Name _____ Date _____ Class _____

LESSON 3-1

Reteach

Using Graphs and Tables to Solve Linear Systems

A **linear system** of equations is a set of two or more linear equations. To **solve a linear system**, find all the ordered pairs (x, y) that make both equations true. Use a table and a graph to solve a system of equations.

$\begin{cases} y + x = 2 \\ y - 2x = 5 \end{cases}$ Solve each equation for y. $\rightarrow \begin{cases} y = -x + 2 \\ y = 2x + 5 \end{cases}$

Make a table of values for each equation.

$y = -x + 2$	
x	y
-2	4
-1	3
0	2
1	1

$\leftrightarrow$

$y = 2x + 5$	
x	y
-2	1
-1	3
0	5
1	7

When $x = -1$, $y = 3$ for both equations.

On a graph, the point where the lines intersect is the solution.

Use the table to draw the graph of each equation.

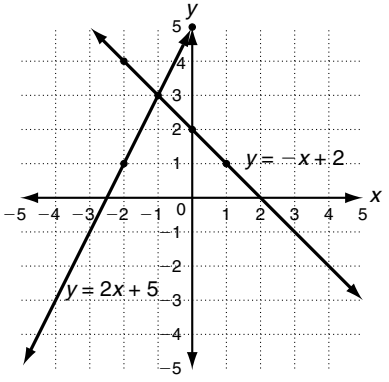
The lines appear to intersect at $(-1, 3)$.

Substitute $(-1, 3)$ into the original equations to check.

$y + x = 2$ $\qquad$ $y - 2x = 5$

$3 + (-1) \stackrel{?}{=} 2$ $\qquad$ $3 - 2(-1) \stackrel{?}{=} 5$

$2 = 2$✓ $\qquad$ $5 = 5$✓



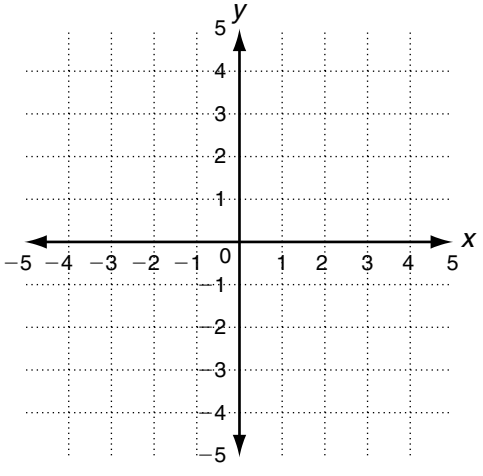
Solve the system using a table and a graph. Give the ordered pair that solves both equations.

1. $\begin{cases} x + y = 1 \\ 2x - y = 5 \end{cases}$

Solution: _____

$y = -x + 1$	
x	y
0	
1	
2	
3	

$y = 2x - 5$	
x	y
0	
1	
2	
3	



Name _____ Date _____ Class _____

LESSON 3-1

Reteach

Using Graphs and Tables to Solve Linear Systems (continued)

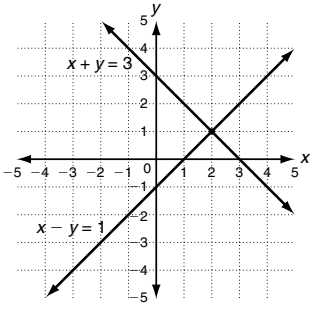
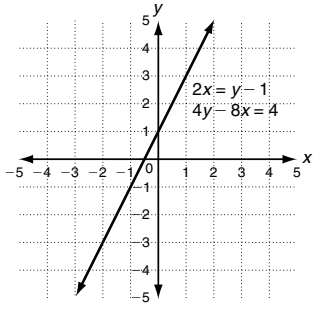
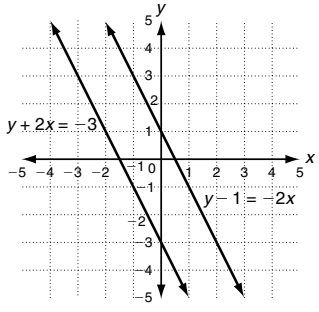
To classify a linear system:

Step 1 Write each equation in the form $y = mx + b$.

Step 2 Compare the slopes and y-intercepts.

