Name _____ Date _____ Class _____

LESSON 3-5

Reteach

Solving Inequalities with Variables on Both Sides

Variables must be collected on the same side of an inequality before the inequality can be solved. If you collect the variables so that the variable term is positive, you will not have to multiply or divide by a negative number.

Solve $x > 8(x - 7)$.

Collect the variables on the left.

$x > 8(x - 7)$

$x > 8x - 56$ *Distribute.*

$-8x \quad -8x$ *Add $-8x$ to both sides.*

$-7x > -56$

$\frac{-7x}{-7} > \frac{-56}{-7}$ *Divide both sides by -7.*

$x < 8$ *Reverse the sign.*

Solve $x > 8(x - 7)$.

Collect the variables on the right.

$x > 8(x - 7)$

$x > 8x - 56$ *Distribute.*

$-x \quad -x$ *Add $-x$ to both sides.*

$0 > 7x - 56$

$+56 \quad +56$

$56 > 7x$

$\frac{56}{7} > \frac{7x}{7}$ *Divide both sides by 7.*

$8 > x$

$x < 8$

Notice that if you want to have the variable on the left to make graphing solutions easier, you may still need to switch the inequality sign, even if you did not multiply or divide by a negative number.

Write the first step you would take to solve each inequality if you wanted to keep the variable positive.

1. $6y < 10y + 1$ _____

2. $4p - 2 \geq 3p$ _____

3. $5 - 3r \leq 6r$ _____

Solve each inequality.

4. $8c + 4 > 4(c - 3)$

5. $5(x - 1) < 3x + 10 - 8x$

6. $-8 + 4a - 12 > 2a + 10$

Name _____ Date _____ Class _____

LESSON 3-5

Reteach

Solving Inequalities with Variables on Both Sides (continued)

An inequality with infinite solutions is called an identity.

Solve $-2x - 5 \leq 4x + 8 - 6x$.

$-2x - 5 \leq 4x + 8 - 6x$

$-2x - 5 \leq -2x + 8$ *Combine like terms.*

$+2x \quad +2x$ *Add 2x to each side.*

$-5 \leq 8$ ✓ *True statement. This is an identity.*

The solution is the set of all real numbers.

Check any value of x:

Try $x = 3$.

$-2x - 5 \leq 4x + 8 - 6x$

$-2(3) - 5 \overset{?}{\leq} 4(3) + 8 - 6(3)$

$-6 - 5 \overset{?}{\leq} 12 + 8 - 18$

$-11 \overset{?}{\leq} 2$ ✓

An inequality with no solutions is called a contradiction.

Solve $3(x - 4) > 7 + 3x$.

$3(x - 4) > 7 + 3x$

$3x - 12 > 7 + 3x$ *Distribute.*

$-3x \quad -3x$ *Add $-3x$ to each side.*

$-12 > 7$ ✘ *False statement. This is a contradiction.*

There are no solutions.

Check any value of x:

Try $x = 2$.

$3(x - 4) > 7 + 3x$

$3(2 - 4) \overset{?}{>} 7 + 3(2)$

$3(-2) \overset{?}{>} 7 + 6$

$-6 \overset{?}{>} 13$ ✘

Solve each inequality.

7. $t + 5 < t + 5$

8. $x + 5 \leq x + 5$

9. $4y + 3(y - 2) < 7y$

10. $10n - 4 \leq 5(2n + 1)$

11. $9x + 3 - 5x \geq 2(2x + 5)$

LESSON 3-5

Practice A

Solving Inequalities with Variables on Both Sides

Fill in the blanks to solve each inequality.

1. $2x \le 3x + 8$

$-3x \quad -3x$

$-1x \le 8$

$\div(-1) \quad \div(-1)$

$x \ge -8$

2. $8y > -2(3y - 7)$

$8y > -6y + 14$

$+6y \quad +6y$

$14y > 14$

$\div 14 \quad \div 14$

$y > 1$

3. $3(5n + 6) < 10n - 4$

$15n + 18 < 10n - 4$

$-10n \quad -10n$

$5n + 18 < -4$

$-18 \quad -18$

$5n < -22$

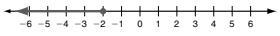
$\div 5 \quad \div 5$

$n < -4\frac{2}{5}$

Solve each inequality and graph the solutions.

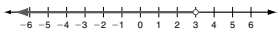
4. $5x \ge 7x + 4$

$x \le -2$



5. $3(b - 5) < -2b$

$b < 3$



Identify each inequality as an identity (all real numbers are solutions) or contradiction (no solutions).

