Name _____ Date _____ Class _____

Reteach

Solving Two-Step and Multi-Step Inequalities

When solving inequalities with more than one step, use inverse operations to isolate the variable. The order of the inverse operations is the order of the operations in reverse. You can check your solution by substituting the endpoint and another point in the solution back into the original inequality.

Solve $-5x + 3 < 23$ and graph the solutions.

$-5x + 3 < 23$

$\underline{ -3 \quad -3}$ *Add -3 to each side.*

$-5x < 20$

$\dfrac{-5x}{-5} < \dfrac{20}{-5}$ *Divide both sides by -5.*

$x > -4$ *Reverse the inequality sign.*

Check:

Try -4.	Try 6.
$-5x + 3 < 23$	$-5x + 3 < 23$
$-5(-4) + 3 \overset{?}{<} 23$	$-5(6) + 3 \overset{?}{<} 23$
$20 + 3 \overset{?}{<} 23$	$-30 + 3 \overset{?}{<} 23$
$23 \overset{?}{<} 23$ ✗	$-27 \overset{?}{<} 23$ ✓

The endpoint -4 is not a solution. The open circle on the graph is correct. The value 6 is a solution. The direction of the inequality symbol is correct.
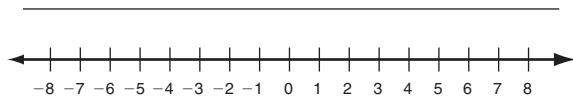
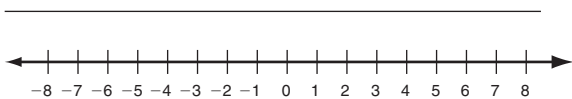**Solve each inequality and graph the solutions.**

1. $-3e - 10 \le -4$

_____

2. $\dfrac{c}{2} + 8 > 11$

_____

3. $15 \le 3 - 4s$

_____

4. $\dfrac{3}{4}j + 1 > 4$

_____

Holt Algebra 1
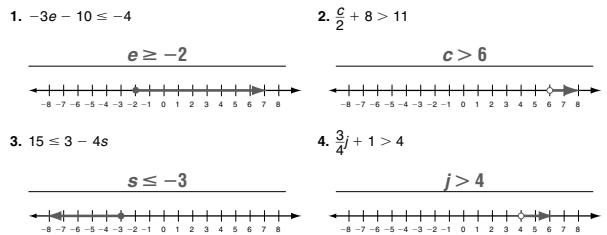
LESSON **Reteach**
3-4 ***Solving Two-Step and Multi-Step Inequalities (continued)***

Solving inequalities may require using the Distributive Property, combining like terms, or clearing fractions. Remember that you can clear fractions by multiplying both sides of the inequality by the least common denominator (LCD).

Solve $\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$.

$$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$$

$$6\left(\frac{3}{2}x + \frac{1}{6}\right) \geq 6\left(\frac{5}{3}\right) \quad \textit{Multiply both sides by the LCD 6.}$$

$$6\left(\frac{3}{2}x\right) + 6\left(\frac{1}{6}\right) \geq 6\left(\frac{5}{3}\right) \quad \textit{Distribute 6.}$$

$$9x + 1 \geq 10$$

$$\underline{-1 \quad\quad -1} \quad \textit{Add} -1 \textit{ to both sides.}$$

$$9x \geq 9$$

$$\frac{9x}{9} \geq \frac{9}{9} \quad \textit{Divide both sides by 9.}$$

$$x \geq 1$$

Check:

Try 1.

$$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$$

$$\frac{3}{2}(1) + \frac{1}{6} \overset{?}{>} \frac{5}{3}$$

$$\frac{3}{2} + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$$

$$\frac{9}{6} + \frac{1}{6} \overset{?}{\geq} \frac{10}{6}$$

$$\frac{10}{6} \overset{?}{\geq} \frac{10}{6} \quad \checkmark$$

Try 2.

$$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$$

$$\frac{3}{2}(2) + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$$

$$3 + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$$

$$3\frac{1}{6} \overset{?}{\geq} 1\frac{2}{3} \quad \checkmark$$

The endpoint 1 is a solution. The value 2 is a solution. The direction of the inequality symbol is correct.

