Name ________________________________ Date __________ Class ___________

LESSON 3-2

Reteach

Using Algebraic Methods to Solve Linear Systems

To use the **substitution method** to solve a system of linear equations:

1. Solve one equation for one variable.
2. Substitute this expression into the other equation.
3. Solve for the other variable.
4. Substitute the value of the known variable in the equation in Step 1.
5. Solve for the other variable.
6. Check the values in both equations.

$$\begin{cases} y = x + 2 \\ 2x + y = 17 \end{cases}$$

Use this equation. It is solved for *y*.

Use the substitution method when the coefficient of one of the variables is 1 or -1.

$$2x + y = 17$$

$$2x + (x + 2) = 17 \quad \textit{Substitute } x + 2 \textit{ for } y.$$

$$3x + 2 = 17 \quad \textit{Simplify and solve for } x.$$

$$3x = 15$$

$$x = 5$$

Substitute $x = 5$ into $y = x + 2$ and solve for y: $y = x + 2$

$$y = 5 + 2$$

$$y = 7$$

The solution of the system is the ordered pair $(5, 7)$.

Check using both equations: $y = x + 2$; $7 \stackrel{?}{=} (5) + 2$; $7 = 7$✓

$2x + y = 17$; $2(5) + 7 \stackrel{?}{=} 17$; $17 = 17$✓

Use substitution to solve each system of equations.

1. $\begin{cases} y = 2x - 5 \\ 3x + y = 10 \end{cases}$

Use $y = 2x - 5$.

$3x +$ ________ $= 10$

____

____

____

Ordered pair solution: ______________

2. $\begin{cases} 3x + 2y = 1 \\ x - y = 2 \end{cases}$

Solve for x: $x - y = 2$.

$x =$ ______________

$3($________$) + 2y = 1$

____

____

Ordered pair solution: ______________

Name ______________________ Date __________ Class __________

LESSON 3-2

Reteach

Using Algebraic Methods to Solve Linear Systems (continued)

To use the **elimination method** to solve a system of linear equations:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value for the known variable into one of the original equations.
4. Solve for the other variable.
5. Check the values in both equations.

Use the elimination method when the coefficients of one of the variables are the same or opposite.

The y terms have opposite coefficients, so add.

$$\begin{cases} 3x + 2y = 7 \\ 5x - 2y = 1 \end{cases}$$

$$\begin{array}{r} 3x + 2y = 7 \\ \underline{+\ 5x - 2y = 1} \end{array} \quad \textit{Add the equations.}$$

$$8x \qquad = 8 \quad \textit{Solve for x.}$$

$$x = 1$$

Substitute $x = 1$ into $3x + 2y = 7$ and solve for y:

$$\begin{aligned} 3x + 2y &= 7 \\ 3(1) + 2y &= 7 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

The solution to the system is the ordered pair $(1, 2)$.

Check using both equations:

$$\begin{array}{rr} 3x + 2y = 7 & 5x - 2y = 1 \\ 3(1) + 2(2) \stackrel{?}{=} 7 & 5(1) - 2(2) \stackrel{?}{=} 1 \\ 7 = 7 \checkmark & 1 = 1 \checkmark \end{array}$$

Use elimination to solve each system of equations.

3. $\begin{cases} 2x + y = 1 \\ -2x - 3y = 5 \end{cases}$

$$\begin{array}{r} 2x + y = 1 \\ \underline{+(-2x - 3y = 5)} \end{array}$$

$-2y =$ ____

$y =$ ____

____

Ordered pair solution: ______________

4. $\begin{cases} 3x + 4y = 13 \\ 5x - 4y = -21 \end{cases}$

$$\begin{array}{r} 3x + 4y = 13 \\ \underline{+\ 5x - 4y = -21} \end{array}$$

____

$x =$ ____

____

Ordered pair solution: ______________

LESSON 3-2
Practice A
Using Algebraic Methods to Solve Linear Systems

Use substitution to solve each system of equations.

1. $\begin{cases} y = x - 3 \\ x + 2y = 6 \end{cases}$

a. Substitute $x - 3$ for y in $x + 2y = 6$. Then solve the equation for x.