Step 3 Classify by the number of solutions of the system.

Remember: m = slope and b = y-intercept.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
The lines have **different slopes** and intersect at one point.	The lines have the **same slope and *y*-intercept.** Their graph is the same line.	The lines have the **same slope and different *y*-intercepts.** The lines are parallel.
$\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ Solve each equation for y. $\begin{cases} y = -x + 3;\ m = -1 \\ y = x - 1;\ m = 1 \end{cases}$ The slopes are different. The system has one solution and is independent.	$\begin{cases} 2x = y - 1 \\ 4y - 8x = 4 \end{cases}$ Solve each equation for y. $\begin{cases} y = 2x + 1;\ m = 2,\ b = 1 \\ y = 2x + 1;\ m = 2,\ b = 1 \end{cases}$ The slopes and the y-intercepts are the same. The system has infinitely many solutions and is dependent.	$\begin{cases} y + 2x = -3 \\ y - 1 = -2x \end{cases}$ Solve each equation for y. $\begin{cases} y = -2x - 3;\ m = -2,\ b = -3 \\ y = -2x + 1;\ m = -2,\ b = 1 \end{cases}$ The slopes are the same but the y-intercepts are different. The system has no solution and is inconsistent.
		

Classify each system and determine the number of solutions.

2. $\begin{cases} y + x = 2 \\ y + 1 = -x \end{cases}$

$y =$ _____ , $m =$ ____, $b =$ ____

$y =$ _____, $m =$ ____, $b =$ ____

Number of solutions: _____

_____

3. $\begin{cases} y + 1 = 3x \\ 2y - 6x = -2 \end{cases}$

$y =$ _____, $m =$ ____, $b =$ ____

$y =$ _____, $m =$ ____, $b =$ ____

Number of solutions: _____

_____

LESSON 3-1

Practice A

Using Graphs and Tables to Solve Linear Systems

Does the given ordered pair solve the system of equations? Substitute each value for *x* and *y* into the equations. Write *yes* or *no*.

1. $(2, -1)$ $\begin{cases} 3x + y = 3 \\ x - y = 5 \end{cases}$

$3(2) + (-1) \stackrel{?}{=} 3$

$(2) - (-1) \stackrel{?}{=} 5$

No

2. $(4, 5)$ $\begin{cases} x - 6y = -26 \\ 2x + y = 13 \end{cases}$

Yes

3. $(-3, -7)$ $\begin{cases} -x + 2y = 1 \\ 4x - 3y = 19 \end{cases}$

No

Use a table and a graph to solve the system.

4. $\begin{cases} y = x + 1 \\ x = 2y + 2 \end{cases}$

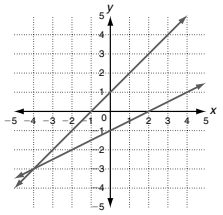
a. Make a table of values for each equation.

$y = x + 1$

x	y
-2	-1
-1	0
0	1
1	2
2	3

$x = 2y + 2$

x	y
-2	-2
-1	$-\frac{3}{2}$
0	-1
1	$-\frac{1}{2}$
2	0

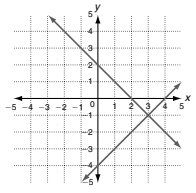


b. Use the values in the tables to graph each equation.

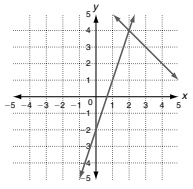
c. Which ordered pair solves both equations? $(-4, -3)$

Use a graph to solve each system.

5. $\begin{cases} x + y = 2 \\ x - y = 4 \end{cases}$ $(3, -1)$



6. $\begin{cases} y = 3x - 2 \\ x + y = 6 \end{cases}$ $(2, 4)$



LESSON 3-1

Practice B

Using Graphs and Tables to Solve Linear Systems

Classify each system, and determine the number of solutions.

1. $\begin{cases} y = -4x + 7 \\ 12x + 3y = 21 \end{cases}$

Consistent, dependent; infinitely many solutions

2. $\begin{cases} 5y = x - 10 \\ y = \frac{x}{5} + 3 \end{cases}$

Inconsistent; no solutions

3. $\begin{cases} x + 6y = -2 \\ 12x - 6y = 0 \end{cases}$

Consistent, independent; one solution

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations. If it is not, give the correct solution.