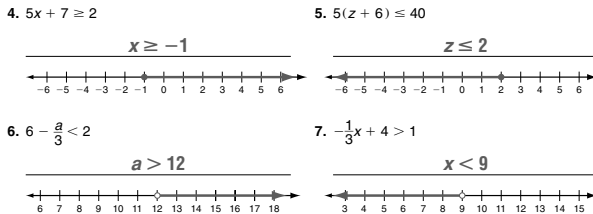**Solve each inequality.**

5. $-\frac{5}{6}x + 3 < \frac{1}{2}$

6. $2(b - 7) + -4b \geq 30 - 18$

7. $\frac{2}{3}(g + 4) - g > 1$

8. $-\frac{3}{5} + \frac{8}{5}k - (3k - 2) \leq 0$

Holt Algebra 1

Solving Two-Step and Multi-Step Inequalities

Fill in the blanks to solve each inequality.

1. $2x - 5 \leq 7$

$\underline{+\ 5} \quad \underline{+\ 5}$

$2x \leq \underline{12}$

$\div\ 2 \quad \div\ 2$

$x \leq \underline{6}$

2. $-3(k-1) < 15$

$-3k + \underline{3} < 15$

$\underline{-\ 3}\ \underline{-\ 3}$

$-3k < \underline{12}$

$\div (-3) \div (-3)$

$k \underline{\geq} \underline{-4}$

3. $\frac{1}{2}n + \frac{5}{6} > \frac{2}{3}$

$\underline{6} \left(\frac{1}{2}n + \frac{5}{6}\right) > \underline{6} \left(\frac{2}{3}\right)$

$3n + 5 > \underline{4}$

$\underline{-\ 5}\ \underline{-\ 5}$

$3n > \underline{-1}$

$\div\ \underline{3} \div\ \underline{3}$

$n\ \underline{>}\ \underline{-\frac{1}{3}}$

Solve each inequality and graph the solutions.

4. $5x + 7 \geq 2$

$x \geq -1$

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

5. $5(z + 6) \leq 40$

$z \leq 2$

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

6. $6 - \frac{a}{3} < 2$

$a > 12$

6 7 8 9 10 11 12 13 14 15 16 17 18

7. $-\frac{1}{3}x + 4 > 1$

$x < 9$

3 4 5 6 7 8 9 10 11 12 13 14 15

Write and solve an inequality for each problem.

8. Ted needs an average of at least 70 on his three history tests. He has already scored 85 and 60 on two tests. What is the minimum grade Ted needs on his third test?

$\frac{85 + 60 + x}{3} \geq 70;\ x \geq 65;$ at least 65

9. A VHS tape holds at most 360 minutes. A tape already has a 120-minute movie on it. How many 30-minute sitcoms can be recorded on the remaining tape?

$30s + 120 \leq 360;\ s \leq 8;$ 0, 1, 2, 3, 4, 5, 6, 7, or 8 sitcoms

Holt Algebra 1

Solving Two-Step and Multi-Step Inequalities

Solve each inequality and graph the solutions.

1. $-3a + 10 < -11$

$a > 7$

1 2 3 4 5 6 7 8 9 10 11 12 13

2. $4x - 12 \geq 20$

$x \geq 8$

2 3 4 5 6 7 8 9 10 11 12 13 14

3. $\frac{2k - 3}{-5} > 7$

$k < -16$

−22 −21 −20 −19 −18 −17 −16 −15 −14 −13 −12 −11 −10

4. $-\frac{1}{5}z + \frac{2}{3} \leq 2$

$z \geq -6\frac{2}{3}$

−12 −11 −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0

5. $6(n - 8) \geq -18$

$n \geq 5$

−1 0 1 2 3 4 5 6 7 8 9 10 11

6. $10 - 2(3x + 4) < 11$

$x > -1\frac{1}{2}$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5

7. $7 + 2c - 4^2 \leq -9$

$c \leq 0$

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

8. $15p + 3(p - 1) > 3(2^3)$

$p > 1\frac{1}{2}$

−4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Write and solve an inequality for each problem.

9. A full-year membership to a gym costs $325 upfront with no monthly charge. A monthly membership costs $100 upfront and $30 per month. For what numbers of months is it less expensive to have a monthly membership?

$30m + 100 < 325;\ m < 7.5;$ 1, 2, 3, 4, 5, 6, or 7 months

10. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side. What are the possible values of x for this triangle?