$x = 4$

b. Substitute your value for x in $y = x - 3$ and solve for y.

$y = 1$

c. Write the solution as an ordered pair. (4, 1)

2. $\begin{cases} x = 5 - y \\ 2x + 5y = 16 \end{cases}$ (3, 2)

3. $\begin{cases} y = 3x + 2 \\ 2x + 3y = 17 \end{cases}$ (1, 5)

4. $\begin{cases} x - y = 2 \\ y = 4x + 1 \end{cases}$ (−1, −3)

Use elimination to solve each system of equations.

5. $\begin{cases} 4x - 5y = 7 \\ 3x - 4y = 6 \end{cases}$

a. Multiply the first equation by −3 and the second equation by 4.

$\begin{cases} -12x + 15y = -21 \\ 12x - 16y = 24 \end{cases}$

b. Add the two equations, which eliminates x. Solve for y.

$y = -3$

c. Substitute your value for y into the first equation. Solve for x.

Write the solution as an ordered pair. (−2, −3)

6. $\begin{cases} 5x + y = 19 \\ -2x - y = -7 \end{cases}$ (4, −1)

7. $\begin{cases} -x + 3y = 12 \\ 6x - y = -21 \end{cases}$ (−3, 3)

8. $\begin{cases} 2x + 3y = 4 \\ 4x - 2y = -8 \end{cases}$ (−1, 2)

LESSON 3-2
Practice B
Using Algebraic Methods to Solve Linear Systems

Use substitution to solve each system of equations.

1. $\begin{cases} x = 7y - 4 \\ 2x - 3y = 14 \end{cases}$ (10, 2)

2. $\begin{cases} y - 3x = 5 \\ 2x = 3y + 6 \end{cases}$ (−3, −4)

3. $\begin{cases} 3x - 4y = 20 \\ y - 2x = 0 \end{cases}$ (−4, −8)

Use elimination to solve each system of equations.

4. $\begin{cases} x + 6y = 1 \\ 3x + 5y = -10 \end{cases}$ (−5, 1)

5. $\begin{cases} 3x + 4y = 6 \\ 2x + 3y = 3 \end{cases}$ (6, −3)

6. $\begin{cases} 3x - 5y = 1 \\ 2x + 3y = -12 \end{cases}$ (−3, −2)

Use substitution or elimination to solve each system of equations.

7. $\begin{cases} x + y = 13 \\ 2x - 3y = 1 \end{cases}$ (8, 5)

8. $\begin{cases} 9x + 2y = 5 \\ 3x - y = -10 \end{cases}$ (−1, 7)

9. $\begin{cases} 2x + y = 1 \\ x = 5 + y \end{cases}$ (2, −3)

10. $\begin{cases} x = -8y \\ x + y = 14 \end{cases}$ (16, −2)

11. $\begin{cases} 2x + 4y = 12 \\ -3x + 3y = 63 \end{cases}$ (−12, 9)

12. $\begin{cases} 5x - 2y = -1 \\ 3x - y = -2 \end{cases}$ (−3, −7)

Solve.

13. Bill leaves his house for Makayla's house riding his bicycle at 8 miles per hour. At the same time, Makayla leaves her house heading toward Bill's house walking at 3 miles per hour.

a. Write a system of equations to represent the distance, d, each is from Makayla's house in h hours. They live 8.25 miles apart.

$\begin{cases} d = 8.25 - 8h \\ d = 3h \end{cases}$

b. Solve the system to determine how long they travel before meeting.

0.75 h or 45 min

LESSON 3-2
Practice C
Using Algebraic Methods to Solve Linear Systems

Use substitution or elimination to solve each system of equations.

