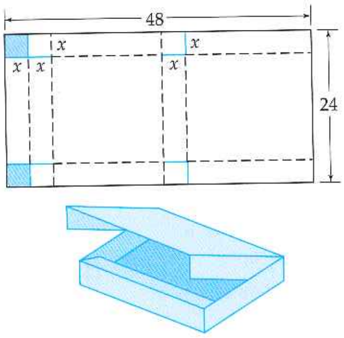
a. What was the murder rate in 1990?
b. In what year was the rate 8 people per 100,000?
c. In what year was the rate the highest?

40. During the first 150 hours of an experiment, the growth rate of a bacteria population at time t hours is $g(t) = -0.0003t^3 + 0.04t^2 + 0.3t + 0.2$ bacteria per hour.
   a. What is the growth rate at 50 hours? at 100 hours?
   b. What is the growth rate at 145 hours? What does this mean?
   c. At what time is the growth rate 0?
   d. At what time is the growth rate -50 bacteria per hour?
   e. Approximately at what time does the highest growth rate occur?

41. An open-top reinforced box is to be made from a 12-by-36-inch piece of cardboard by cutting along the marked lines, discarding the shaded pieces, and folding as shown in the figure. If the box must be less than 2.5 inches high, what size squares should be cut from the corners in order for the box to have a volume of 448 cubic inches?



42. A box with a lid is to be made from a 48-by-24-inch piece of cardboard by cutting and folding, as shown in the figure. If the box must be at least 6 inches high, what size squares should be cut from the two corners in order for the box to have a volume of 1000 cubic inches?



43. In a sealed chamber where the temperature varies, the instantaneous rate of change of temperature with respect to time over an 11-day period is given by $F(t) = 0.0035t^4 - 0.4t^2 - 0.2t + 6$, where time is measured in days and temperature in degrees Fahrenheit (so that rate of change is in degrees per day).
   a. At what rate is the temperature changing at the beginning of the period ($t = 0$)? at the end of the period ($t = 11$)?
   b. When is the temperature increasing at a rate of 4°F per day?
   c. When is the temperature decreasing at a rate of 3°F per day?
   d. When is the temperature decreasing at the fastest rate?

44. *Critical Thinking*
   a. If c is a zero of

$$f(x) = 5x^4 - 4x^3 + 3x^2 - 4x + 5,$$

   show that $\frac{1}{c}$ is also a zero.
   b. Do part **a** with $f(x)$ replaced by $g(x)$.

$$g(x) = 2x^6 + 3x^5 + 4x^4 - 5x^3 + 4x^2 + 3x + 2$$

   c. Let

$$f(x) = a_{12}x^{12} + a_{11}x^{11} + \cdots + a_2x^2 + a_1x + a_0.$$

   If c is a zero of f, what conditions must the coefficients a_i satisfy so that $\frac{1}{c}$ is also a zero?

40. a. 77.7 bacteria per hour; 130.2 bacteria per hour
b. -29.8875 bacteria per hour, so the population is shrinking
c. approximately 140.485 hours
d. approximately 147.765 hours
e. approximately 92.493 hours

41. 2 by 2 in.

42. approximately 6.63564 in. by 6.63564 in.

43. a. 6° per day at the beginning; 6.6435° per day at the end
b. Day 2.0330 and day 10.7069
c. Day 5.0768 and day 9.6126
d. Day 7.6813

44. a. $f\left(\frac{1}{c}\right) = 5\left(\frac{1}{c}\right)^4 - 4\left(\frac{1}{c}\right)^3 + 3\left(\frac{1}{c}\right)^2 - 4\left(\frac{1}{c}\right) + 5$

$$= \frac{5 - 4c + 3c^2 - 4c^3 + 5c^4}{c^4}$$

$$= \frac{0}{c^4} = 0$$

b. $f\left(\frac{1}{c}\right) = 2\left(\frac{1}{c}\right)^6 + 3\left(\frac{1}{c}\right)^5 + 4\left(\frac{1}{c}\right)^4 - 5\left(\frac{1}{c}\right)^3 + 4\left(\frac{1}{c}\right)^2 + 3\left(\frac{1}{c}\right) + 2$

$$= \frac{2 + 3c + 4c^2 - 5c^3 + 4c^4 + 3c^5 + 2c^6}{c^6}$$

$$= \frac{0}{c^6} = 0$$

c. $a_0 = a_{12}$, $a_1 = a_{11}$, $a_2 = a_{10}$, etc.