4. $(-4, 8)$ $\begin{cases} y = -2x \\ 3x + y = -4 \end{cases}$ It is the solution.

5. $(11, 3)$ $\begin{cases} y = x - 8 \\ x + 4y = -2 \end{cases}$ $(6, -2)$

6. $(4, 1)$ $\begin{cases} y = 5x - 1 \\ 8 = 4x + y \end{cases}$ $(1, 4)$

7. $(5, -5)$ $\begin{cases} x + y = 10 \\ x - y = 0 \end{cases}$ $(5, 5)$

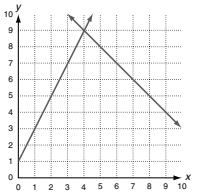
8. $(2, -1)$ $\begin{cases} 2x + 3y = -8 \\ 3x - 4y = 5 \end{cases}$ $(-1, -2)$

9. $(0, 3)$ $\begin{cases} 3x + 5y = 15 \\ x - y = -3 \end{cases}$ It is the solution.

Solve by graphing a system of equations.

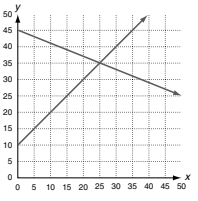
10. A puppy pen is 1 foot longer than twice its width. John wants to increase the length and width by 5 feet each to enlarge the area by 90 square feet. What will be the area of the new pen?

126 square feet



11. Keesha has 10 more quarters than dimes, which, together, total $11.25. How many coins does she have in quarters and dimes?

35 quarters + 25 dimes = 60 coins



LESSON 3-1

Practice C

Using Graphs and Tables to Solve Linear Systems

Match each system of equations with the corresponding conditions that it satisfies.

1. The system is consistent and dependent. → $\begin{cases} y = \frac{2}{3}x + 10 \\ 6y - 4x = 60 \end{cases}$

2. The system is inconsistent. → $\begin{cases} y = \frac{2}{3}x + 10 \\ y = 3x + 1 \end{cases}$

3. The system is consistent and independent. → $\begin{cases} y = \frac{2}{3}x + 10 \\ 3y + 9 = 2x \end{cases}$

Solve.

4. A tub containing 16 gallons of water is draining at a rate of 1 gallon per hour. A basin of 3.5 gallons of water is draining at a rate of 1 gallon every 6 hours.

a. Write a system of equations that represents *y*, the number of gallons left in the container after *x* hours.

$$\begin{cases} y = -x + 16 \\ y = -\frac{1}{6}x + 3.5 \end{cases}$$

b. If both containers began draining at the same time, how soon will the tub and basin hold the same amount of water?

15 h

c. When the amounts are equal, how much water will be in each container?

1 gallon

5. Jenna has $1500 in a savings account. She adds $30 to her account each month. Luis has $2400 in his savings account. He withdraws $30 from his account each month.

a. In how many months will they have the same balance in their savings accounts? 15 months

b. What will be the balance in each account? $1950

LESSON 3-1

Reteach

Using Graphs and Tables to Solve Linear Systems

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$\begin{cases} y + x = 2 \\ y - 2x = 5 \end{cases}$ Solve each equation for y. → $\begin{cases} y = -x + 2 \\ y = 2x + 5 \end{cases}$

Make a table of values for each equation.

$y = -x + 2$	
x	y
-2	4
-1	3
0	2
1	1

↔

$y = 2x + 5$	
x	y
-2	1
-1	3
0	5
1	7

When $x = -1$, $y = 3$ for both equations.

On a graph, the point where the lines intersect is the solution.

Use the table to draw the graph of each equation.

The lines appear to intersect at $(-1, 3)$.

Substitute $(-1, 3)$ into the original equations to check.

$y + x = 2$

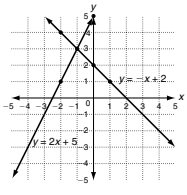
$3 + (-1) \stackrel{?}{=} 2$

$2 = 2$✓

$y - 2x = 5$

$3 - 2(-1) \stackrel{?}{=} 5$

$5 = 5$✓



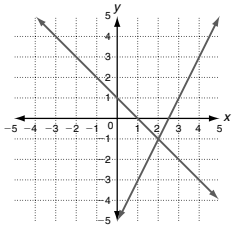
Solve the system using a table and a graph. Give the ordered pair that solves both equations.