6. $10 < -2$ — contradiction

7. $a - 7 \le a$ — identity

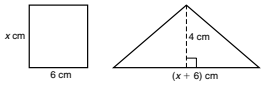
8. $2(z + 3) > 2z$ — identity

Write and solve an inequality for each problem.

9. Jay can buy a stereo either online or at a local store. If he buys online, he gets a 15% discount, but has to pay a $12 shipping fee. At the local store, the stereo is not on sale, but there is no shipping fee. For what regular prices is it cheaper for Jay to buy the stereo online?

$p - 0.15p + 12 < p$; $p > 80$; greater than $80

10. For what values of x is the area of the rectangle greater than the area of the triangle?



$6x > \frac{1}{2}(4)(x + 6)$; $x > 3$

LESSON 3-5

Practice B

Solving Inequalities with Variables on Both Sides

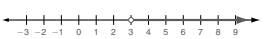
Solve each inequality and graph the solutions.

1. $2x + 30 \ge 7x$

$x \le 6$

2. $2k + 6 < 5k - 3$

$k > 3$

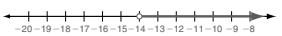


3. $3b - 2 \le 2b + 1$

$b \le 3$

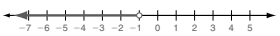
4. $2(3n + 7) > 5n$

$n > -14$



5. $5s - 9 < 2(s - 6)$

$s < -1$

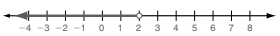


6. $-3(3x + 5) \ge -5(2x - 2)$

$x \ge 25$

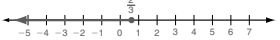
7. $1.4z + 2.2 > 2.6z - 0.2$

$z < 2$



8. $\frac{7}{8}p - \frac{1}{4} \le \frac{1}{2}p$

$p \le \frac{2}{3}$



Solve each inequality.

9. $v + 1 > v - 6$ — all real numbers

10. $3(x + 4) \le 3x$ — no solutions

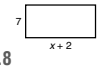
11. $-2(8 - 3x) \ge 6x + 2$ — no solutions

Write and solve an inequality for each problem.

12. Ian wants to promote his band on the Internet. Site A offers website hosting for $4.95 per month with a $49.95 startup fee. Site B offers website hosting for $9.95 per month with no startup fee. For how many months would Ian need to keep the website for Site B to be less expensive than Site A?

$9.95m < 4.95m + 49.95$; $m < 9.99$; for 0 to 9 months

13. For what values of x is the area of the rectangle greater than the perimeter?



$7(x + 2) > 7 + (x + 2) + 7 + (x + 2)$; $x > 0.8$

LESSON 3-5

Practice C

Solving Inequalities with Variables on Both Sides

Solve each inequality.

1. $2x + 1 < 8x - 2$

$x > \frac{1}{2}$

2. $4(3p + 5) \ge -2p$

$p \ge -1\frac{3}{7}$

3. $-2s + 3 \ge -7s$

$s \ge -\frac{3}{5}$

4. $\frac{1}{2}(5 - 2x) > -x + 1$ — all real numbers

5. $5(n - 2) < 4(2n + 6) + 2$

$n > -12$

6. $\frac{2}{3}y + 6 < \frac{2}{3}y - 6$ — no solutions

7. $\frac{3x}{8} + 4 \le 0.2x + 5$

$x \le 5\frac{5}{7}$

8. $-z + 20 > z + 20$

$z < 0$

9. $2a + 10 \le 2(-2a + 3) + 6a$ — no solutions

10. $5b + 20 > -2 + 3b$

$b > -11$

11. $6(k - 5) > 3k - 26$

$k > 1\frac{1}{3}$

12. $0.42d < 152.5 + 0.17d$

$d < 610$

For 13–17, use the table at right. The table gives the populations of Toledo, Ohio, and Lexington, Kentucky, during the last three U.S. Censuses.

	1980	1990	2000
Toledo, OH	350,000	330,000	310,000
Lexington, KY	200,000	230,000	260,000

13. About how much did the population of Toledo change each **year**? (Note: *Not* each decade!) — decreased 2000 people each year

14. Write an expression for the population of Toledo any number of years after 1980. — $350{,}000 - 2000y$

15. About how much did the population of Lexington change each **year**? — increased 3000 people each year

16. Write an expression for the population of Lexington any number of years after 1980. — $200{,}000 + 3000y$

17. Assuming the patterns in the table continue, write and solve an inequality to find the years in which the population of Lexington will be greater than the population of Toledo. — $350{,}000 - 2000y < 200{,}000 + 3000y$; $y > 30$ for years after 2010

LESSON 3-5

Reteach

Solving Inequalities with Variables on Both Sides

Variables must be collected on the same side of an inequality before the inequality can be solved. If you collect the variables so that the variable term is positive, you will not have to multiply or divide by a negative number.