(3x) cm
(x + 5) cm
40 cm

$x + 5 + 3x > 40;\ x > 8.75$

Holt Algebra 1

Solving Two-Step and Multi-Step Inequalities

Solve each inequality.

1. $4 \geq -3x + 5$

$x \geq \frac{1}{3}$

2. $-2.5(y - 6) \leq -4.5$

$y \geq 7.8$

3. $5 > 12a - (3 - 4a)$

$a < \frac{1}{2}$

4. $5 < \frac{7 - 3h}{-4}$

$h > 9$

5. $\frac{x - 2}{3} \geq \frac{1}{2}$

$x \geq 3\frac{1}{2}$

6. $\frac{3}{8}n - 2 < 1\frac{1}{5}$

$n < 5\frac{13}{15}$ or $n < 5.8\overline{6}$

7. $-2p - 3\frac{1}{4} \leq -1$

$p \geq -1\frac{1}{8}$ or $p \geq -1.125$

8. $2^2 - (3^2)x + 14x > \frac{4^3}{2}$

$x > 5\frac{3}{5}$ or $x > 5.6$

9. $\frac{3}{4}w - \frac{1}{2}(2w + 1) < \frac{2}{5}$

$w > -3\frac{3}{5}$ or $w > -3.6$

Write, solve, and graph an inequality for each statement.

10. Two is greater than the sum of 3 and one-fourth of a number.

inequality: $2 > 3 + \frac{1}{4}x$

solution: $x < -4$

graph: −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2

11. The product of −2 and the sum of a number and 3 is at least 0.

inequality: $-2(x + 3) \geq 0$

solution: $x \leq -3$

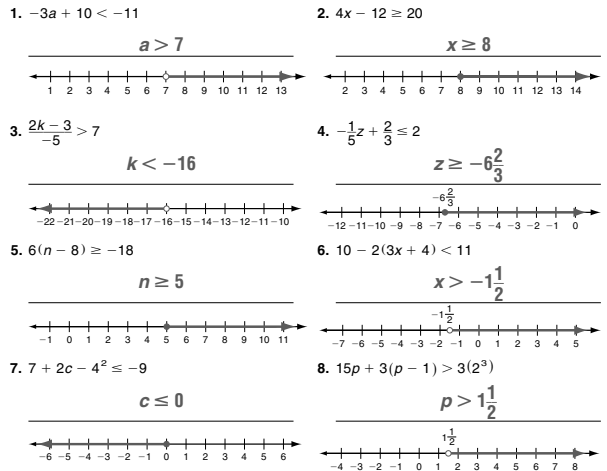graph: −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3

Write and solve an inequality for each problem.

12. Ann's grade is the average of four scores: three chapter tests and a final that counts as two chapter tests. She scored 72, 90, and 75 on the chapter tests. What score does she need on the final to have a grade of at least 80?

$\frac{72 + 90 + 75 + 2f}{5} \geq 80;\ f \geq 81.5;$ at least 81.5

13. Trey is limiting his diet to no more than 600 calories per meal. For lunch, he had a can of iced tea and two bowls of soup. The tea had 140 calories. What are the possible number of calories in each bowl of soup?

$140 + 2s \leq 600;\ s \leq 230;$ no more than 230 calories

Holt Algebra 1

Solving Two-Step and Multi-Step Inequalities

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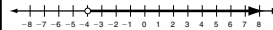
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$-5x + 3 < 23$

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$-5x < 20$

$\frac{-5x}{-5} < \frac{20}{-5}$ Divide both sides by −5.

$x > -4$ Reverse the inequality sign.

−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Check:

Try −4.

$-5x + 3 < 23$

$-5(-4) + 3 \overset{?}{<} 23$

$20 + 3 \overset{?}{<} 23$

$23 \overset{?}{<} 23$ ✗

Try 6.

$-5x + 3 < 23$

$-5(6) + 3 \overset{?}{<} 23$

$-30 + 3 \overset{?}{<} 23$

$-27 \overset{?}{<} 23$ ✓

The endpoint −4 is not a solution. The open circle on the graph is correct. The value 6 is a solution. The direction of the inequality symbol is correct.

Solve each inequality and graph the solutions.