1. $\begin{cases} x + y = 1 \\ 2x - y = 5 \end{cases}$

Solution: $(2, -1)$

$y = -x + 1$	
x	y
0	1
1	0
2	-1
3	-2

$y = 2x - 5$	
x	y
0	-5
1	-3
2	-1
3	1



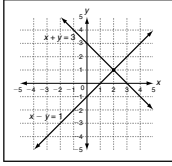
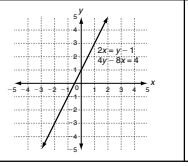
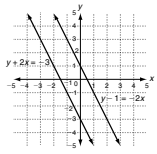
LESSON 3-1

Reteach

Using Graphs and Tables to Solve Linear Systems (continued)

To classify a linear system:
Step 1 Write each equation in the form $y = mx + b$.
Step 2 Compare the slopes and y-intercepts.
Step 3 Classify by the number of solutions of the system.

Remember: m = slope and b = y-intercept.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
The lines have **different slopes** and intersect at one point.	The lines have the **same slope and y-intercept.** Their graph is the same line.	The lines have the **same slope and different y-intercepts.** The lines are parallel.
$\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$	$\begin{cases} 2x = y - 1 \\ 4y - 8x = 4 \end{cases}$	$\begin{cases} y + 2x = -3 \\ y - 1 = -2x \end{cases}$
Solve each equation for y.	Solve each equation for y.	Solve each equation for y.
$\begin{cases} y = -x + 3;\ m = -1 \\ y = x - 1;\ m = 1 \end{cases}$	$\begin{cases} y = 2x + 1;\ m = 2,\ b = 1 \\ y = 2x + 1;\ m = 2,\ b = 1 \end{cases}$	$\begin{cases} y = -2x - 3;\ m = -2,\ b = -3 \\ y = -2x + 1;\ m = -2,\ b = 1 \end{cases}$
The slopes are different.	The slopes and the y-intercepts are the same.	The slopes are the same but the y-intercepts are different.
The system has one solution and is independent.	The system has infinitely many solutions and is dependent.	The system has no solution and is inconsistent.
		

Classify each system and determine the number of solutions.

2. $\begin{cases} y + x = 2 \\ y + 1 = -x \end{cases}$

$y =$ $-x + 2$, $m =$ -1, $b =$ 2

$y =$ $-x - 1$, $m =$ -1, $b =$ -1

Number of solutions: none

inconsistent

3. $\begin{cases} y + 1 = 3x \\ 2y - 6x = -2 \end{cases}$

$y =$ $3x - 1$, $m =$ 3, $b =$ -1

$y =$ $3x - 1$, $m =$ 3, $b =$ -1

Number of solutions: infinitely many

dependent

LESSON 3-1

Challenge

System Classifications

A system of linear equations can be consistent and independent, consistent and dependent, or inconsistent. **Use the following system of equations for Exercises 1–3.**

$$\begin{cases} x - 3y = 12 \\ 3x + by = c \end{cases}$$

1. Find values of b and c so that the system is consistent and dependent. $b = -9,\ c = 36$

2. Find values of b and c so that the system is inconsistent. $b = -9,\ c \neq 36$

3. Find values of b and c so that the system is consistent and independent. $b \neq -9$

Use this system of equations for Exercises 4–7.

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \\ a_3x + b_3y = c_3 \end{cases}$$

4. Describe how the graph of these 3 equations looks when the system has exactly one solution.

3 lines intersect at a single point

5. Describe how the graph of these 3 equations looks when the system has an infinite number of solutions. 3 lines that coincide

6. Describe how the graph of these 3 equations looks when the system has no solution. 3 parallel lines

7. Explain why the system of equations must be consistent if the 3 constant terms, c_1, c_2 and c_3 are all zero.

Possible answer: In order to be inconsistent, the equations must be parallel and have different y-intercepts. If the constant terms are equal to 0, then all y-intercepts are 0, therefore there are no parallel lines and the system must be consistent.

Use this system of equations for Exercise 8.

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

8. Under what conditions will this system be consistent and independent?

Possible answer: To be independent, the slopes cannot be equal. This can be stated as $\frac{-a}{b} \neq \frac{-c}{d}$, $-ad \neq -bc$, or $-ad + bc \neq 0$.