Solve $x > 8(x - 7)$.

Collect the variables on the left.

$x > 8(x - 7)$	
$x > 8x - 56$	*Distribute.*
$-8x \quad -8x$	*Add $-8x$ to both sides.*
$-7x > -56$	
$\frac{-7x}{-7} > \frac{-56}{-7}$	*Divide both sides by -7.*
$x < 8$	*Reverse the sign.*

Solve $x > 8(x - 7)$.

Collect the variables on the right.

$x > 8(x - 7)$	
$x > 8x - 56$	*Distribute.*
$-x \quad -x$	*Add $-x$ to both sides.*
$0 > 7x - 56$	
$+56 \quad +56$	
$56 > 7x$	
$\frac{56}{7} > \frac{7x}{7}$	*Divide both sides by 7.*
$8 > x$	
$x < 8$	

Notice that if you want to have the variable on the left to make graphing solutions easier, you may still need to switch the inequality sign, even if you did not multiply or divide by a negative number.

Write the first step you would take to solve each inequality if you wanted to keep the variable positive.

1. $6y < 10y + 1$ — add $-6y$ to both sides

2. $4p - 2 \ge 3p$ — add $-3p$ to both sides

3. $5 - 3r \le 6r$ — add $3r$ to both sides

Solve each inequality.

4. $8c + 4 > 4(c - 3)$ — $c > -4$

5. $5(x - 1) < 3x + 10 - 8x$ — $x < \frac{3}{2}$

6. $-8 + 4a - 12 > 2a + 10$ — $a > 15$

LESSON 3-5

Reteach

Solving Inequalities with Variables on Both Sides (continued)

An inequality with infinite solutions is called an identity.

Solve $-2x - 5 \leq 4x + 8 - 6x$.

$-2x - 5 \leq 4x + 8 - 6x$		
$-2x - 5 \leq -2x + 8$		*Combine like terms.*
$+2x \quad +2x$		*Add 2x to each side.*
$-5 \leq 8$ ✓		*True statement. This is an identity.*

The solution is the set of all real numbers.

Check any value of x:

Try $x = 3$.

$-2x - 5 \leq 4x + 8 - 6x$

$-2(3) - 5 \stackrel{?}{\leq} 4(3) + 8 - 6(3)$

$-6 - 5 \stackrel{?}{\leq} 12 + 8 - 18$

$-11 \stackrel{?}{\leq} 2$ ✓

An inequality with no solutions is called a contradiction.

Solve $3(x - 4) > 7 + 3x$.

$3(x - 4) > 7 + 3x$		
$3x - 12 > 7 + 3x$		*Distribute.*
$-3x \quad -3x$		*Add $-3x$ to each side.*
$-12 > 7$ ✘		*False statement. This is a contradiction.*

There are no solutions.

Check any value of x:

Try $x = 2$.

$3(x - 4) > 7 + 3x$

$3(2 - 4) \stackrel{?}{>} 7 + 3(2)$

$3(-2) \stackrel{?}{>} 7 + 6$

$-6 \stackrel{?}{>} 13$ ✘

Solve each inequality.

7. $t + 5 < t + 5$

no solutions

8. $x + 5 \leq x + 5$

all real numbers

9. $4y + 3(y - 2) < 7y$

all real numbers

10. $10n - 4 \leq 5(2n + 1)$

all real numbers

11. $9x + 3 - 5x \geq 2(2x + 5)$

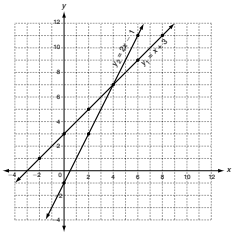
no solutions

LESSON 3-5

Challenge

Above and Below

The grid at right shows the graphs of two functions, $y_1 = x + 3$ and $y_2 = 2x - 1$. These functions can be graphed by creating a table of ordered pairs for each function and then plotting the ordered pairs. The ordered pairs of each function form straight lines as shown.



1. For what value of x is $y_1 = y_2$? What happens to the graphs at this x-value?

$x = 4$; the lines intersect

2. For what values of x is $y_1 > y_2$? What happens to the graphs for these x-values?

$x < 4$; the line for y_1 is above (greater than) the line for y_2

3. The equation $y_1 = x + 3$ tells you that y_1 is the same as the expression $x + 3$. Likewise, $y_2 = 2x - 1$ tells you that y_2 is the same as $2x - 1$. Use this information to rewrite $y_1 > y_2$ using the variable x.

y_1	$>$	y_2
$x + 3$	$>$	$2x - 1$

4. Solve the inequality in problem 3. How do the solutions relate to your answer for problem 2?

$x < 4$; same solutions as problem 2

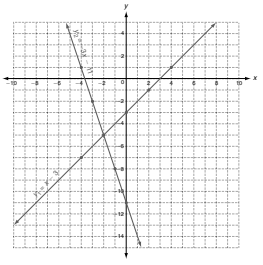
5. Explain how you could use the graph to solve this inequality: $2x - 1 \geq x + 3$.