1. $\begin{cases} x = y - 5.2 \\ 2x + 3y = 9.6 \end{cases}$ (−1.2, 4)

2. $\begin{cases} 3x - 4y = 5 \\ x = y + \frac{1}{2} \end{cases}$ $\left(-3, -3\frac{1}{2}\right)$

3. $\begin{cases} x + 4y = \frac{1}{4} \\ 4x - 3y = 39 \end{cases}$ $\left(8\frac{1}{4}, -2\right)$

4. $\begin{cases} 2x + 20y = 3 \\ 2x = -7y - 10 \end{cases}$ $\left(-8\frac{1}{2}, 1\right)$

5. $\begin{cases} x + y = 5 \\ 3x + 2y = 4 \end{cases}$ (−6, 11)

6. $\begin{cases} 3x + 4y = 35 \\ 4x - 2y = 21 \end{cases}$ $\left(7, 3\frac{1}{2}\right)$

7. $\begin{cases} 3\frac{1}{4}x + 3y = 42 \\ 5x = 4y \end{cases}$ $\left(6, 7\frac{1}{2}\right)$

8. $\begin{cases} 5x - 5y = 6 \\ 4x + 7y = -4 \end{cases}$ $\left(\frac{2}{5}, -\frac{4}{5}\right)$

9. $\begin{cases} 2x - 8y = 24 \\ x - 21 = 16y \end{cases}$ $\left(9, -\frac{3}{4}\right)$

Solve.

10. Cora bought 4 pounds of nuts and 2 pounds of raisins for $23.50. Mark bought 2 pounds of nuts and 4 pounds of raisins for $18.50.

a. Write a system of equations that represents the price of the nuts, n, and the price of the raisins, r. $\begin{cases} 4n + 2r = 23.5 \\ 2n + 4r = 18.5 \end{cases}$

b. Solve the system. How much should a pound of nuts and a pound of raisins cost together? $7.00

11. Kate and Riley are reading the same book. Kate reads $\frac{1}{3}$ page per minute, and Riley reads $\frac{3}{4}$ page per minute. Kate has already read 70 pages, while Riley has read 30 pages. If they both resume reading together, eventually Riley will catch up to Kate.

a. On what page will that occur?

102

b. How many minutes have they read when Riley catches up?

96

LESSON 3-2
Reteach
Using Algebraic Methods to Solve Linear Systems

To use the **substitution method** to solve a system of linear equations:
1. Solve one equation for one variable.
2. Substitute this expression into the other equation.
3. Solve for the other variable.
4. Substitute the value of the known variable in the equation in Step 1.
5. Solve for the other variable.
6. Check the values in both equations.

$\begin{cases} y = x + 2 \\ 2x + y = 17 \end{cases}$

Use this equation. It is solved for y.

Use the substitution method when the coefficient of one of the variables is 1 or −1.

$2x + y = 17$

$2x + (x + 2) = 17$ *Substitute $x + 2$ for y.*

$3x + 2 = 17$ *Simplify and solve for x.*

$3x = 15$

$x = 5$

Substitute $x = 5$ into $y = x + 2$ and solve for y: $y = x + 2$

$y = 5 + 2$

$y = 7$

The solution of the system is the ordered pair (5, 7).

Check using both equations: $y = x + 2$; $7 \stackrel{?}{=} (5) + 2$; $7 = 7$✓

$2x + y = 17$; $2(5) + 7 \stackrel{?}{=} 17$; $17 = 17$✓

Use substitution to solve each system of equations.

1. $\begin{cases} y = 2x - 5 \\ 3x + y = 10 \end{cases}$

Use $y = 2x - 5$.

$3x + \underline{2x - 5} = 10$

$5x - 5 = 10$

$x = 3$

$y = 2(3) - 5 = 1$

Ordered pair solution: (3, 1)

2. $\begin{cases} 3x + 2y = 1 \\ x - y = 2 \end{cases}$

Solve for x: $x - y = 2$.

$x = \underline{y + 2}$

$3(\underline{y + 2}) + 2y = 1$

$y = -1$

$x = -1 + 2 = 1$

Ordered pair solution: (1, −1)

LESSON 3-2

Reteach

Using Algebraic Methods to Solve Linear Systems (continued)

To use the **elimination method** to solve a system of linear equations:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value for the known variable into one of the original equations.
4. Solve for the other variable.
5. Check the values in both equations.

Use the elimination method when the coefficients of one of the variables are the same or opposite.