1. $-3e - 10 \leq -4$

$e \geq -2$

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2. $\frac{c}{2} + 8 > 11$

$c > 6$

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3. $15 \leq 3 - 4s$

$s \leq -3$

−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

4. $\frac{3}{4}j + 1 > 4$

$j > 4$

−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Holt Algebra 1

Holt Algebra 1

Reteach
Solving Two-Step and Multi-Step Inequalities (continued)

Solving inequalities may require using the Distributive Property, combining like terms, or clearing fractions. Remember that you can clear fractions by multiplying both sides of the inequality by the least common denominator (LCD).

Solve $\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$.

$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$

$6\left(\frac{3}{2}x + \frac{1}{6}\right) \geq 6\left(\frac{5}{3}\right)$ Multiply both sides by the LCD 6.

$6\left(\frac{3}{2}x\right) + 6\left(\frac{1}{6}\right) \geq 6\left(\frac{5}{3}\right)$ Distribute 6.

$9x + 1 \geq 10$

$\underline{\quad -1 \quad -1}$ Add -1 to both sides.

$9x \geq 9$

$\frac{9x}{9} \geq \frac{9}{9}$ Divide both sides by 9.

$x \geq 1$

Check:

Try 1.

$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$

$\frac{3}{2}(1) + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$

$\frac{3}{2} + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$

$\frac{9}{6} + \frac{1}{6} \overset{?}{\geq} \frac{10}{6}$

$\frac{10}{6} \overset{?}{\geq} \frac{10}{6}$ ✓

Try 2.

$\frac{3}{2}x + \frac{1}{6} \geq \frac{5}{3}$

$\frac{3}{2}(2) + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$

$3 + \frac{1}{6} \overset{?}{\geq} \frac{5}{3}$

$3\frac{1}{6} \overset{?}{\geq} 1\frac{2}{3}$ ✓

The endpoint 1 is a solution. The value 2 is a solution. The direction of the inequality symbol is correct.

Solve each inequality.

5. $-\frac{5}{6}x + 3 < \frac{1}{2}$

6. $2(b - 7) + -4b \geq 30 - 18$

$x > 3$

$b \leq -13$

7. $\frac{2}{3}(g + 4) - g > 1$

8. $-\frac{3}{5} + \frac{8}{5}k - (3k - 2) \leq 0$

$g < 5$

$k \geq 1$

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## Challenge
Solving Multi-Step Inequalities

As you have already seen, when you solve a linear equation, you find a definite value. If the variable is equal to 3, then the number line is divided into three parts: the number 3, those numbers greater than 3, and those numbers less than 3. When you deal with multiple inequalities, you divide the number line into even more parts.

In Exercises 1–3, use the number line below.
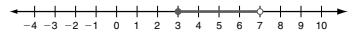
1. Draw a graph showing the solutions to $x \geq 3$ and $x < 7$.
2. What integers satisfy both of these inequalities?

   $3, 4, 5, 6$

3. What inequalities describe your response in Exercise 2?

   $x \geq 3$ and $x \leq 6$ where x is an integer.

On a certain freeway, the minimum allowable speed is 40 miles per hour, and the maximum allowable speed is 55 miles per hour.

Use the information above to answer Exercises 4–6.

4. Write an inequality for each driving situation.

   a. A motorist is driving too fast.

   $x > 55$

   b. A motorist is driving too slow.

   $x > 0$ and $x < 40$

5. Is it technically correct to say that the speed limit is exactly 55 miles per hour?

   No, since there is a lower limit as well as an upper limit

6. Write multiple inequalities for each situation.

   a. The driver's speed is within the speed limits.

   $x \geq 40$ and ≤ 55

   b. The driver's speed is outside of the speed limits.

   $x > 0$ and $x < 40$ or $x > 55$

People use multiple inequalities when they deal with real-estate situations.

7. John and Marsha would like to buy a house priced at $185,000, plus or minus $5,000.

   a. What are the maximum and minimum house price that they are willing to pay?

   maximum: $190,000
   minimum: $180,000

   b. Write a pair of inequalities describing their price range.

   $x \geq \$180,000$ and $x \leq \$190,000$

8. Use inequalities to describe the following sentence: "We sell houses priced at $80,000, plus or minus $5000, to houses priced at $250,000, plus or minus $5000."