LESSON 3-1

Problem Solving

Using Graphs and Tables to Solve Linear Systems

Solve.

1. After the lesson, Carl takes the wakeboarding class to the *Glass Cafe*. He pays $26 for 8 large and 4 small juice drinks. A large glass costs $1 more than a small glass.

a. Write a linear system of equations to find the cost of each size drink.

$$\begin{cases} 4x + 8y = 26 \\ x + 1 = y \end{cases}$$

b. Write one equation at the top of each table and complete the table.

c. What is the cost of each size drink?

Small: $1.50; large: $2.50

$4x + 8y = 26$	
x	y
1	2.75
1.5	2.50
2	2.25
2.5	2
3	1.75

$x + 1 = y$	
x	y
1	2
1.5	2.5
2	3
2.5	3.5
3	4

2. Sandy rented a jet ski for $95 plus $15 per hour. Pauline rented a jet ski for $80 plus $20 per hour.

a. Write a linear system of equations to find the number of hours for which the rental cost is the same.

$$\begin{cases} y = 95 + 15x \\ y = 80 + 20x \end{cases}$$

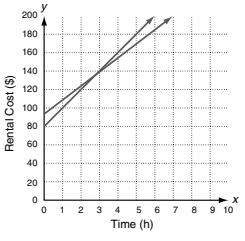
b. Graph the system.

c. For what number of hours would Sandy and Pauline pay the same to rent a jet ski?

3 h

d. How much would it cost to rent the jet ski for this amount of time?

$140



Choose the letter for the best answer.

3. Juan started with 50 gallons of water in his pool, and he is filling it at a rate of 10 gallons per minute. His next-door neighbor Sam started with 20 gallons of water in his pool, and he is filling it at a rate of 15 gallons per minute. Which system of equations could you use to find when the pools will contain the same amount of water?

A $\begin{cases} y = 50 + 15x \\ y = 20 + 10x \end{cases}$ **C** $\begin{cases} y = 50 - 15x \\ y = 20 - 10x \end{cases}$

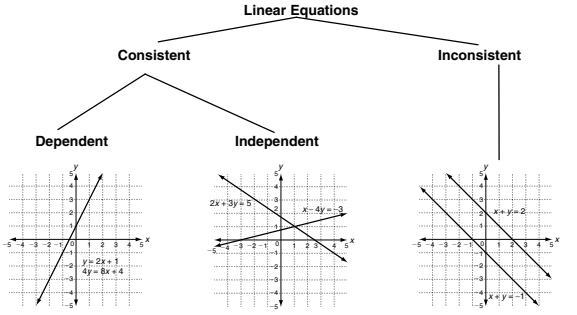
(B) $\begin{cases} y = 50 + 10x \\ y = 20 + 15x \end{cases}$ **D** $\begin{cases} y = 50 - 10x \\ y = 20 - 15x \end{cases}$

LESSON 3-1

Reading Strategies

Use Graphic Aids

A system of linear equations can be classified.



Both lines the same
Infinitely many solutions

Lines intersect
One solution

Lines parallel
No solutions

Ring all the terms that describe the system.

1. $\begin{cases} y = -x + 1 \\ y = x + 3 \end{cases}$

(a.) Consistent
b. Inconsistent
c. Dependent
(d.) Independent

2. $\begin{cases} x + y = 5 \\ x + y = 2 \end{cases}$

a. Consistent
(b.) Inconsistent
c. Dependent
d. Independent

3. $\begin{cases} 3x - y = 3 \\ 3y = 9x - 9 \end{cases}$

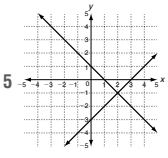
(a.) Consistent
b. Inconsistent
(c.) Dependent
d. Independent

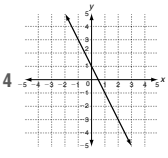
Draw a line from each system of equations to the matching graph.

4. $\begin{cases} 2x + y = 1 \\ 2y = -4x + 2 \end{cases}$

5. $\begin{cases} y = x - 3 \\ y = -x + 1 \end{cases}$

6. $\begin{cases} x - y = 5 \\ 2x + y = 4 \end{cases}$

5 

4 

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