$\begin{cases} 3x + 2y = 7 \\ 5x - 2y = 1 \end{cases}$

The y terms have opposite coefficients, so add.

$$\begin{array}{rl} 3x + 2y = 7 & \\ +\ 5x - 2y = 1 & \textit{Add the equations.} \\ \hline 8x \qquad = 8 & \textit{Solve for x.} \\ x = 1 & \end{array}$$

Substitute $x = 1$ into $3x + 2y = 7$ and solve for y: $3x + 2y = 7$

$3(1) + 2y = 7$

$2y = 4$

$y = 2$

The solution to the system is the ordered pair $(1, 2)$.

Check using both equations:

$3x + 2y = 7$	$5x - 2y = 1$
$3(1) + 2(2) \stackrel{?}{=} 7$	$5(1) - 2(2) \stackrel{?}{=} 1$
$7 = 7$ ✓	$1 = 1$ ✓

Use elimination to solve each system of equations.

3. $\begin{cases} 2x + y = 1 \\ -2x - 3y = 5 \end{cases}$

$2x + y = 1$
$+(-2x - 3y = 5)$

$-2y =$ 6

$y =$ −3

$x = 2$

Ordered pair solution: $(2, -3)$

4. $\begin{cases} 3x + 4y = 13 \\ 5x - 4y = -21 \end{cases}$

$3x + 4y = 13$
$+\ 5x - 4y = -21$

$8x = -8$

$x =$ −1

$y = 4$

Ordered pair solution: $(-1, 4)$

LESSON 3-2

Challenge

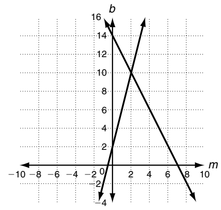
Using Linear Systems to Find the Equation of a Line

Linear systems of equations can be used to find the equation of a line. Determine the equation of the line passing through points $(-4, 2)$ and $(2, 14)$ using $y = mx + b$. Substituting the x- and y-coordinates for the values of x and y in the slope-intercept form of the line gives the system.

$\begin{cases} 2 = -4m + b \\ 14 = 2m + b \end{cases}$

Solve this system to find the slope and y-intercept of the line passing through points $(-4, 2)$ and $(2, 14)$. Finding $m = 2$ and $b = 10$ from the table or the graph allows you to write the equation of the line $y = 2x + 10$.

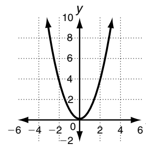
	$2 = -4m + b$	$14 = 2m + b$
m	b	b
0	2	14
1	6	12
2	10	10
3	14	8
4	18	6



Use a system of equations to find the equation of the line passing through the given points.

1. $(5, 7)$ and $(1, 19)$ — $y = -3x + 22$
2. $(-2, 4)$ and $(2, 8)$ — $y = x + 6$
3. $(3, -5)$ and $(5, 1)$ — $y = 3x - 14$
4. $(1, 1)$ and $(5, -9)$ — $y = -2.5x + 3.5$
5. $(-1, 8)$ and $(1, -8)$ — $y = -8x$

The equation of a parabola in standard form is $y = ax^2 + bx + c$. There are three constants, a, b, and c. Three points not on a line will determine a unique parabola.



Find the equation of the parabola passing through the given points.

6. $(0, 1)$, $(1, 0)$, and $(2, 1)$ — $y = x^2 - 2x + 1$
7. $(1, 5)$, $(2, 4)$, and $(4, 8)$ — $y = x^2 - 4x + 8$

LESSON 3-2

Problem Solving

Using Algebraic Methods to Solve Linear Systems

Shanae mixes feed for various animals at the zoo so that the feed has the right amount of protein. Feed X is 18% protein. Feed Y is 10% protein. Use this data for Exercises 1–4.

1. How much of each feed should Shanae mix to get 50 lb of feed that is 15% protein?
   a. Write a linear system of equations. $\begin{cases} 0.18x + 0.10y = (0.15)50 \\ x + y = 50 \end{cases}$
   b. Solve the system. How much of each feed should she mix?
   31.25 lb of Feed X and 18.75 lb of Feed Y
2. Shanae has 15 lb of Feed Y left. She wants to make a mixture that is 12% protein. She needs to know how much of Feed X to use, and how much of the mixture she can make.
   a. Write a linear system of equations. $\begin{cases} 0.18x + (0.10)(15) = (0.12)z \\ x + 15 = z \end{cases}$
   b. How much of Feed X should she use? 5 lb of Feed X
   c. How much of the mixture will she make? 20 lb of the mixture

Choose the letter for the best answer.