   $x \geq \$75,000$ and $x \leq \$245,000$

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Problem Solving
Solving Two-Step and Multi-Step Inequalities

Write and solve an inequality for each situation.

1. Jillene is playing in a basketball tournament and scored 24 points in her first game. If she averages over 20 points for both games, she will receive a trophy. How many points can Jillene score in the second game and receive a trophy?

   $\frac{p + 24}{2} > 20$;

   $p > 16$

2. Marcus has accepted a job selling cell phones. He will be paid $1500 plus 15% of his sales each month. He needs to earn at least $2430 to pay his bills. For what amount of sales will Marcus be able to pay his bills?

   $1500 + 0.15s \geq 2430$;

   $s \geq 6200$

3. A 15-foot-tall cedar tree is growing at a rate of 2 feet per year beneath power lines that are 58 feet above the ground. The power company will have to prune or remove the tree before it reaches the lines. How many years can the power company wait before taking action?

   $15 + 2y < 58$;

   $y < 21.5$

4. Binh brought $23 with her to the county fair. She purchased a $5 T-shirt and now wants to buy some locally grown plants for $2.50 each. What are the numbers of plants that she can purchase with her remaining money?

   $5 + 2.5p \leq 23$; $p \leq 7.2$;

   0, 1, 2, 3, 4, 5, 6, or 7 plants

Benedict, Ricardo, and Charlie are considering opportunities for summer work. The table below shows the jobs open to them and the pay for each. Use this information to answer questions 5–7.

5. Benedict has saved $91 from last year and would like to baby-sit to earn enough to buy a mountain bike. A good quality bike costs at least $300. What numbers of hours h can Benedict baby-sit to reach his goal?

   A $h \geq 14$ Ⓒ $h \geq 38$

   B $h \geq 23$ D $h \geq 71$

Job	Pay
Mowing Lawns	$15 per lawn
Baby-Sitting	$5.50 per hour
Tutoring	$9 per session

6. Ricardo has agreed to tutor for the school. He owes his older brother $59 and would like to end the summer with at least $400 in savings. How many sessions s can Ricardo tutor to meet his goal?

   F $s \geq 31$ Ⓗ $s \geq 51$

   G $s \geq 38$ J $s \geq 83$

7. Charlie has agreed to mow his neighbor's lawn each week and will also baby-sit some hours. If he makes $100 or more each week, his parents will charge him rent. How many hours h should Charlie agree to baby-sit each week to avoid paying rent?

   Ⓐ $h \leq 15$ C $h \leq 21$

   B $h \geq 15$ D $h \geq 21$

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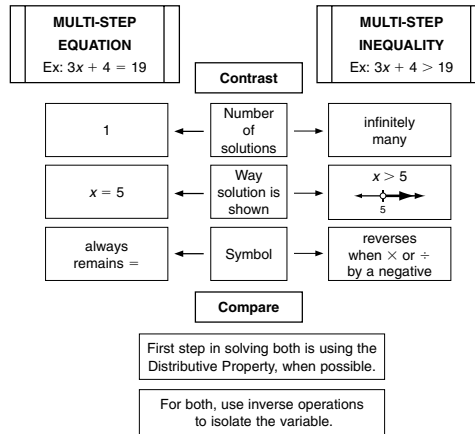## Reading Strategies
Compare and Contrast

The chart below summarizes the similarities and differences between multi-step equations and multi-step inequalities when there is a variable on one side.

MULTI-STEP EQUATION — Ex: $3x + 4 = 19$

Contrast

MULTI-STEP INEQUALITY — Ex: $3x + 4 > 19$

1	Number of solutions → infinitely many
$x = 5$	Way solution is shown → $x > 5$
always remains =	Symbol → reverses when × or ÷ by a negative

Compare

First step in solving both is using the Distributive Property, when possible.

For both, use inverse operations to isolate the variable.

Answer each question.

1. Why are solutions to inequalities graphed on number lines?

   to show the infinitely many solutions

2. Show what the solution $x = 5$ looks like when graphed on a number line.

3. What one value is the solution to the equation $3(x + 4) = 9$?

   -1

4. List five values that are solutions to the inequality $3(x + 4) < 9$.

   Possible answer: $-2, -3, -4, -5, -6$

5. Do $-2x + 3 = 7$ and $-2x + 3 \geq 7$ have any solutions in common?

   yes

   If so, what? -2

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