Possible answer: Find the x-values for which the line for y_2 is either above (greater than) or intersects (equal to) the line for y_1. The solutions are $x \geq 4$.

6. Generate ordered pairs for the functions below. For each, plot the points, connect them with straight lines, and label as in the example above.

$y_1 = x - 3$	
x	y
-4	-7
-2	-5
0	-3
2	-1
4	1

$y_2 = -3x - 11$	
x	y
-4	1
-3	-2
-2	-5
-1	-8
0	-11



Use the graph to solve $x - 3 \leq -3x - 11$.

$x \leq -2$

LESSON 3-5

Problem Solving

Solving Inequalities With Variables on Both Sides

Write and solve an inequality for each situation.

1. Rosa has decided to sell pet rocks at an art fair for $5 each. She has paid $50 to rent a table at the fair and it costs her $2 to package each rock with a set of instructions. For what numbers of sales will Rosa make a profit?

$5r > 50 + 2r$; $r > 17$

2. Jamie has a job paying $25,000 and expects to receive a $1000 raise each year. Wei has a job paying $19,000 a year and expects a $1500 raise each year. For what span of time is Jamie making more money than Wei?

$25{,}000 + 1000y > 19{,}000 + 1500y$; $y < 12$

3. Sophia types 75 words per minute and is just starting to write a term paper. Patton already has 510 words written and types at a speed of 60 words per minute. For what numbers of minutes will Sophia have more words typed than Patton?

$75m > 510 + 60m$; $m > 34$

4. Keith is racing his little sister Pattie and has given her a 15 foot head start. She runs 5 ft/sec and he is chasing at 8 ft/sec. For how long can Pattie stay ahead of Keith?

$15 + 5s > 8s$; $s < 5$

The table below shows the population of four cities in 2004 and the amount of population change from 2003. Use this table to answer questions 5–6.

City	Population (2004)	Population Change (from 2003)
Lakewood, CO	141,301	−830
Vallejo, CA	118,349	−1155
Carrollton, TX	117,823	+1170
Manchester, NH	109,310	+261

5. If the trends in this table continue, after how many years y will the population of Manchester, NH, be more than the population of Vallejo, CA? Round your answer to the nearest tenth of a year.

A $y > 0.2$ **C** $y > 34.6$

(B) $y > 6.4$ **D** $y > 78.6$

6. If the trends in this table continue, for how long x will the population of Carrollton, TX be less than the population of Lakewood, CO? Round your answer to the nearest tenth of a year.

(F) $x < 11.7$ **H** $x < 20.1$

G $x < 14.6$ **J** $x < 28.3$

LESSON 3-5

Reading Strategies

Analyze Choices

When solving an inequality with variables on both sides, you have a choice: Collect the variables on the left, or collect the variables on the right. There are advantages to each, depending on the inequality. Either way, you should get the same answer.

$x - 7 > 4x + 2$

$-4x \quad -4x$

$-3x - 7 > 2$

$+7 \quad +7$

$\frac{-3x}{-3} > \frac{9}{-3}$

$x < -3$

Collect variables on the left.

Analysis

With this choice, we divide by a negative. So we must remember to change the direction of the inequality symbol.

$x - 7 > 4x + 2$

(Collect variables on the left. / Collect variables on the right.)

$x - 7 > 4x + 2$

$-x \quad -x$

$-7 > 3x + 2$

$-2 \quad -2$

$\frac{-9}{3} > \frac{3x}{3}$

$-3 > x$

$x < -3$

Collect variables on the right.

Analysis

With this choice, the solution is "backwards". We must flip the solution so that we can read it from left to right.

Answer each question.

1. Kirby often forgets to switch the inequality symbol when dividing by a negative. When solving $3x - 10 \geq x$, should Kirby collect the variables on the left or the right? Why?

Left; so he divides by a positive number.

2. Rewrite the inequality $8 < t$ so that the variable is on the left.

$t > 8$

3. Solve the inequality $-2(n + 3) > -4n + 8$ by collecting variables on the left, then by collecting variables on the right. Which seems better in this case? Why?

$n > 7$; left; you do not have to switch the inequality symbol.