3. Raul mixes 12 lb of Feed X with 20 lb of Feed Y. Which equation gives the percent of protein (c) in the mixture?
   A $12(0.18) + 20(0.10) = 32c$
   B $32[12(0.18) + 20(0.10)] = c$
   C $12(0.18) + 20(0.10) = c$
   D $[12(0.18) + 20(0.10)]c = 32$
4. Alonzo needs to know how much of Feed X and Feed Y to mix to get 25 lb of a mixture that is 12% protein. Which equation can be used as part of a system of equations to find the solution?
   A $(0.10 + 0.18)(x + y) = (0.12)25$
   B $(0.18)x + (0.10)y = (0.12)25$
   C $25(0.18 + 0.10) = (0.12)x$
   D $10 + 18 = (0.12)25$
5. Billie reorders Feed X and Feed Y. Feed X costs \$58 per 100 lb. Feed Y costs \$45 per 100 lb. The order comes to \$470 for 900 lb. How much of each did she order?
   A Feed X: 350 lb; Feed Y: 550 lb
   B Feed X: 400 lb; Feed Y: 500 lb
   C Feed X: 450 lb; Feed Y: 540 lb
   D Feed X: 500 lb; Feed Y: 400 lb
6. Shanae earns \$8.00 per hour during the daytime and \$9.50 per hour in the evenings after 6 P.M. Last week she earned \$314.00 for 37 hours. How many daytime and evening hours did she work?
   A 35 daytime; 2 evening
   B 30 daytime; 7 evening
   C 25 daytime; 12 evening
   D 20 daytime; 17 evening

LESSON 3-2

Reading Strategies

Understand Vocabulary

There are two ways you can solve a system of equations algebraically.

Substitution	**Elimination**
Use substitution when you can easily solve one equation for one variable.	Use elimination to add or subtract equations to remove one of the variables.
Memory tip: You can *substitute* salad for fries with your order.	**Memory tip:** The Tigers were *eliminated* from the basketball tournament.
For the system $\begin{cases} x - y = 4 \\ 2x - 3y = 7 \end{cases}$ it is easy to solve $x - y = 4$ for x:	For the system $\begin{cases} 5x - 2y = -9 \\ 3x + 2y = 1 \end{cases}$ if you add the 2 equations together, the y-term is *eliminated* because $-2y + 2y = 0$.
$x = \boxed{4 + y}$	Addition gives $8x = -8$, so $x = -1$.
Then *substitute* for x in the second equation and solve for y:	Finally, solve for y.
$2x - 3y = 7$	$3x + 2y = 1$
$2(\boxed{4 + y}) - 3y = 7$	$3(-1) + 2y = 1$
$8 + 2y - 3y = 7$	$-3 + 2y = 1$
$-y = -1$ or $y = 1$	$2y = 1 + 3 = 4$
Finally, solve for x:	$y = 2$
$x = 4 + y$	The solution to the system is $(-1, 2)$.
$x = 4 + 1 = 5$	
The solution to the system is $(5, 1)$.	

Tell which method you would use to solve each system of equations and explain why.

1. $\begin{cases} 2x + y = 3 \\ 3x + 4y = 9 \end{cases}$ Possible answer: substitution because I can easily solve the first equation for y
2. $\begin{cases} 3x - 4y = 9 \\ -3x + 5y = -9 \end{cases}$ Possible answer: elimination because I can eliminate x from the system by adding the two equations together
3. $\begin{cases} -2x + 5y = 3 \\ x - 2y = 0 \end{cases}$ Possible answer: substitution because I can easily solve the second equation for x
4. $\begin{cases} 3x - y = -1 \\ 4x + \frac{1}{2}y = 1 \end{cases}$ Possible answer: elimination because I can eliminate y from the system by multiplying the second equation by 2, then adding the equations